Accelerator Physics Lecture Modules

1. Introduction/Timing
2. Beam propagation basics
3. Simple Harmonic Oscillator

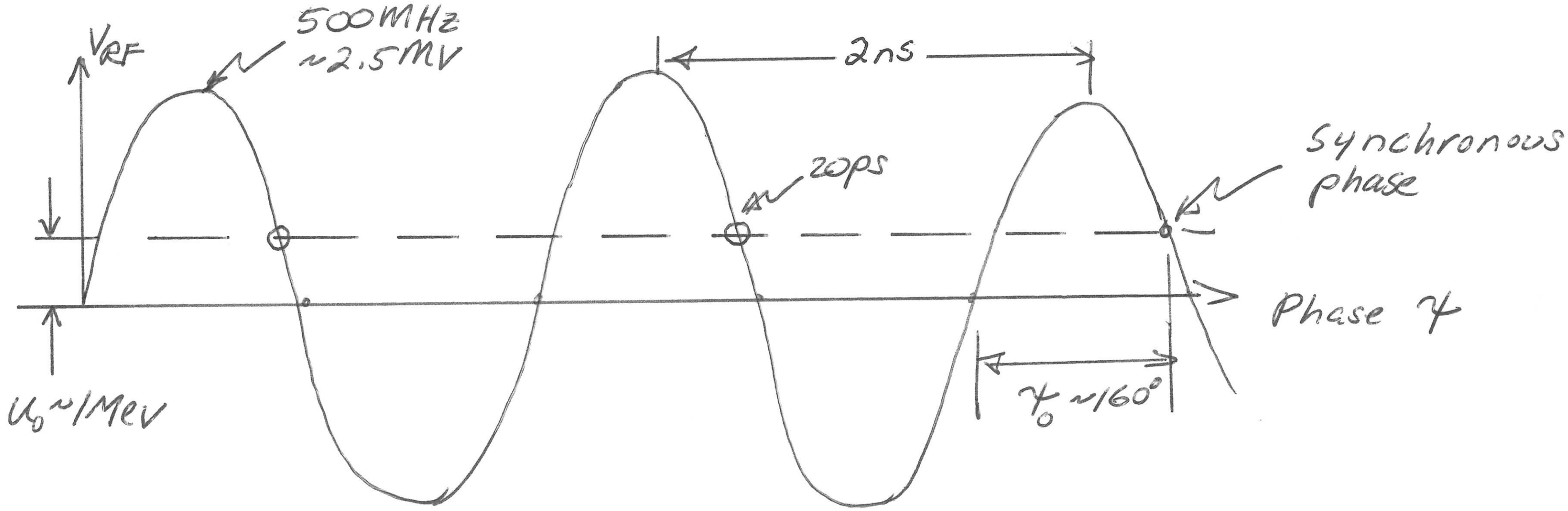
4. Betatron oscillations (transverse motion)
5. Dispersion
6. Closed Orbit Distortion

→ 7. Synchrotron oscillations
8. Properties of synchrotron radiation

9. Radiation damping
10. Equilibirum emittance

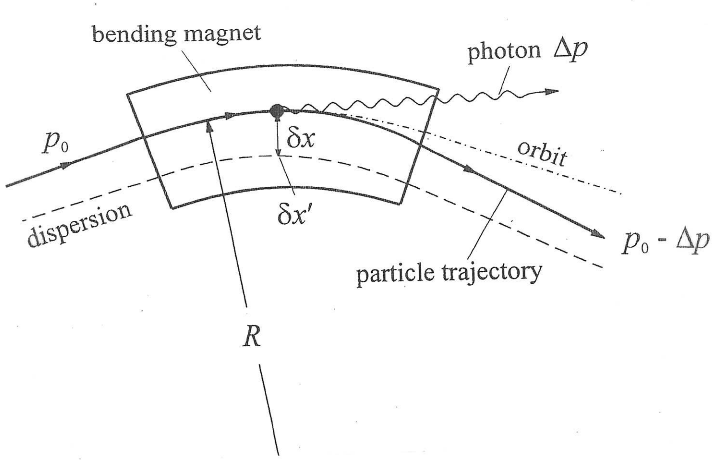
Longitudinal Oscillations

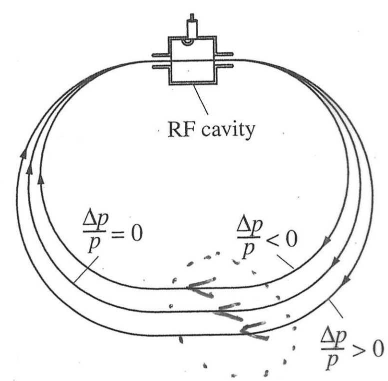
(synchrotron oscillations, phase focusing)

