Accelerator Physics Lecture Modules

1. Introduction/Timing
2. Beam propagation basics
3. Simple Harmonic Oscillator

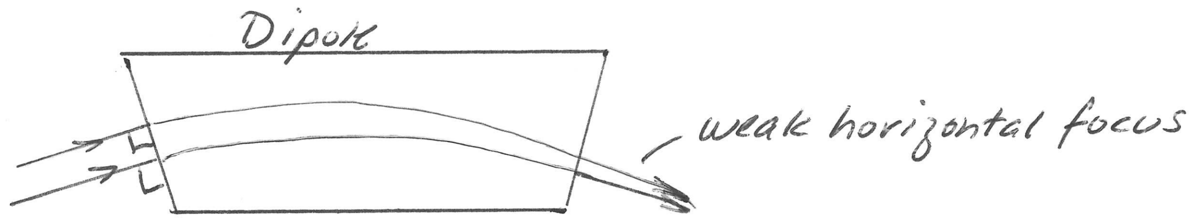
4. Betatron oscillations (transverse motion)
5. Dispersion
6. Closed Orbit Distortion

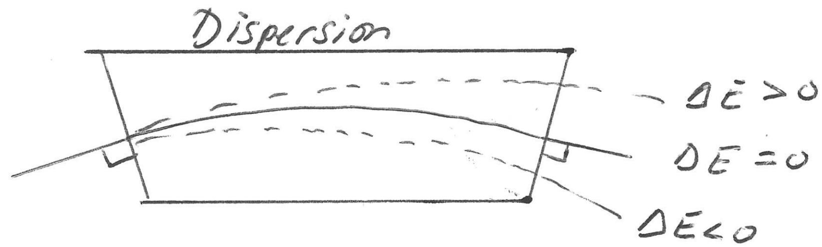
7. Synchrotron oscillations
8. Properties of synchrotron radiation

