Accelerator Physics Lecture Modules

1. Introduction/Timing
2. Beam propagation basics
3. Simple Harmonic Oscillator

4. Betatron oscillations (transverse motion)
5. Dispersion
6. Closed Orbit Distortion

7. Synchrotron oscillations
8. Properties of synchrotron radiation

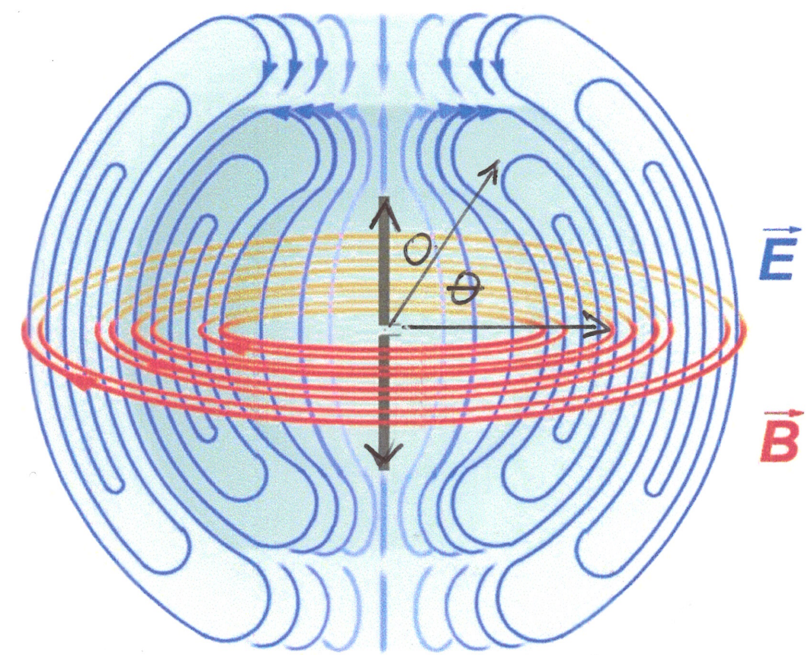
9. Radiation damping
10. Equilibirum emittance

Synchrotron Radiation – The Dipole Oscillator

Examine field emission from a dipole (Radio Antenna)

- Charge acceleration is vertical
- Propagating field components <u>transverse</u>
- Poynting's Vector $S = \frac{c}{4\pi}\left[\vec{E} \times \vec{B}\right]$ W/m^2
- $E_\perp = \frac{q \cdot a}{c^2 R} \cdot \cos\theta$

$$\boxed{S = \frac{q^2 a^2}{4\pi c^3 R^2} \cos^2\theta \quad W/m^2}$$



Wiki

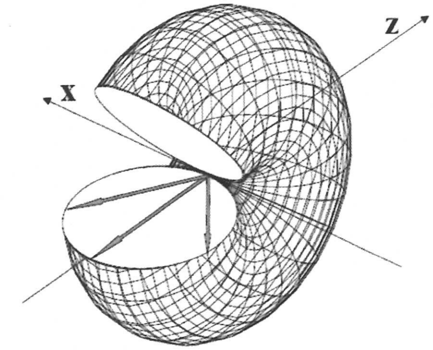
For synchrotron radiation

$$F = \gamma m a = \frac{q \vec{v} \times \vec{B}}{c} \rightarrow a = \frac{qcB}{\gamma m c}$$

Dipole Oscillator Power

3-D field pattern looks like a doughnut

SR emission in particle system



$$S \propto \frac{q^2 a^2}{R^2} \cos^2\theta \quad W/m^2$$

$$a = \frac{q B}{\gamma m c}$$ ← strong field (B), light particle (mc)

Integrate over doughnut surface to get Watts (Joules/sec)

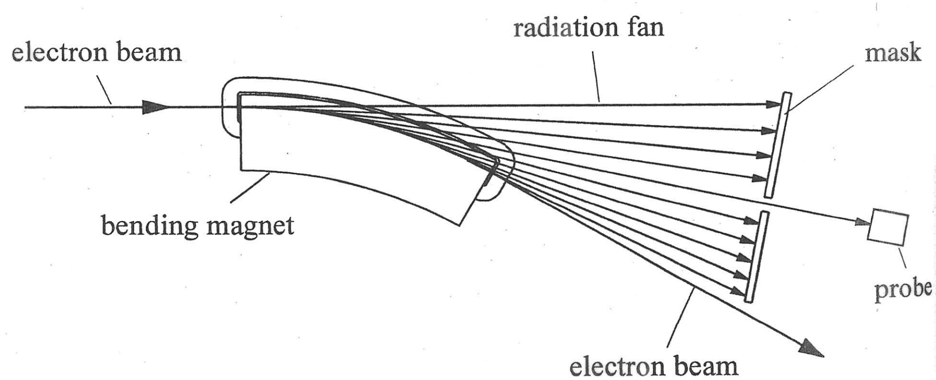
$$P_{rad} \propto E^2 B^2 \quad \text{or} \quad P_{rad} \propto \frac{E^4}{R^2}$$

E: particle energy; B: magnetic field

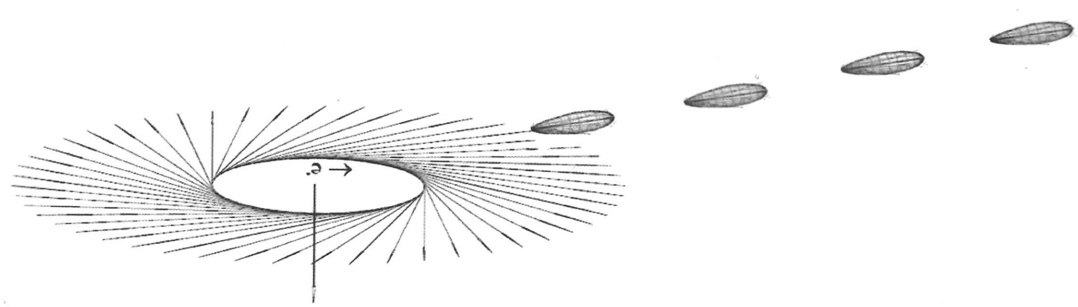
For a given bend radius $\boxed{P_{rad} \propto E^4}$