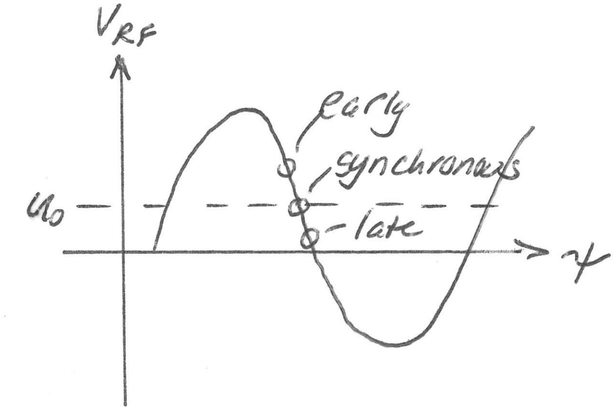


Single electrons emit photon 'quanta'

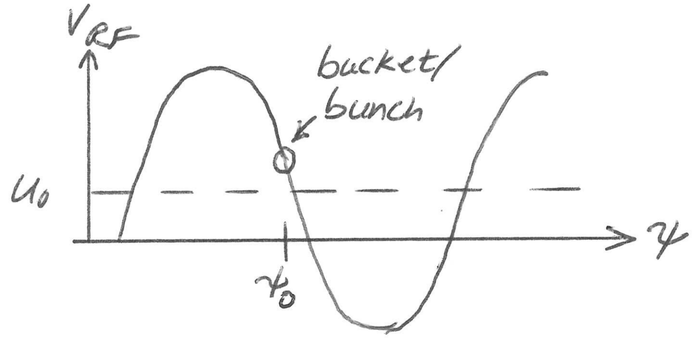
Recoil excites synchrotron, betatron oscillations