9. Radiation damping
10. Equilibirum emittance

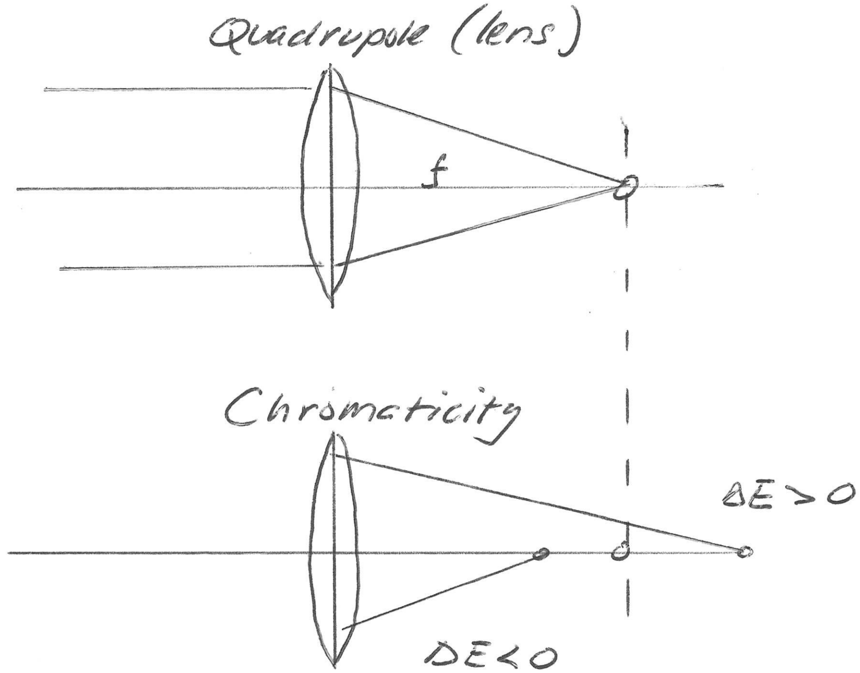
Basic Optical Elements

Bend

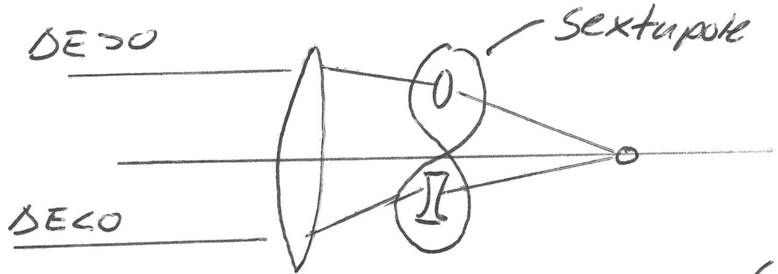


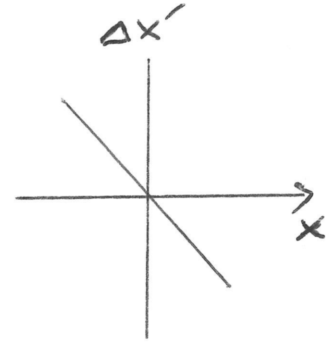


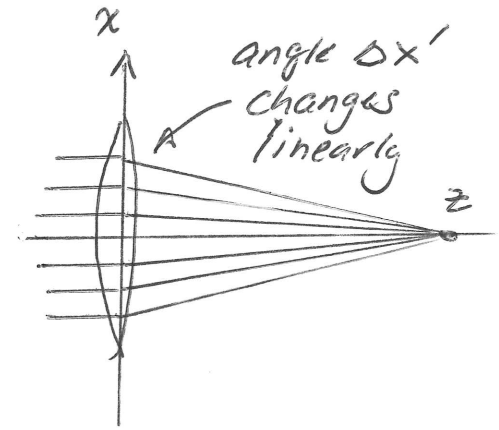
Focus



Higher - Order Elements







- Quadrupole lens has a linear gradient
- Sextupole lens has quadradic gradient
- Octupole cubic
- Decapole quartic

Charged Particle Motion

Lorentz Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Static electric field $E \sim \frac{1MV}{m} = 10^6\ V/m$

electron moves at velocity $v \sim c$

$F_E = F_B$

$10^6 = q \cdot c \cdot B$

$10^6 = 3\times10^8 \cdot B \rightarrow B = 10^{-2}$ Tesla (100 Gauss)

$\therefore$ Use magnetic fields to bend and focus

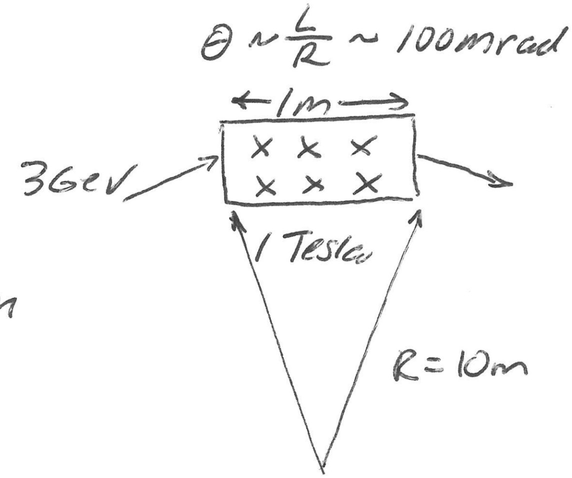
Example: Dipole Magnet

$E = 3$ GeV beam, $B = 1$ Tesla

Centripetal = Lorentz

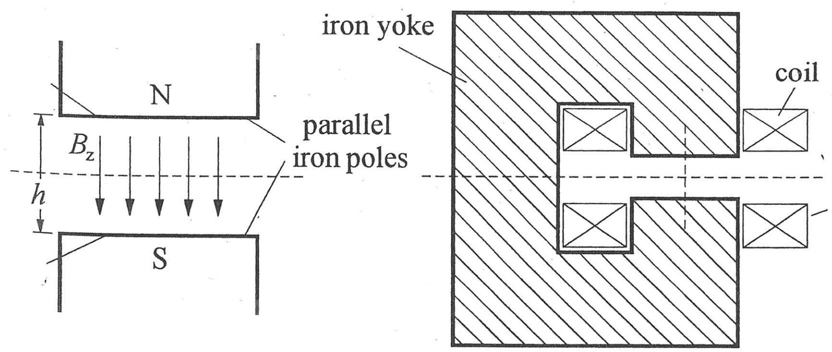
$\frac{\gamma m v^2}{R} = qvB$, where $\gamma m v^2 \approx E$