Synchrotron Radiation in a Storage Ring

Dipole bending magnets supply transverse acceleration

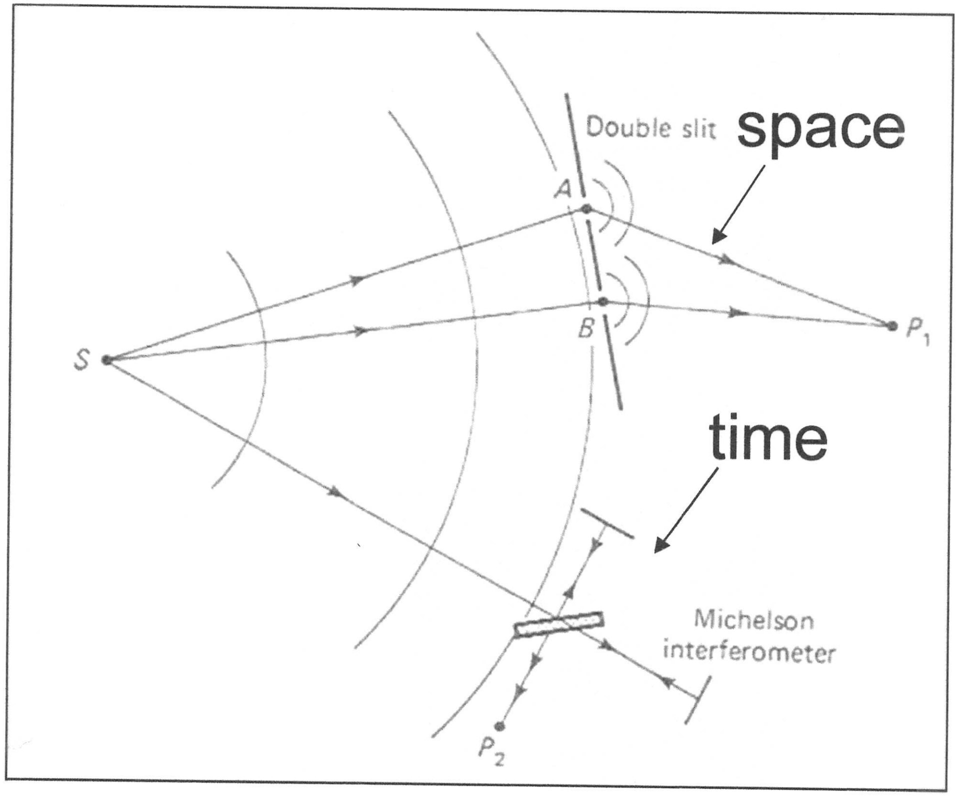


SR emission over 2π circumference with pulse structure



Note: individual photon emission events are quantized like an atom
radiation is <u>longitudinally</u> <u>incoherent</u>

Space and Time Correlation (Coherence)



At the beach...

SR Emission in the Laboratory Frame

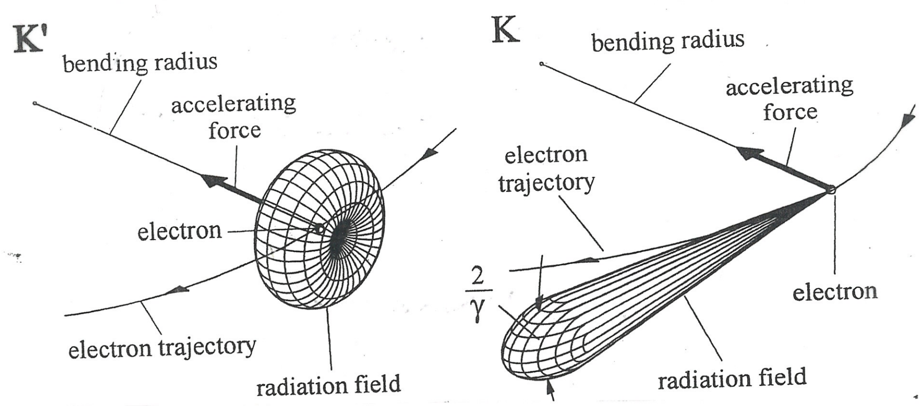
Particle frame – doughnut dipole pattern

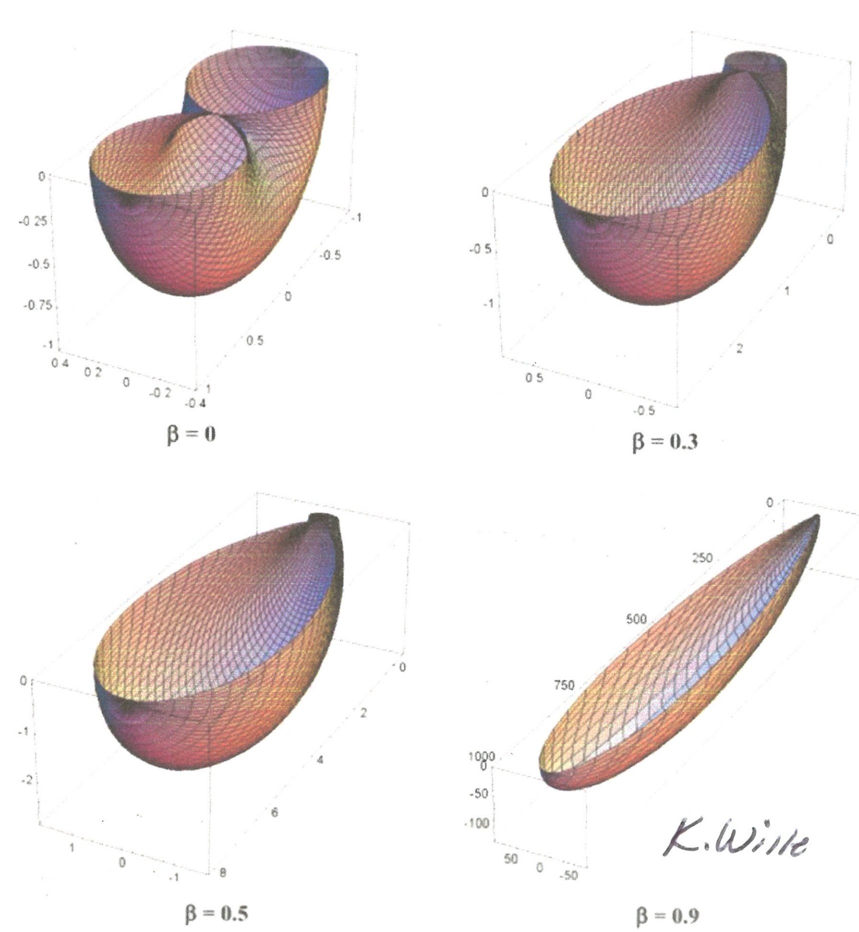
Laboratory frame – highly Lorentz-transformed 'beam of light'