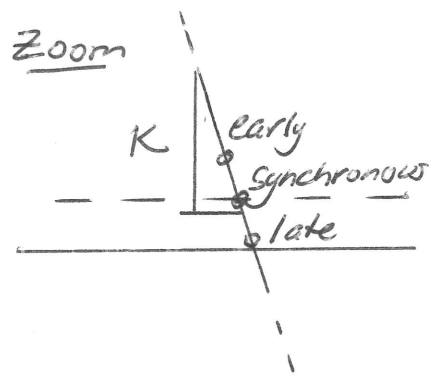


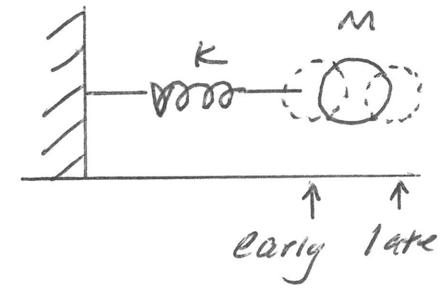




Longitudinal Oscillations - Phase Space



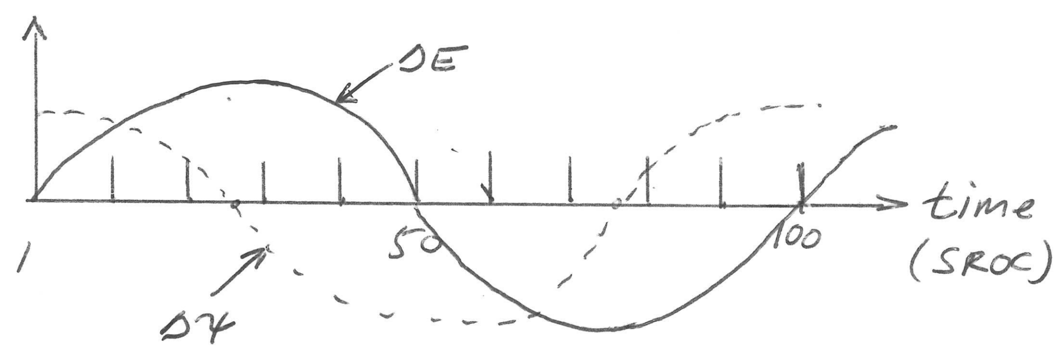


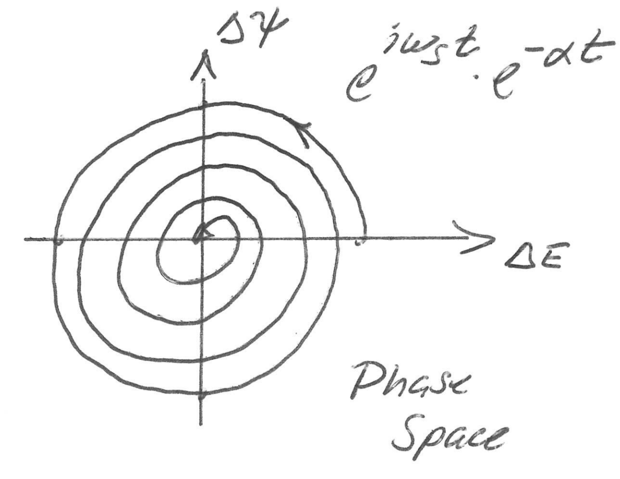


In the linear approximation expect $\psi'' + 2\alpha\psi' + \omega_s^2\psi = 0$

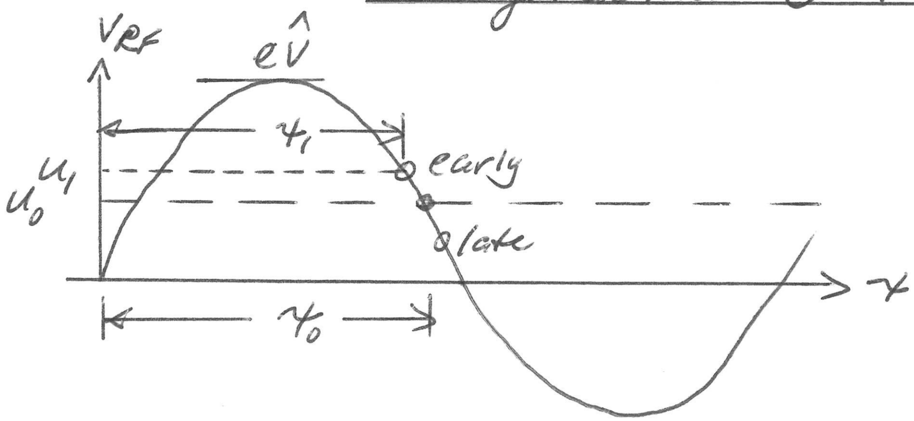
damping ~5ms — synchrotron frequency ~10KHz

Classical derivation in terms of $\Delta\psi$ or ΔE $(\Delta E, \Delta\psi)$ canonical coordinates





Longitudinal Oscillations - Equation of Motion



Synchronous Particle : $E_0 = e\hat{V}\sin\psi_0 - U_0$

Off-Energy Particle : $E_1 = e\hat{V}\sin\psi_1 - U_1$

Deviation :

$$\Delta E = E_1 - E_0$$

$$\Delta E = e\hat{V}(\sin\psi_1 - \sin\psi_0) - (U_1 - U_0)$$

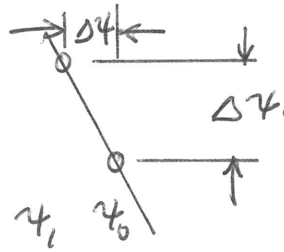
Discrete Differentiate :

$$\Delta E' = \frac{e\hat{V}}{T_0}(\sin\psi_1 - \sin\psi_0) - \left(\frac{U_0 - U_1}{T_0}\right)$$

① ②

$$\boxed{\Delta E' = \frac{e\hat{V}}{T_0} \cdot \Delta\psi\cos\psi_0 - \frac{dU_0}{dE}\Delta E}$$

① $\sin\psi_1 - \sin\psi_0 \sim \Delta\psi\cos\psi_0$



$\frac{\text{rise}}{\text{run}}$

② $U_0 - U_1 \sim U_0 - \left(U_0 + \frac{dU_0}{dE}\Delta E\right) = \frac{-dU_0}{dE}\Delta E$

↑ damping term

Equation of Motion

$$\Delta E' = \underbrace{\frac{e\hat{V}}{T_0}\Delta\psi\cos\psi_0}_{RF} - \underbrace{\frac{dU_0}{dE}\Delta E}_{SR}$$

Differentiate again

$$\Delta E'' = \frac{e\hat{V}}{T_0}\Delta\psi'\cos\psi_0 - \frac{1}{T_0}\frac{dU_0}{dE}\Delta E' = 0$$