$R = \frac{E}{qvB} = \frac{3\times10^9}{(ev)(3\times10^8)(1)} = 10m$



Multipole Magnets

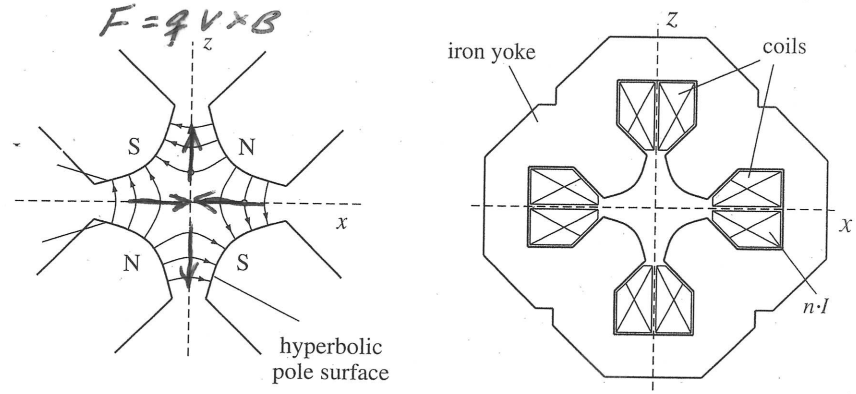
<u>Dipole</u>



$$R = \frac{E_{beam}}{qcB}$$

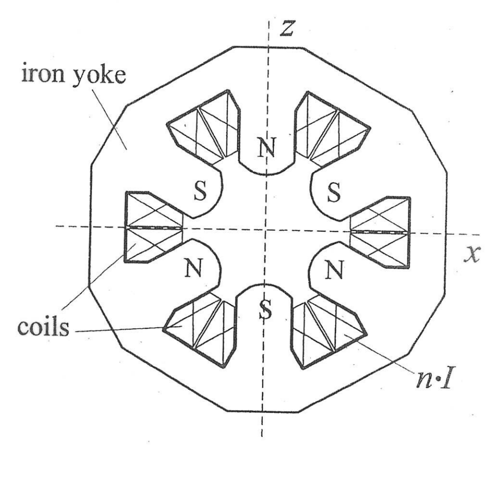
<u>Quadrupole</u>

$F = qv \times B$



Opposite focus
in x-y planes
QF, QD

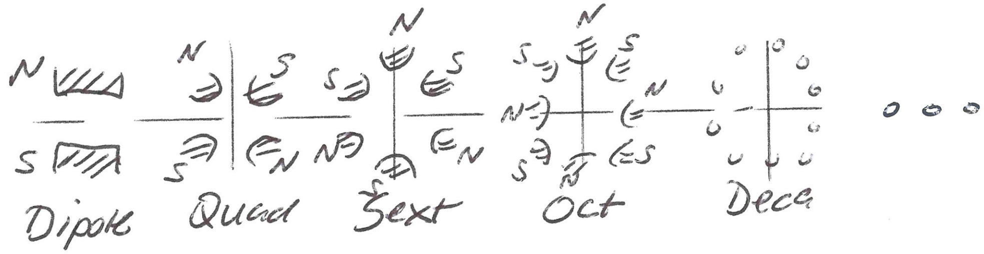
<u>Sextupole</u>



Chromatic
Correction
SF, SD

K. Wille

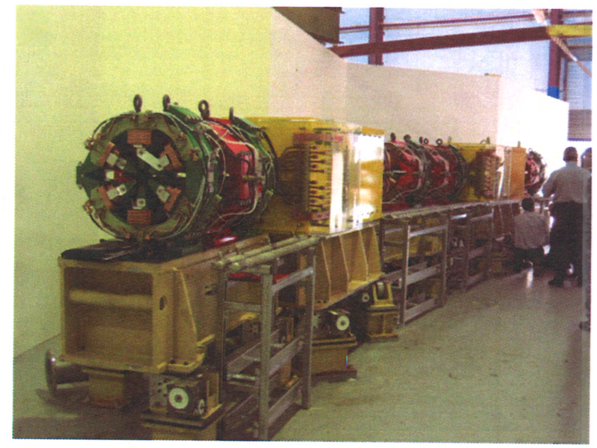
Multipole Magnets

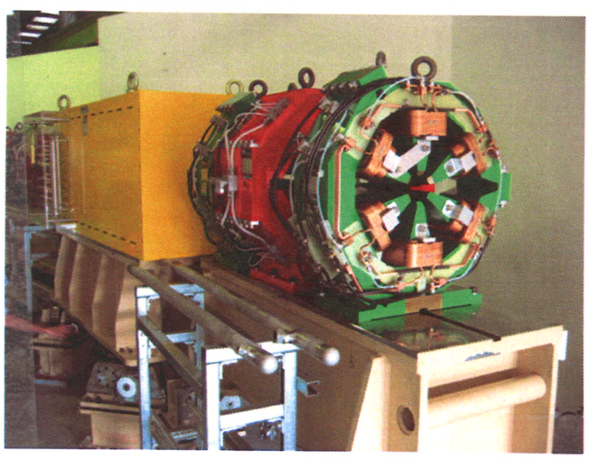


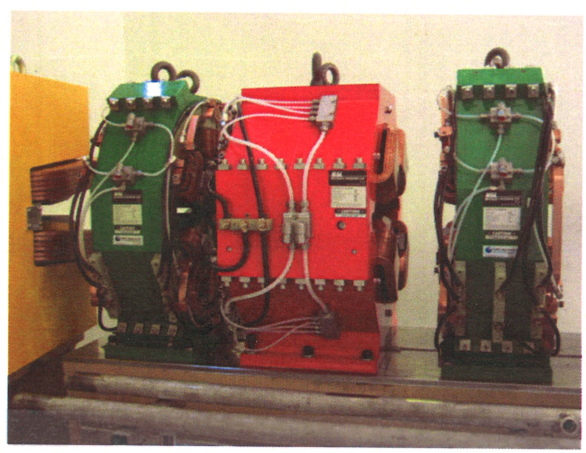
$$B(x) = B_0 + \frac{dB}{dx}\Delta x + \frac{1}{2}\frac{d^2B}{dx^2}\Delta x^2 + \frac{1}{3!}\frac{d^3B}{dx^3}\Delta x^3 + \cdots$$