$\gamma = 1957 \cdot E(\text{GeV}) \cong 6000$ for 3GeV e^- machines

$\frac{1}{\gamma} = \frac{1}{6000} \cong 160\,\mu\text{radian}$ (1.6mm at 10m)

Power density goes way up





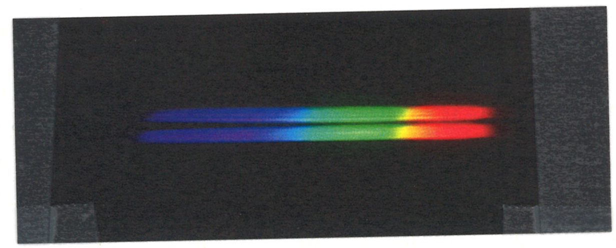
Properties of the SR Beam

1) SR has a forward-directed angular distribution

$\frac{dP}{d\Omega}$ (power into solid angle)

2) SR field has a spectral distribution

$\frac{dP}{d\omega}$ (power into frequency ω)



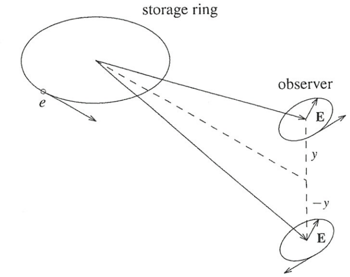
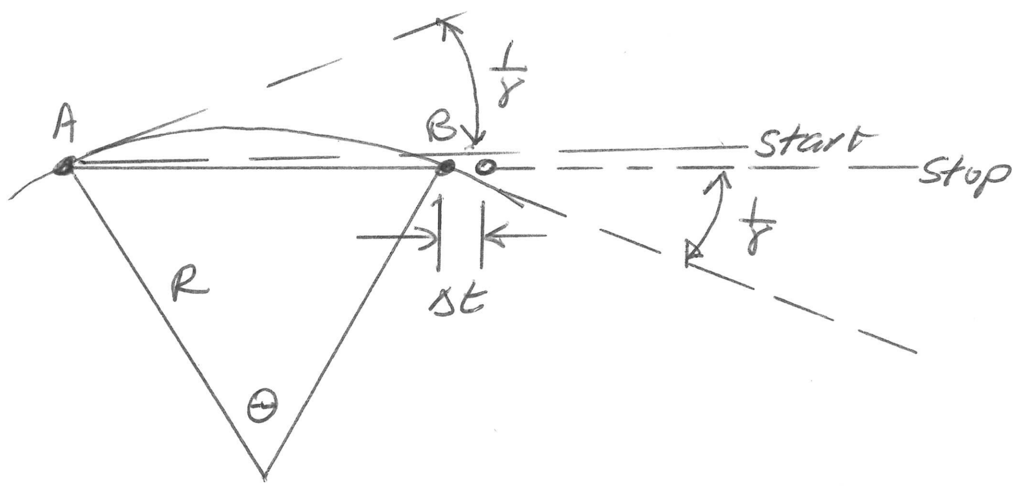


Fig. 1.6. Linear and elliptical polarization of synchrotron radiation.

3) SR field is <u>polarized</u>

- pure horizontal in the accelerator plane
- elliptically polarized above and below the midplane

Characteristic Emission Frequency



EM pulse from one electron
(classical treatment)

electron   photon

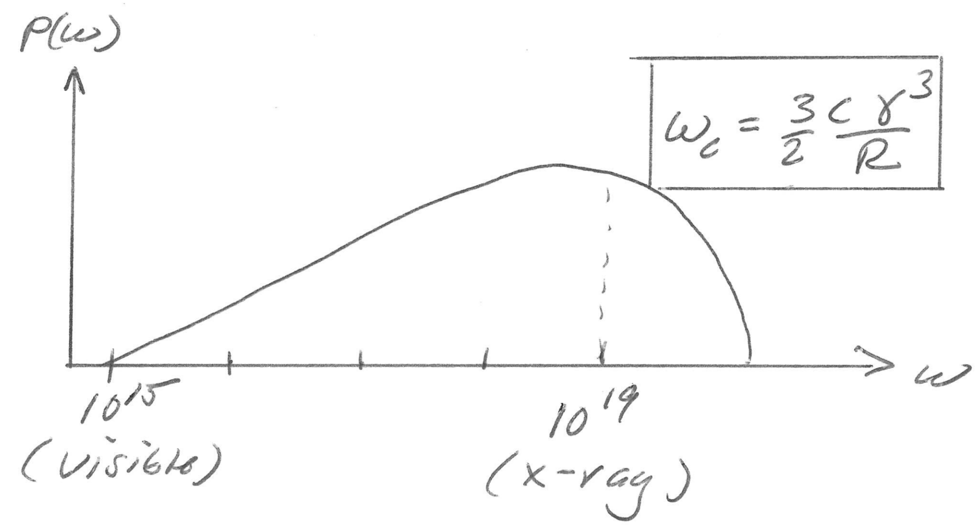
$$\Delta t = \frac{L}{c} = \frac{\overset{\frown}{AB}}{v} - \frac{\overline{AB}}{c}$$

$$\Delta t = \frac{R\theta}{v} - \frac{R\sin\theta}{c}$$

$$\theta \sim \frac{1}{\gamma} \qquad \sin\theta \sim \theta = \frac{1}{\gamma}$$

$$\Delta t \sim \frac{R}{c\gamma^3} \sim \frac{7}{3\times10^8 (6000)^3} \sim 10^{-19}\,s$$

