Understanding Signals from Beams

Contributed to the ACAS 2016 School

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Outline

Common Control Room Measurements

- Particle Currents
  - Total
  - Current distribution (bunch by bunch currents)
- Orbits
- Tunes
  - Betatron
  - Synchrotron
- Bunch Profile
  - Transverse
  - Longitudinal
- Bunch Motion, Signatures of Instabilities
  - Single-bunch
  - Multi-bunch
  - Intra-bunch

Time and Frequency Domains

Fourier transforms

A function f(x) may be Fourier transformed into a function F(s),

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xs} dx \tag{1}$$

and likewise a function F(s) can be transformed into a function f(x)

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{i2\pi xs} ds \tag{2}$$

The Laplace transform is related to the Fourier Transform but involves an integral from 0 to infinity

Time and Frequency Domains

Discrete Fourier Transform

For systems involving discrete samples of data, such as from sampling circuits or from samples taken from circulating bunches, the discrete-time Fourier transform is similar

$$F(\nu) = \frac{1}{N} \sum_{\tau=0}^{N-1} f(\tau) e^{-i2\pi(\nu/N)\tau} \tag{3}$$

$$f(\tau) = \sum_{\nu=0}^{N-1} F(\nu) e^{i2\pi(\nu/N)\tau} \tag{4}$$

There is a related transform, the Z transform, which is the discrete-time equivalent of the Laplace transform

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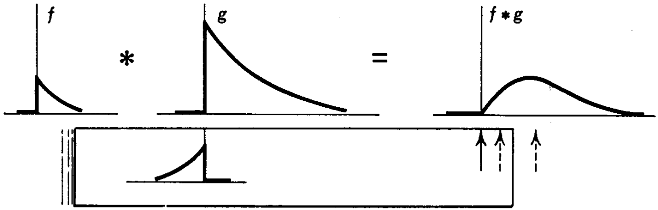
Time and Frequency Domains

Convolution of two functions

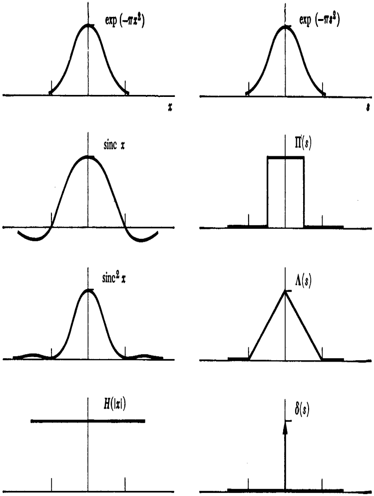
The convolution of two functions f(x) and g(x) is defined as $f(x) \star g(x)$

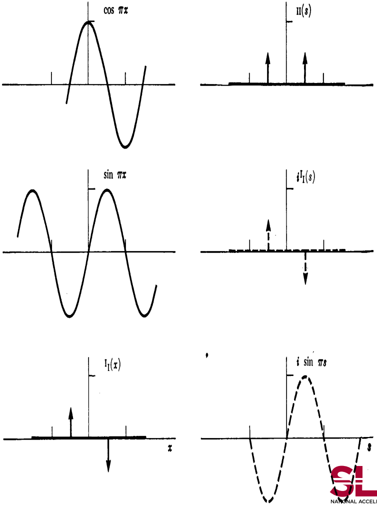
$$f(x) \star g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du \qquad (5)$$

In pictorial form



Common Transform Pairs (from Bracewell)





Linear Time Invariant Systems

If a system converts an input $u(t)$ into an output $y(t)$

$$y(t) = L\left[u(t)\right] \tag{6}$$

the system is linear if for two constants a1 and a2

$$L\left[a_1 u_1 + a_2 u_2\right] = a_1 L\left[u_1(t)\right] + a_2 L\left[u_2(t)\right]. \tag{7}$$

The response of two inputs is the superposition of the individual outputs. If an input is only a single frequency ω, the output can only contain that single frequency ω.