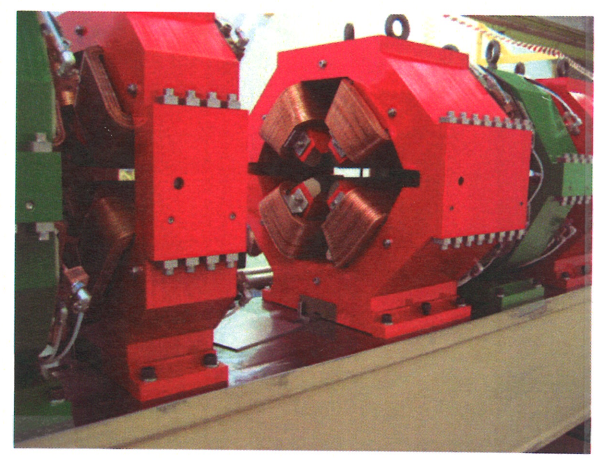
Taylor Series Expansion

Magnet Raft Assembly - Australian Synchrotron



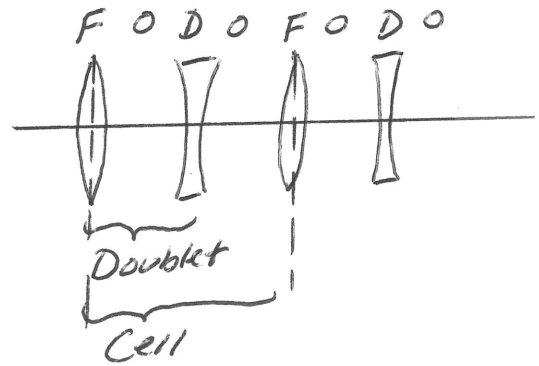


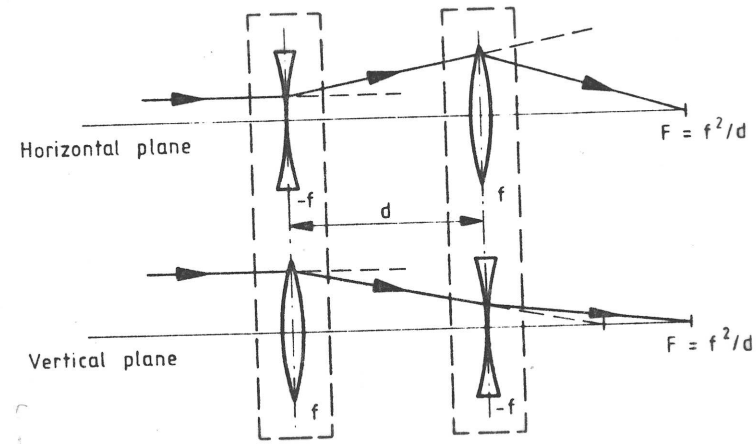




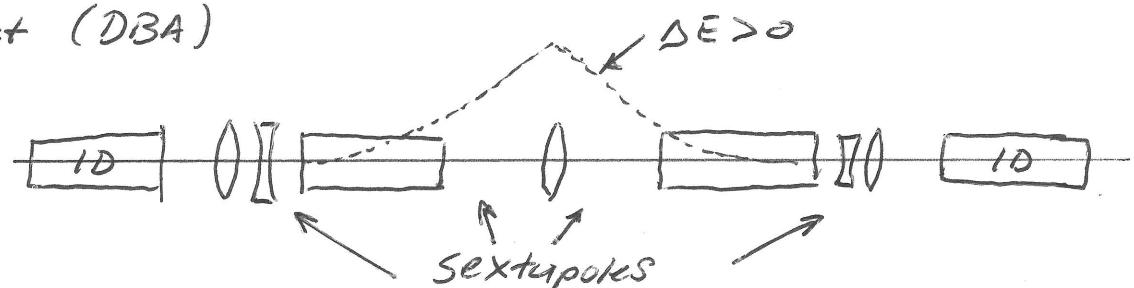
Magnet Structures

F-O-D-O
(Colliders, transport)