$\uparrow$ $\Delta\psi' = \frac{\Delta\psi}{T_0}$ $\qquad$ $\uparrow$ $2\alpha_s$

$$\Rightarrow \Delta E'' + \underbrace{\left(\frac{1}{T_0}\frac{dU_0}{dE}\right)}_{2\alpha_{damp}}\Delta E' + \underbrace{\left(\frac{e\hat{V}}{T_0}\frac{h\omega_{rev}\cos\psi_0}{\alpha E_0}\right)}_{\Omega_s^2}\Delta E = 0$$

From dispersion calculation

$$\frac{\Delta E}{E_0} = \alpha\frac{\Delta T}{T_0}$$

(α ← momentum compaction factor) $\quad$ where $\Delta\psi = h\cdot\omega_{rev}\cdot\Delta T$

Solves

$$\Delta\psi = \frac{\Delta E}{E_0}\cdot\frac{T_0}{\alpha}\cdot h\omega_{rev}$$

$$\Delta\psi' = \frac{\Delta\psi}{T_0} = \frac{\Delta E}{E}\cdot\frac{h\omega_{rev}}{\alpha}$$

Sort it out

$$\Omega = \omega_{rev}\sqrt{\frac{e\hat{V}\cos\psi_0\,\alpha}{2\pi E_0}}$$

$$\alpha_{damp} = \frac{1}{2T_0}\frac{dU_0}{dE}$$

For proton machines $\alpha \rightarrow \left(\alpha_0 - \frac{1}{\gamma^2}\right)$

$$\Omega = 10^7\sqrt{\frac{10^6\cdot 1\cdot 10^{-3}}{10\cdot 10^9}} \sim 10^7\cdot\sqrt{10^{-7}}$$

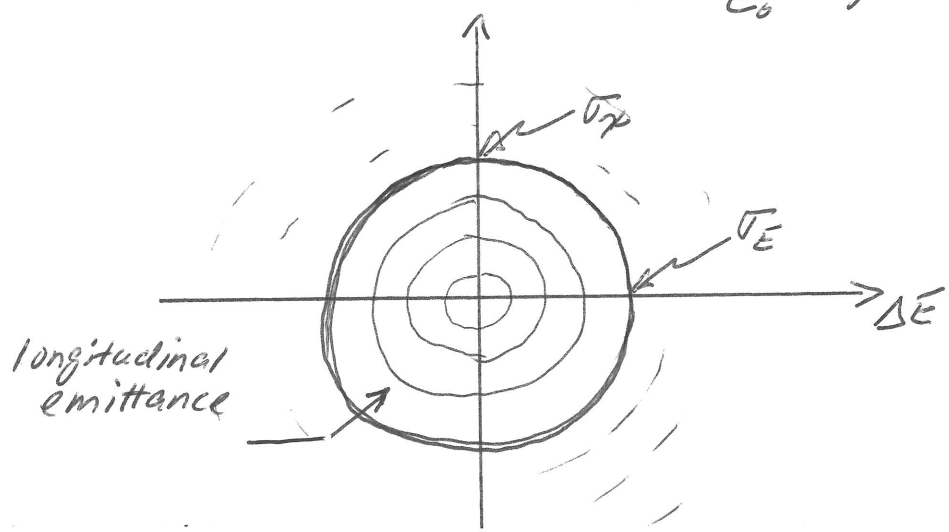
$$\sim 10^4\ Hz$$

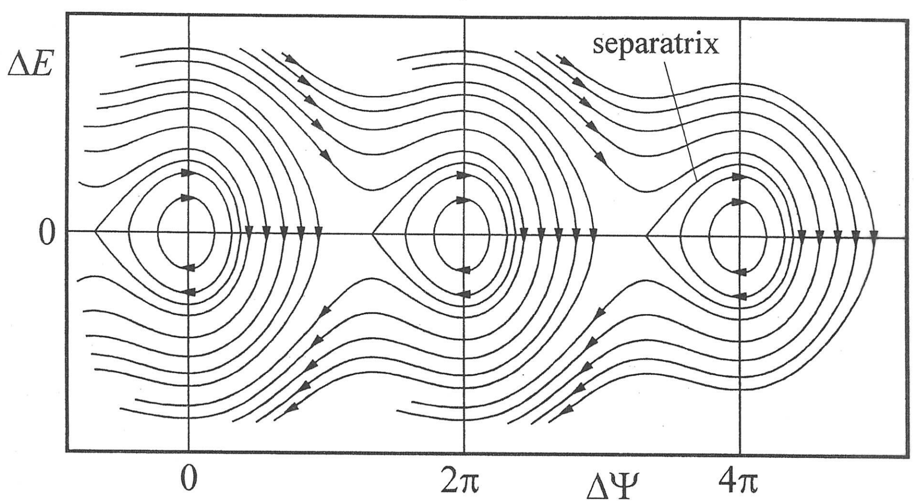
Synchrotron Oscillations

$$\Delta E'' + 2\alpha_s \Delta E' + \Omega^2 \Delta E = 0 \qquad \alpha_s = \frac{1}{2T_0}\frac{dU_0}{dE}$$