$$\Delta f = \frac{1}{\Delta t} = 10^{+19}\,Hz$$



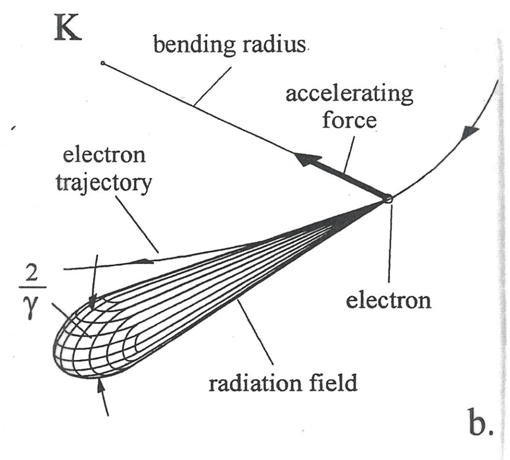
$$\boxed{\omega_c = \frac{3}{2}\frac{c\gamma^3}{R}}$$

The Angular Spectral Power Density: $\frac{d^2P}{d\Omega d\omega}$

Angular

$\frac{dP}{d\Omega}$

$\theta_c \sim \frac{1}{\gamma} \sim 160 \mu rad$



Spectral

$\frac{dP}{d\omega}$

$\omega_c = \frac{3}{2}\frac{c\gamma^3}{R} \sim 10^{19}$

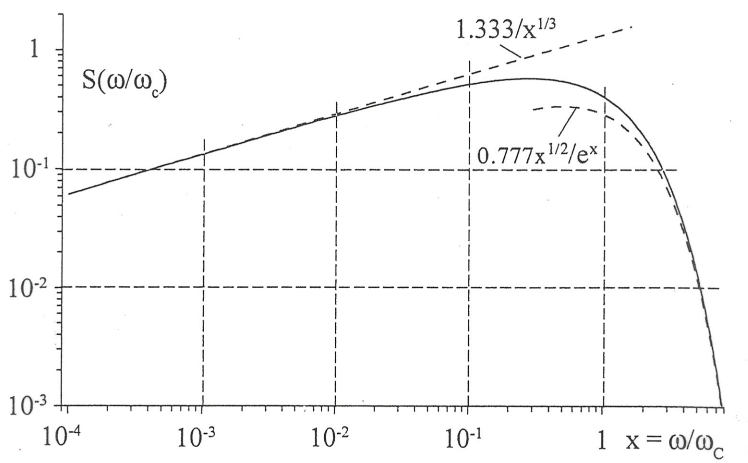
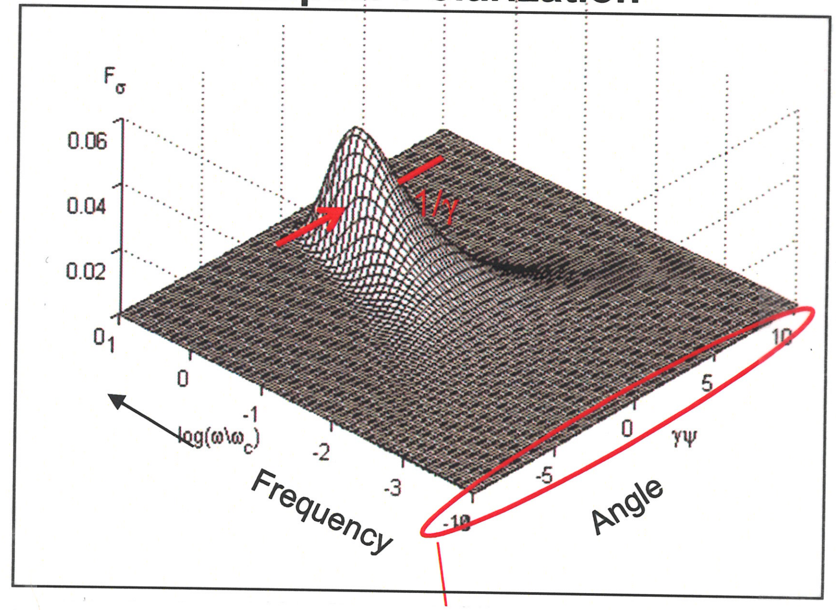
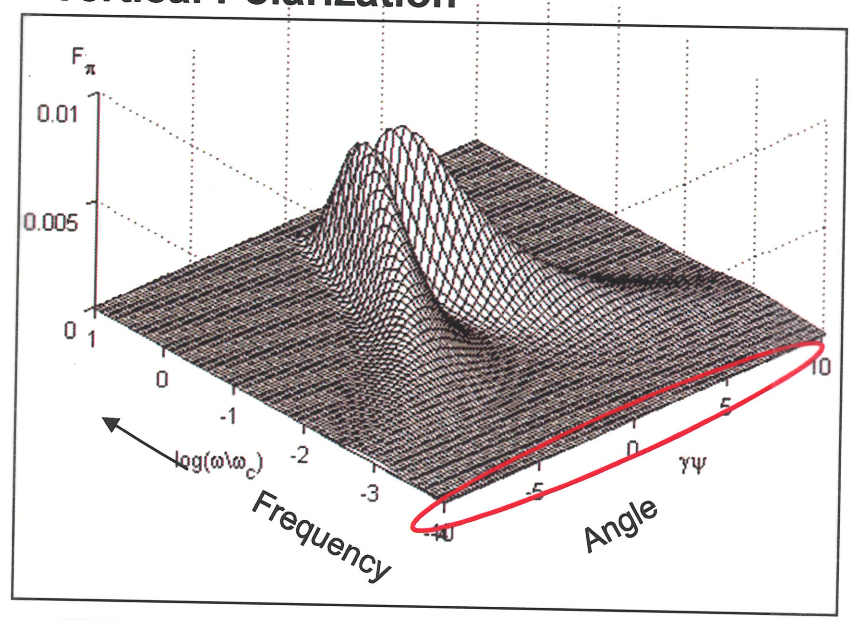


Fig. 9.5. Universal function of the synchrotron radiation spectrum $S(\omega/\omega_c)$