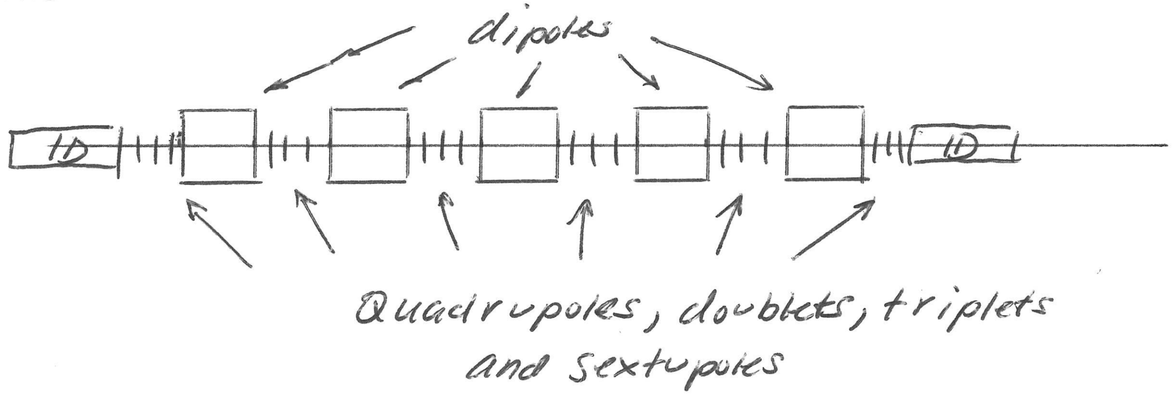




Double-Bend Achromat (DBA)
(1990 - 2010)

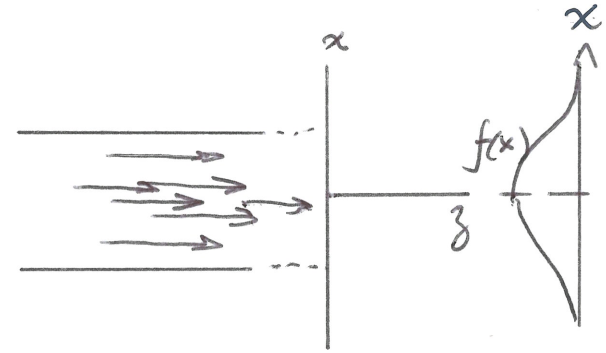


Multi-Bend-Achromat
(>2010)



Beam Propagation Basics

1) Laser-like pencil beam



Gaussian distribution function

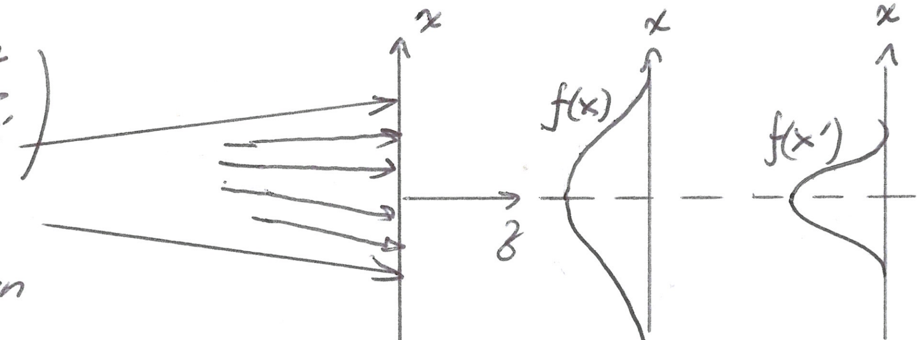
$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

2) Divergent beam

$$f(x,x') = \left(\frac{1}{\sqrt{2\pi}\,\sigma_x} e^{-\frac{x^2}{2\sigma_x^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi}\,\sigma_{x'}} e^{-\frac{x'^2}{2\sigma_{x'}^2}}\right)$$

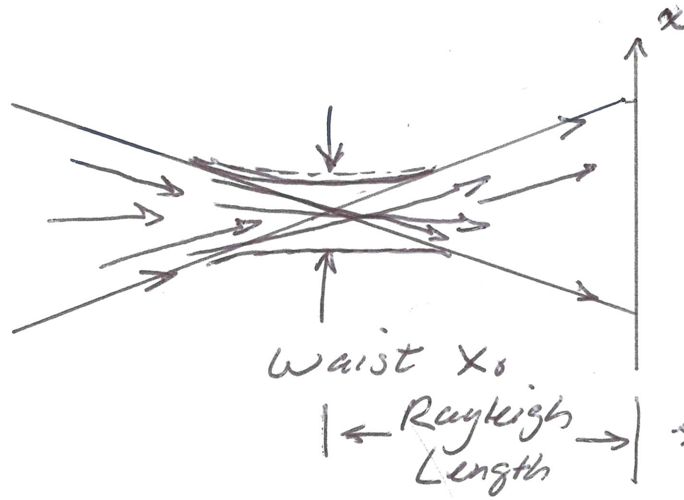
$\sigma_x(z)$, $\sigma_{x'}(z)$

Cross term not shown



Bi-gaussian 'multi-variate distribution function'