$$\Delta E(t) = \Delta E_0 e^{-\alpha_s t} e^{i\Omega t} \qquad \Omega = \omega_{rev}\sqrt{\frac{eV h \cos\psi_0}{2\pi E_0}\alpha}$$

$$\Delta\psi = \left(\frac{T_0 \alpha h \omega_{rev}}{E_0}\right)\Delta E$$



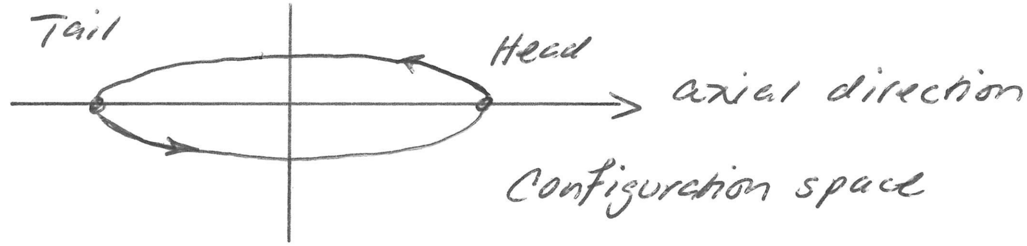


Bi-Gaussian

$$f(E,\psi) = \frac{1}{2\pi\sigma_E\sigma_\psi} e^{-E^2/2\sigma_E^2} e^{-\psi^2/2\sigma_\psi^2}$$

- σ_E and σ_ψ are equilibrium balance between excitation and damping
- ratio $\frac{\sigma_E}{\sigma_\psi}$ depends on momentum compaction 'α'

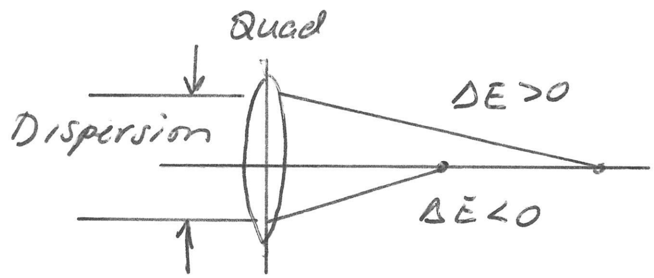
<u>Head-Tail Instability - Broadband, Single Bunch</u>

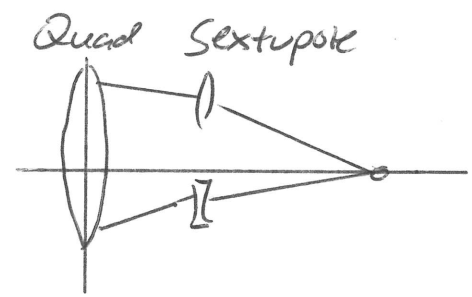


Energy spread causes tune spread

$$\Delta Q = \xi \frac{\Delta E}{E}$$

↑ chrometicity (change of betatron tune with energy)

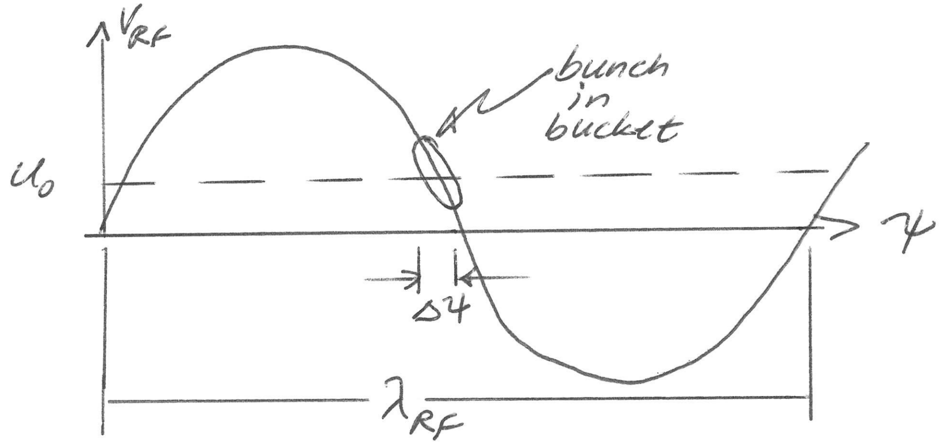




Need <u>positive</u> chromaticity to combat Head-Tail instability

- Requires strong non-linear sextupoles
  - Tune shift with amplitude
    - Loss of dynamic aperture
      - Low beam lifetime, poor injection efficiency