Horizontal Dipole Polarization

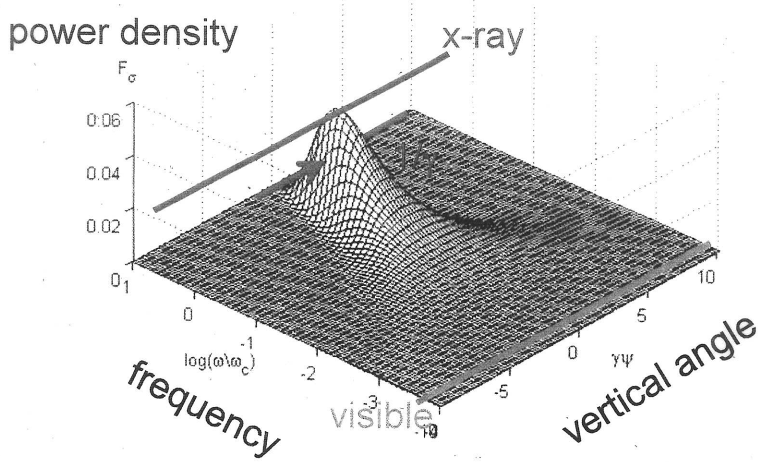


Vertical Polarization



Angular-Spectral Power Density - Schwinger's Equations

σ-mode



$$\frac{d^2P}{d\Omega d\omega} = \frac{P_s\gamma}{\omega_c}\left[F_{s\sigma}(\omega,\psi) + F_{s\pi}(\omega,\psi)\right]$$

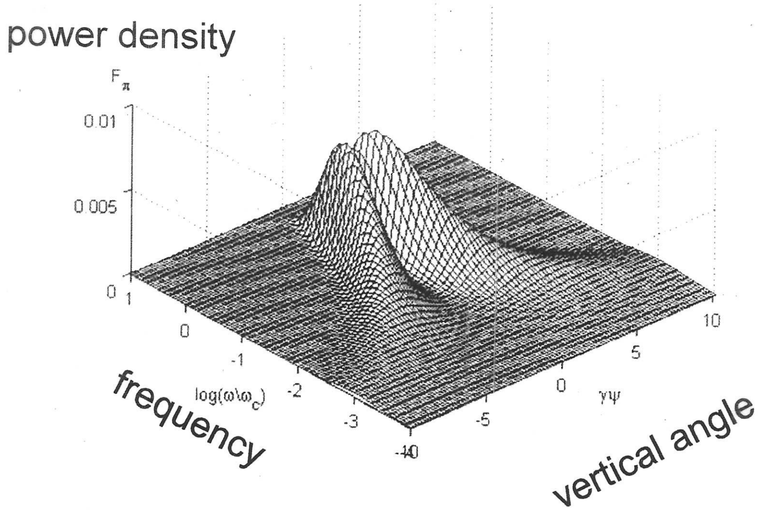
Horizontal Vertical

$$F_{s\sigma}(\omega,\psi) = \left(\frac{3}{2\pi}\right)^3\left(\frac{\omega}{2\omega_c}\right)^2\left(1+\gamma^2\psi^2\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{3/2}\right)$$

$\frac{\omega}{\omega_c}$, $\gamma\psi$

modified Bessel function

π-mode



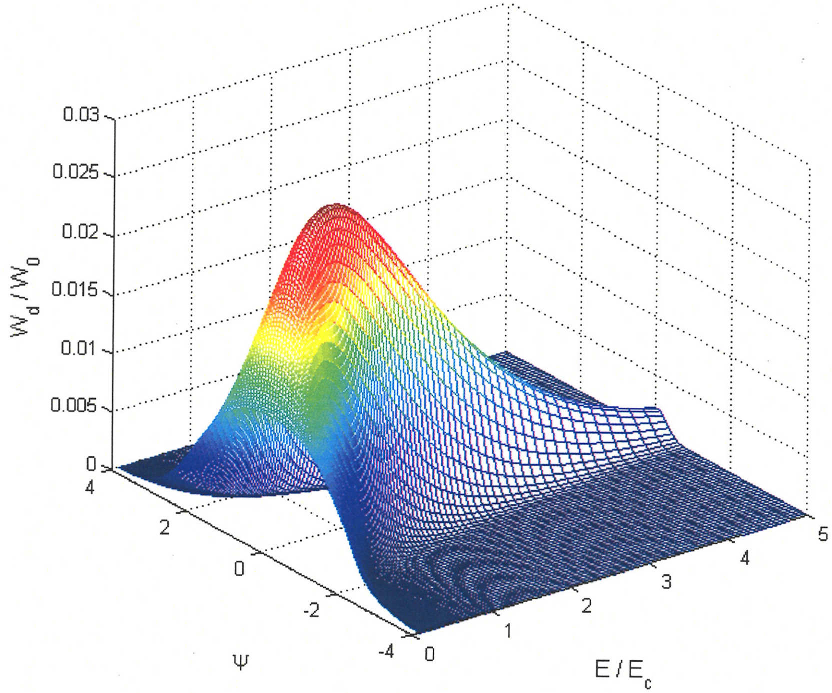
$$F_{s\pi}(\omega,\psi) = \left(\frac{3}{2\pi}\right)^3\left(\frac{\omega}{2\omega_c}\right)^2 \gamma^2\psi^2\left(1+\gamma^2\psi^2\right)K_{1/3}^2\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{3/2}\right)$$