Linear Time Invariant Systems

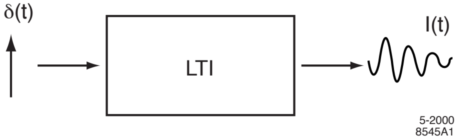
A system is time invariant if for a time delay δ the output has shift invariance, or that

$$L\left[u(t)\right] = y(t) \tag{8}$$

$$L\left[u(t-\delta)\right] = y(t-\delta) \tag{9}$$

Impulse response of LTI system

The impulse response $I(t)$ of a system is found by exciting the system with a δ-function in the time domain.

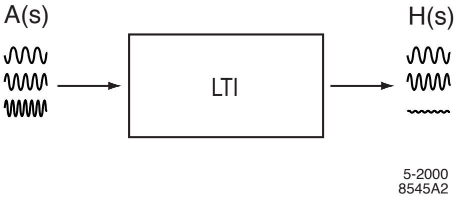


for a general input $u(t)$ the output is a convolution

$$y(t) = u(t) \star I(t) \tag{10}$$

Frequency Response of LTI system

Frequency response $H(s)$ is the transfer function in the frequency domain. Measured by network analyzer via magnitude and phase vs. frequency.



For a general input in the frequency domain $I(s)$ the output $O(s)$ is the product

$$O(s) = H(s)I(s) \tag{11}$$

Frequency Response and Time Response relationship

The time response is also the inverse transform of the product of the Fourier transform of the input $u(t)$ and the frequency response $H(s)$

$$y(t) = u(t) * I(t) \tag{12}$$

$$y(t) = IFT\left[FT(u(t))H(s)\right] \tag{13}$$

For an LTI system, we can measure in either domain, and compute the response via appropriate convolutions, transforms or inverse transforms

The sampling function Ш

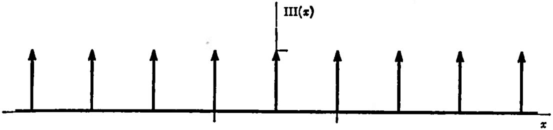


Fig. 5.4 The shah symbol Ш(*x*).

$$Ш(x) = \sum_{n=-\infty}^{n=\infty} \delta(x-n)$$

The Ш is its own Fourier Transform

$$Ш(S) = \sum_{n=-\infty}^{n=\infty} \delta(S-n)$$

For a sampling rate τ

$$Ш(S) = 1/\tau \sum_{n=-\infty}^{n=\infty} \delta(S-n/\tau)$$

The sampling function Ш multiplied by a waveform

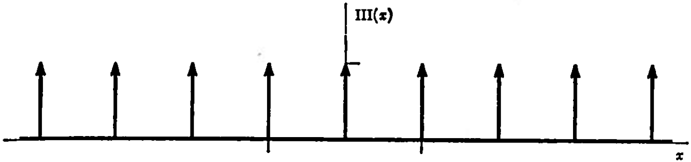


Fig. 5.4 The shah symbol Ш(x).

$$Ш(x) = \sum_{n=-\infty}^{n=\infty} \delta(x-n)$$

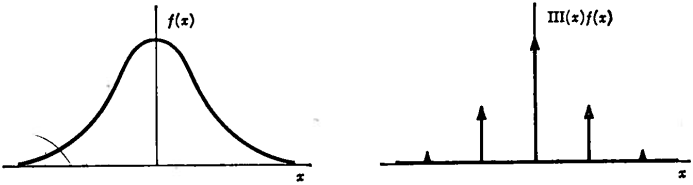


Fig. 5.5 The sampling property of Ш(x).

The sampling function Ш convolved with a spectrum (replicating property)

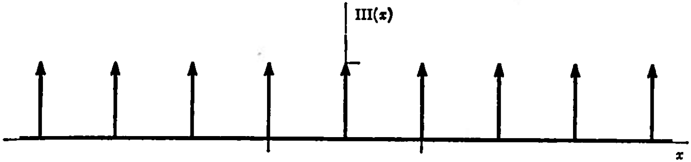


Fig. 5.4 *The shah symbol* Ш(x).