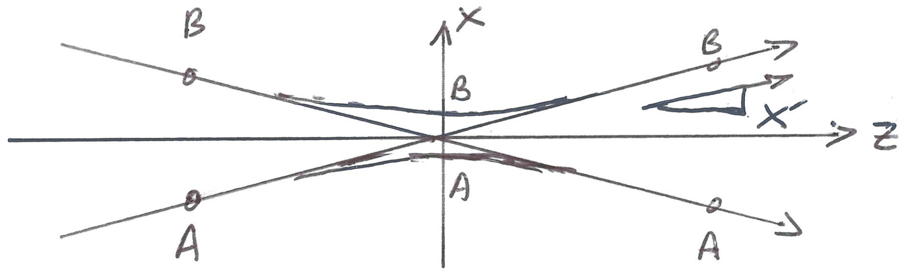
3) Convergent beam



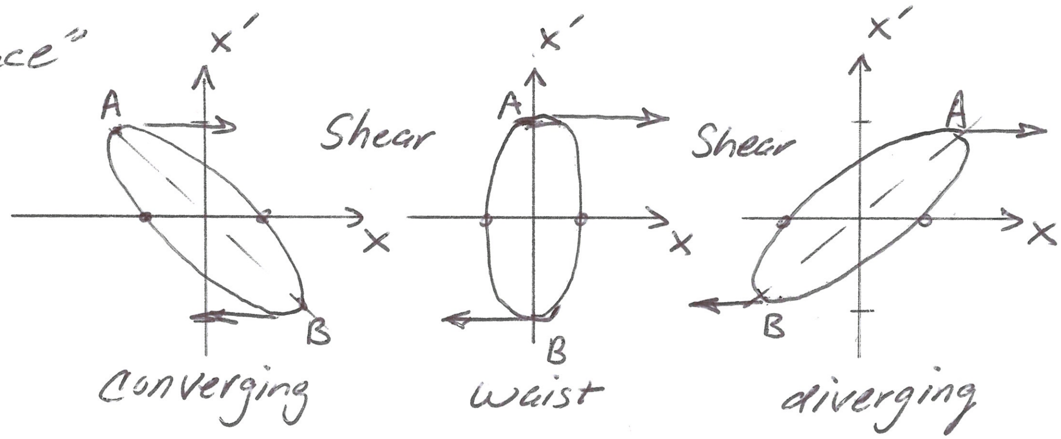
$x_R = \sqrt{2}\, x_0$

Beam Propagation through a Waist

Free-space drift space

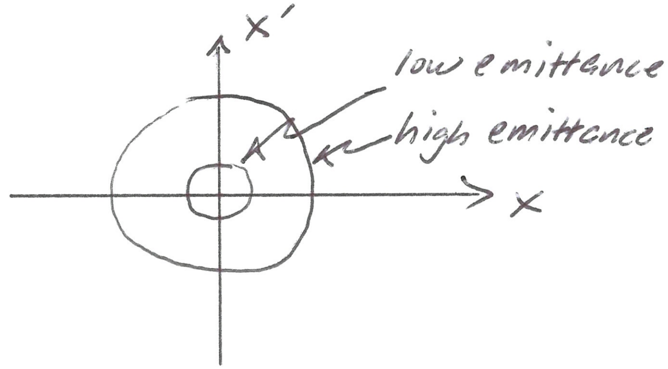


"phase space"



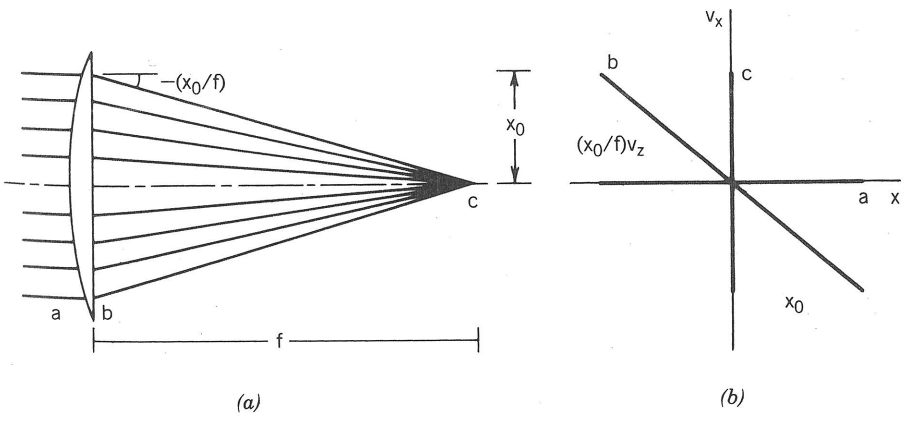
Area in phase-space is <u>emittance</u>

Emittance is a measure of parallism of the beam (laser-like)