$F_\pi = 0$ at $\psi = 0$

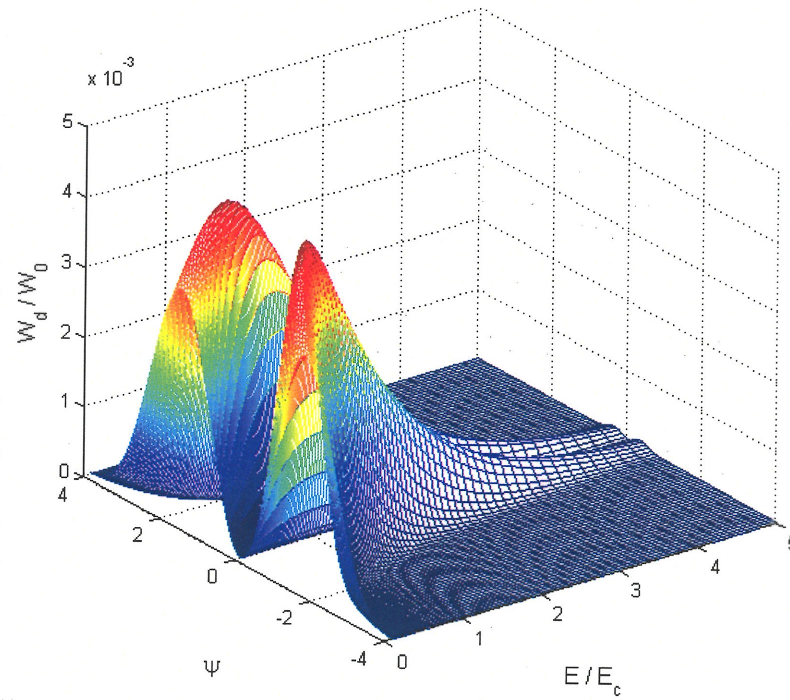
Universal Curves
normalized to ω_c and γ

SR Beam Opening Angle

Horizontal (σ) Polarization



Vertical (π) Polarization



5.5 The angular distribution

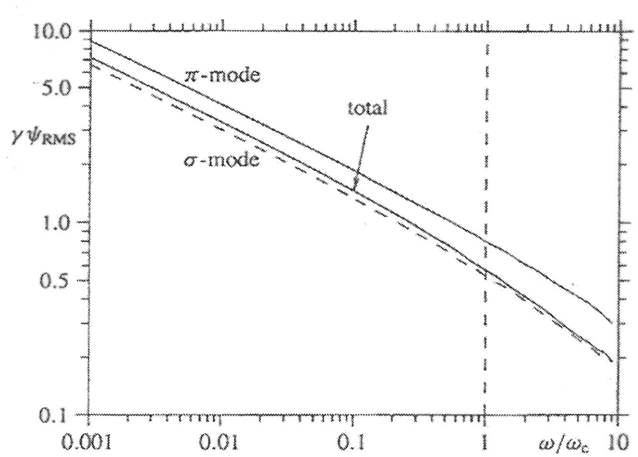


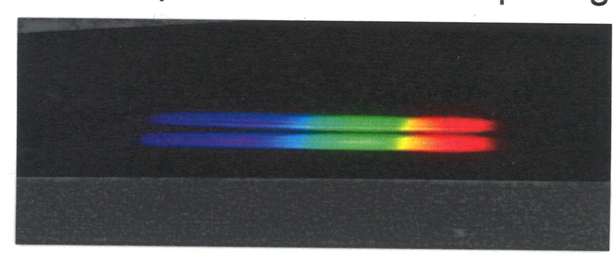
Fig. 5.8. The vertical RMS opening angle versus the frequency.

In the visible $\gamma\psi \sim 20$

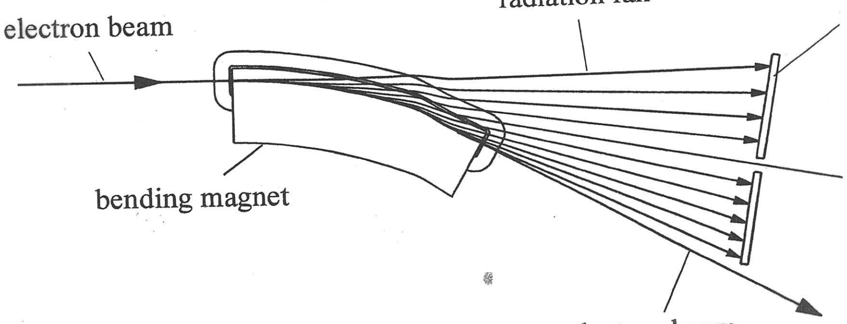
$\gamma \sim \frac{20}{6000} = 3.3$ mrad

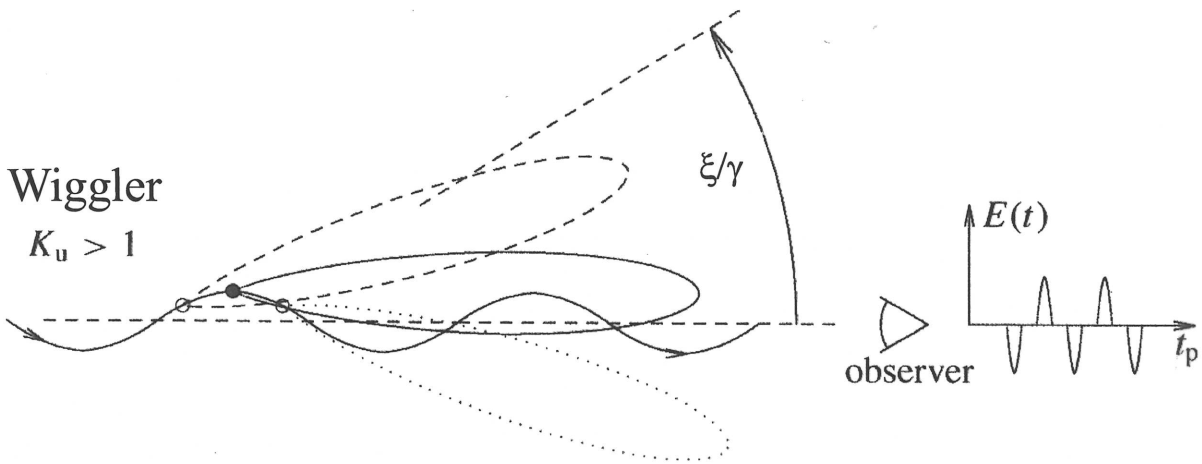
± 30 mm at 10 m

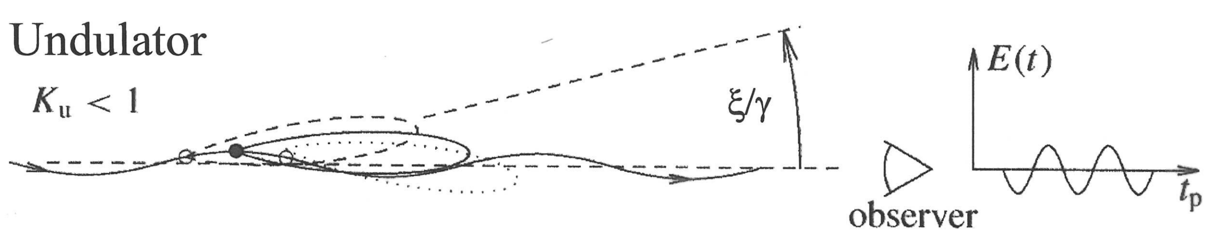
visible component – few mrad opening angle