Bunch Length Calculation



$\Delta\psi = K \cdot \Delta S \qquad K = \frac{2\pi}{\lambda_{RF}}, \quad \Delta\psi = \frac{h\omega_{rev}\alpha}{\Omega}\frac{\Delta E}{E}$

$\Delta S = \sigma_{Bunch} = \frac{\Delta\psi}{K}$

$\sigma_B = \frac{h\omega_{rev}\alpha}{\Omega} \cdot \frac{\Delta E}{E} \cdot \frac{\lambda_{RF}}{2\pi}$

$\sigma_B = 600 \cdot 10^2 \cdot 10^{-3} \cdot 10^{-3} \cdot 10^{-1}$

$\sigma_B = 6 \times 10^{-3}$ (6mm) ✓

$\lambda_{RF} \cong 1\,m$
$h = 600$ (harmonic)
$\frac{\omega_{rev}}{\Omega} \approx 10^2$ (tune)
$\alpha \sim 10^{-3}$ (momentum compaction)
$\frac{\Delta P}{P} \sim 10^{-3}$ (energy spread)

– streak camera –

Synchrotron Oscillations – A Review

1) phase-focusing was a significant breakthrough cyclotron → synchrotron

2) synchronous charge trapped in longitudinal potential well

3) bunch length proportional to spread in phase $\Delta\psi$ ~ 20ps rms

4) typical synchrotron frequency 10 kHz (light source)

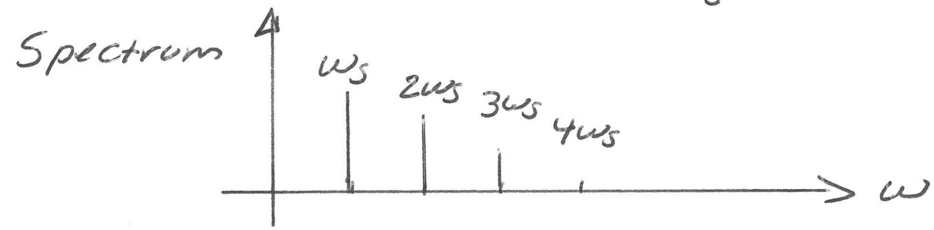
tune $f_s/f_{rev} \sim 10^4/10^6 = 0.01$

damping $\frac{1}{T_0}\frac{du}{dE} \sim 5$ ms

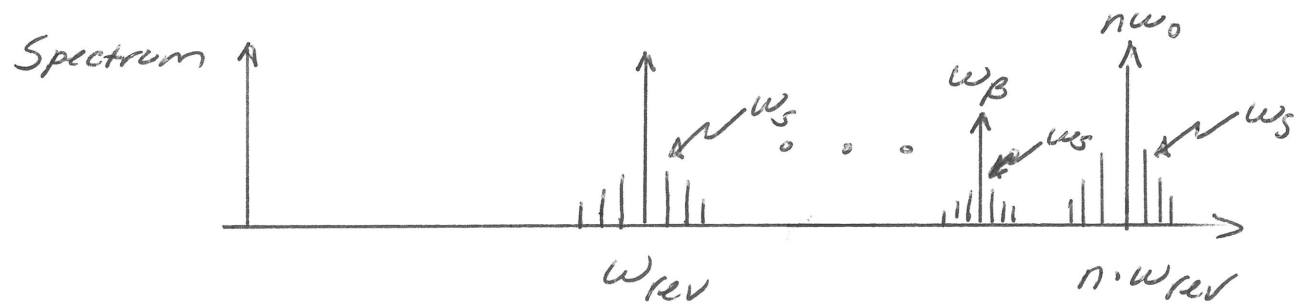
Synchrotron Oscillations in Practice

1) whole bunches execute 'coherent' synchrotron oscillations

+ harmonics ω_s, $2\omega_s$, $3\omega_s$ ...



2) phase oscillations create sidebands at revolution harmonics



3) Synchro-betatron coupling $M_{6\times6} =$