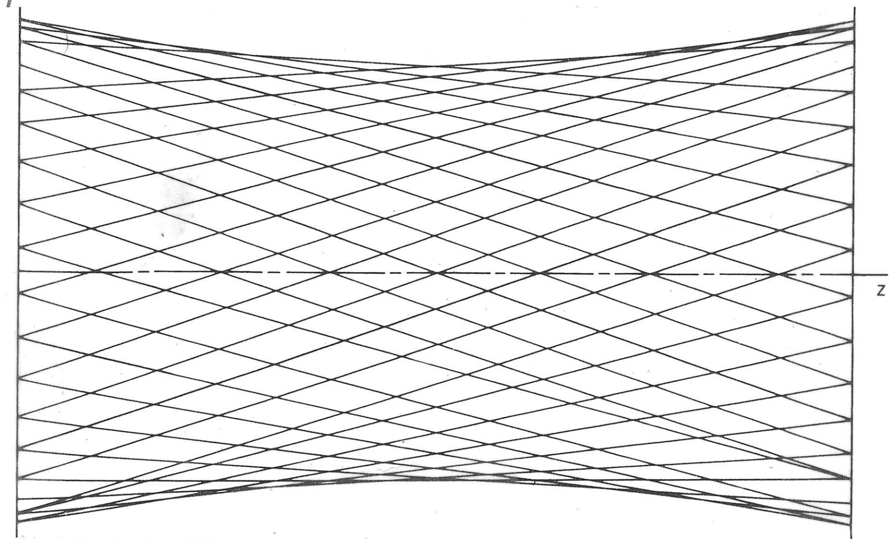


units of meter-radian
(mm-mrad)

Beam Focusing



(a) (b)



Laminar-flow beams have

- no cross-over rays
- low emittance
- tight focus

Non-laminar beams have

- Cross-over (angular dispersion)
- finite emittance
- larger focus

- spread in transverse velocity limits focal spot size
- width of focal spot proportional to transverse velocity spread $\sigma_{x'}$

Humphries

Emittance and Brightness

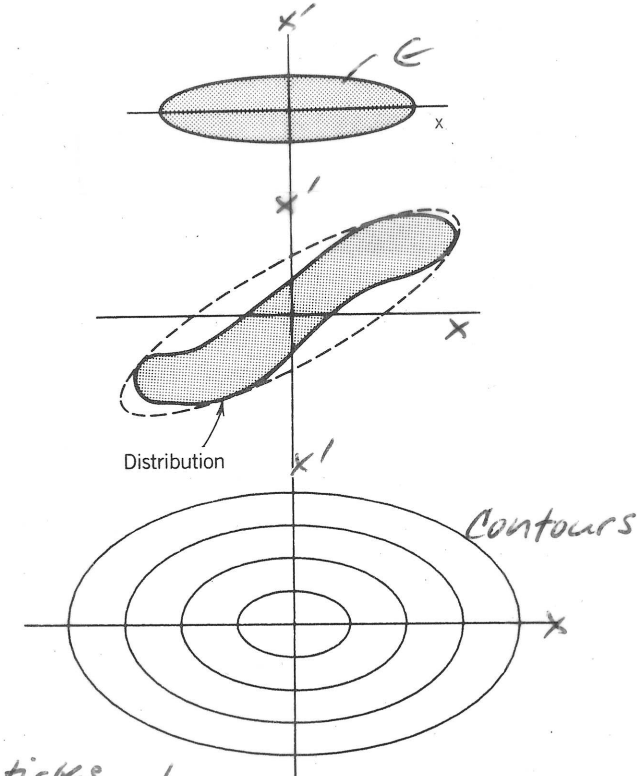
Define an ellipse in phase-space

$$\frac{x^2}{x_0^2} + \frac{x'^2}{x_0'^2} = 1$$

$$\frac{x^2}{x_0^2} + \frac{2xx'}{x_0 x_0'} + \frac{x'^2}{x_0'^2} = 1$$

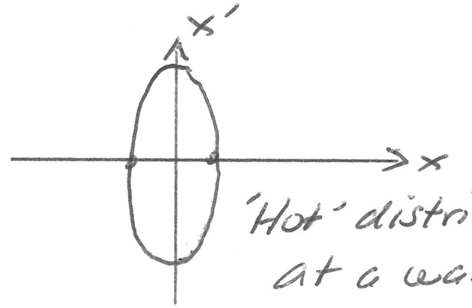
Cross-correlation

$$f(x,x') = A e^{-\frac{\gamma x^2 + 2\alpha x x' + \beta x'^2}{\epsilon}}$$

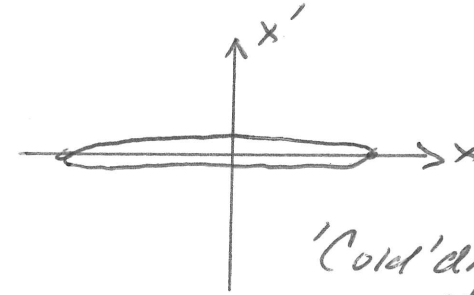


$$\text{Brightness} \sim \frac{\text{particles}}{\text{sec}} \cdot \frac{1}{\sigma_x} \cdot \frac{1}{\sigma_{x'}} \sim \frac{\text{particles}}{\text{sec}} \cdot \frac{1}{\epsilon}$$