SR from Wigglers and Undulators







FEL: SR emission reacts back on beam → microbunching $\sigma_B \sim \lambda_{rad}$

self-amplifies the emission process

Schwinger's Equations – Derivation of Angular Spectral Density

$$\frac{dP}{d\Omega d\omega} = \frac{P_S \gamma}{\omega_c}\left[F_\sigma(\omega,\psi) + F_\pi(\omega,\psi)\right] \quad \left\{ \begin{array}{l} F_{\sigma,\pi} \text{ form factors} \\ \omega \text{ frequency} \\ \gamma \text{ beam energy} \end{array} \right.$$

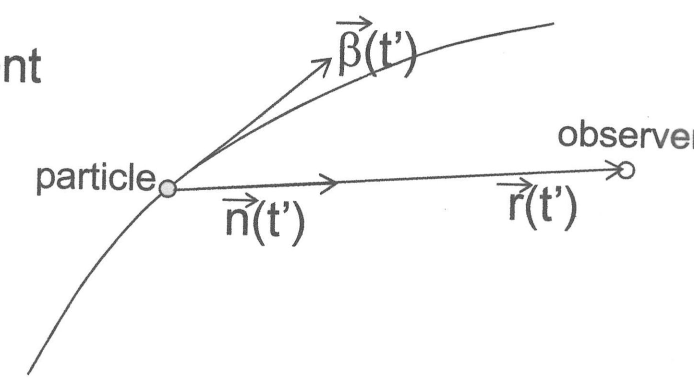
Power scales as E^2

$\longrightarrow$ we need $E_\sigma(\omega,\psi)$ and $E_\pi(\omega,\psi)$

Start with Lienard-Wiechert Potentials

$$V(t) = \int \frac{\rho(t')}{r(t)} dV \Big|_{ret} \qquad \text{scalar potential from charge}$$

$$\mathbf{A}(t) = \int \frac{\mathbf{J}(t')}{r(t)} dV \Big|_{ret} \qquad \text{vector potential from current}$$



$$\mathbf{E} = -\nabla V - \frac{d\mathbf{A}}{dt} \qquad \text{electric field}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \text{magnetic field}$$

Schwinger's Equations

$$\mathbf{E} = -\nabla V - \frac{d\mathbf{A}}{dt}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

After a Jacksonian derivation

$$\mathbf{E}(t) = \frac{1}{4\pi\varepsilon_o c}\left\{\frac{\left[\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})]\times\dot{\boldsymbol{\beta}}\right]}{r(1-\mathbf{n}\cdot\boldsymbol{\beta})^3}\right\}_{\text{ret}}$$

$$\mathbf{B}(t) = \left.\frac{\mathbf{n}\times\mathbf{E}}{c}\right|_{\text{ret}}$$

Poynting Vector (power flux)

'difficulty evaluating the above equations'
'advantageous to calculate Fourier transforms'

Fourier Transform Fields + Small Angle Approximations

$$\tilde{E}_x(\omega) = \frac{-\sqrt{3}e\gamma}{(2\pi)^{3/2}\varepsilon_o c r_c}\cdot\frac{\omega}{2\omega_c}\cdot\left(1+\gamma^2\psi^2\right)K_{2/3}\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{3/2}\right)$$

$$\tilde{E}_y(\omega) = \frac{i\sqrt{3}e\gamma}{(2\pi)^{3/2}\varepsilon_o c r_c}\cdot\frac{\omega}{2\omega_c}\cdot\gamma\psi\left(1+\gamma^2\psi^2\right)K_{1/3}\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{3/2}\right)$$

there are two polarizations

$$\frac{d^2P}{d\Omega d\omega} = \frac{d^2P_\sigma}{d\Omega d\omega} + \frac{d^2P_\pi}{d\Omega d\omega} = \text{Const}\cdot\left(\left|\tilde{E}_x(\omega)\right|^2 + \left|\tilde{E}_y(\omega)\right|^2\right).$$

$$\boxed{\begin{aligned} F_{s\sigma}(\omega,\psi) &= \left(\frac{3}{2\pi}\right)^3\left(\frac{\omega}{2\omega_c}\right)^2\left(1+\gamma^2\psi^2\right)^2 K_{2/3}^2\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{3/2}\right). \\ F_{s\pi}(\omega,\psi) &= \left(\frac{3}{2\pi}\right)^3\left(\frac{\omega}{2\omega_c}\right)^2\gamma^2\psi^2\left(1+\gamma^2\psi^2\right)K_{1/3}^2\left(\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{3/2}\right). \end{aligned}}$$

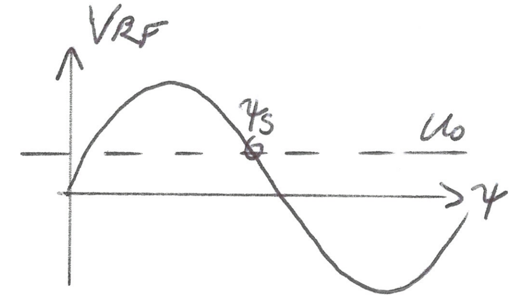
Radiated Power and Energy Loss per Turn

Previously $P_{rad} \propto E^2 B^2$ or $\frac{E^4}{R^2}$

E = Beam Energy
B = Dipole Field
R = Bend radius

P_{rad} = instantaneous power radiated by one electron

Energy loss per revolution



$$U_0 = \oint P_{rad}\,dt = \frac{1}{c}\oint P_{rad}\,ds \propto \frac{E^2}{c}\oint B^2(s)\,ds$$

Bend radius $\frac{1}{R} = \frac{qB}{\gamma m c^2}$

$$\therefore \quad U_0 \propto E^2 \oint \frac{ds}{R^2(s)}$$

synchrotron integral I_2 $(I_1 - I_5)$

Get relativistic factors correct

$$\boxed{U_0 = \frac{e^2 \gamma^4}{3 \epsilon_0 R} = \frac{88.5\, E^4(\text{GeV})}{R(\text{m})}\ \text{keV}}$$

$$U_0 \sim \frac{100 \cdot 100}{10} = \frac{10^4}{10} = 10^3\ \text{keV}$$

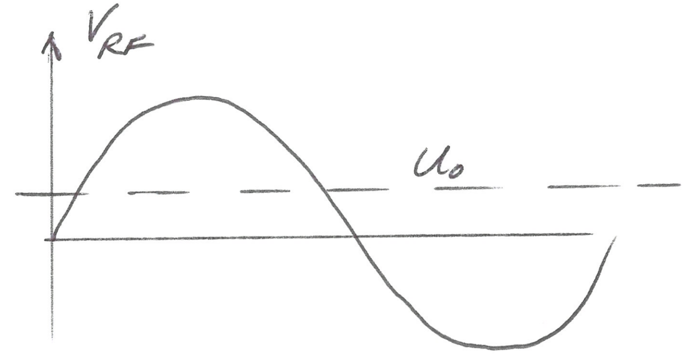
(1 MeV/turn)

SR Power and Energy Loss Scaling

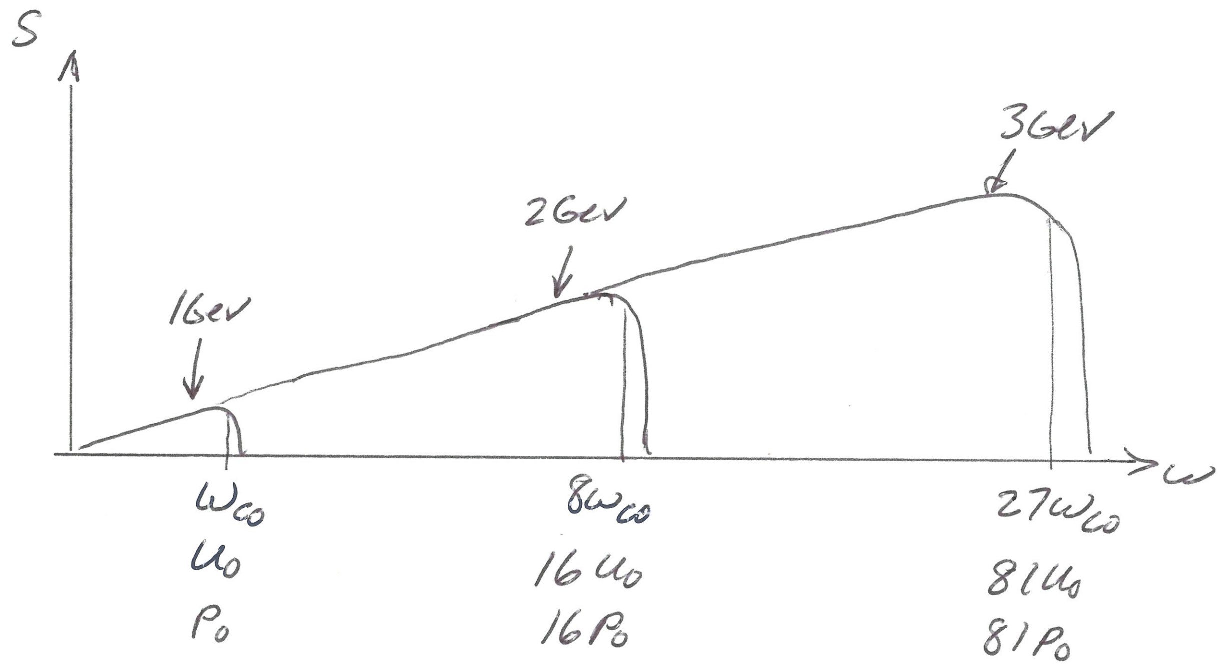
Power $\quad P_{rad} \propto \dfrac{\gamma^4}{R^2} \quad \dfrac{\text{Joules}}{\text{sec}}$

Energy Loss / Turn $\quad U_0 \propto \dfrac{\gamma^4}{R} \quad$ Joule (eV)

$$\left(P = \frac{U}{T_0} \text{ where } T_0 = \frac{2\pi R}{c} \right)$$

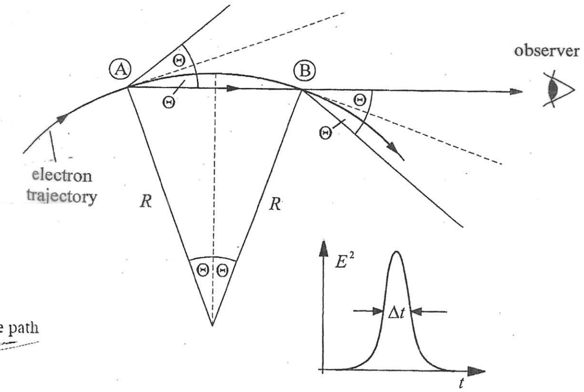


Critical Energy $\quad \omega_c = \dfrac{3}{2} \dfrac{c\gamma^3}{R}$

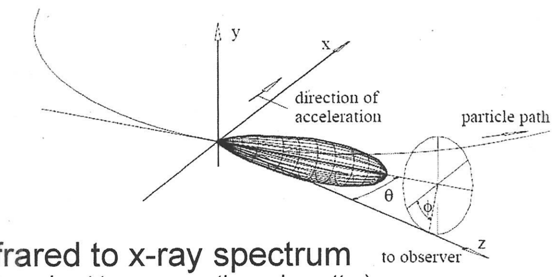


Harder spectrum
more power

SR Review



1) SR spectrum extends into hard x-ray



infrared to x-ray spectrum
(ordinary heat to passage-through-matter)

2) Radiation forward-directed

3) Angular-Spectral power density $\frac{d^2P}{d\Omega d\omega}$
(Schwinger's Equations)

4) Horizontal/Vertical polarization components (or left/right circular)

5) Critical photon energy scales as $\frac{\gamma^3}{R}$

6) Instantaneous radiation power and energy loss/turn scale as γ^4