"Bright beams have good parallelism"
(low angular dispersion)



'Hot' distribution at a waist



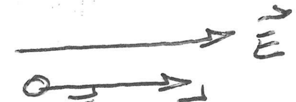
'Cold' distribution at a waist

Humphries

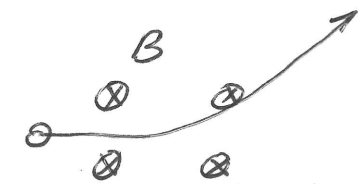
Review I

- Charged particles subject to Lorentz Forces

$$F = q(\vec{E} + \vec{V} \times \vec{B})$$



$\vec{F} = q\vec{E}$

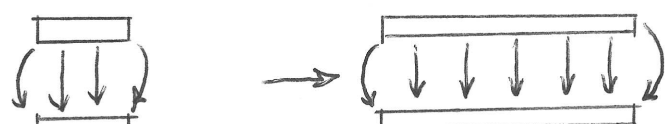


$\vec{F} = q\vec{V} \times \vec{B}$

↑ sign of charge

- Electromagnets/permanent Magnets

Bend, focus, correct aberrations, <u>create</u> aberrations



End View　　　　Side View

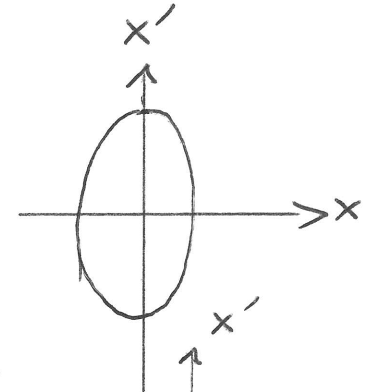
- Quadrupoles focus in one dimension, de-focus in other dimension

Linear Optics ⟷ Linear Algebra
(matrix multiplication)

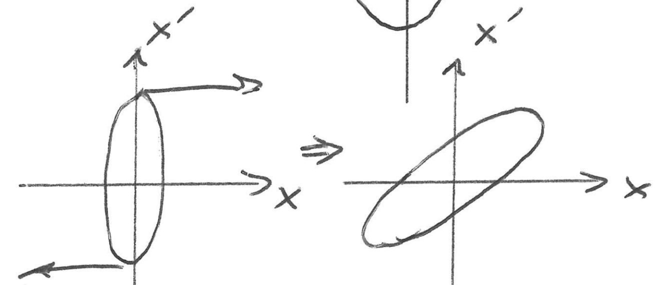
Review II

- Beams consist of $10^6 - 10^{13}$ particles

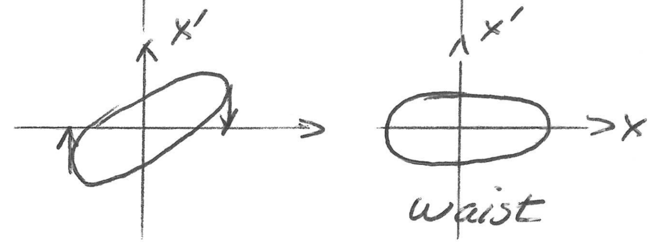
- Distribution in position/angle plotted in phase-space



- Free-space propagation shears distribution

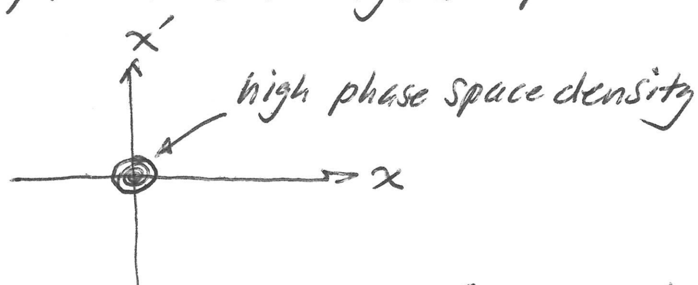


- Focusing shears distribution



- Bright beams: small size, minimum angular spread

$$B \sim \frac{I}{\sigma_x \sigma_{x'}}$$



- Bi-Gaussian distribution $\quad f(x,x') = Ae^{-\frac{\gamma x^2 + 2\alpha x x' + \beta x'^2}{\epsilon}}$

6-0!