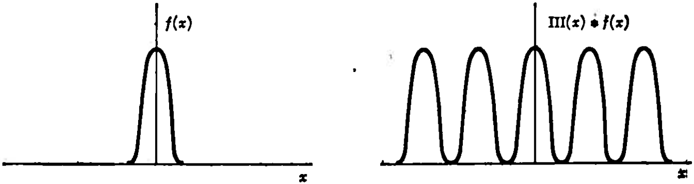
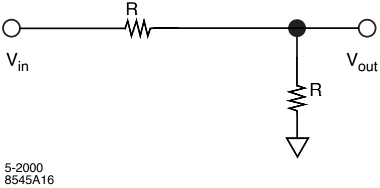


Fig. 5.6 *The replicating property of* Ш(x).

A Quiz on LTI Systems

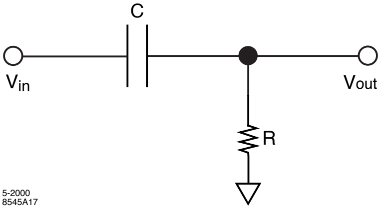
Consider this simple circuit - a voltage divider



Is this an LTI system? Is it ALWAYS an LTI system?

A Quiz on LTI Systems

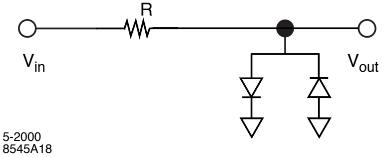
Consider this simple circuit - a high-pass filter



Here the output is frequency dependent. Is this an LTI system?

A Quiz on LTI Systems

Consider this simple circuit - a diode clipper (a limiter)



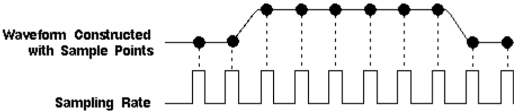
Is this an LTI system? When? What output frequencies are present for an input at ω?. Two signals ω_1 and ω_2? Does it have an Impulse Response $I(t)$?

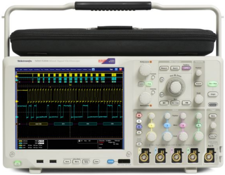
Common Control Room Instrumentation

Most control rooms contain a mix of commercial, general purpose instruments and lab-designed, specialized instruments

Basic Instruments - Time Domain Oscilloscopes

- Real-Time Oscilloscopes
  - Digital or analog
  - Data taken in single continuous triggered sweep
  - Bandwidth to 2 - 8 GHz Common (gets expensive)
  - Resolution (dynamic range) 40 - 50 DB

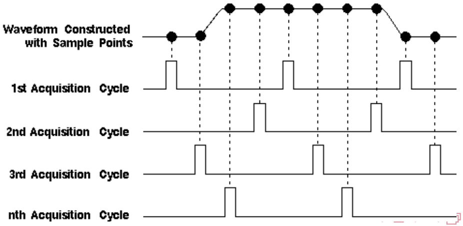




Common Control Room Instrumentation

Higher bandwidths can be achieved by taking several passes through the data

- Equivalent-Time (sampling) Oscilloscopes
  - Digital or analog
  - Data taken over multiple triggered sweeps
  - Bandwidth to 50 + GHz Common (gets expensive)
  - Requires repetitive waveform
  - STABLE trigger (what is risetime on logic signal?)
  - Resolution of sampler, averaging improves SNR
  - Related to boxcar integrator



Common RF Instrumentation - Time Domain reflectometer

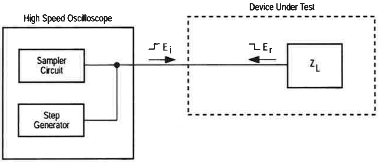


Figure 3. Functional block diagram for a time domain reflectometer

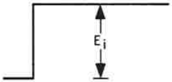
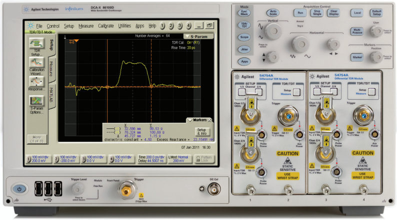


Figure 4. Oscilloscope display when $E_r = 0$



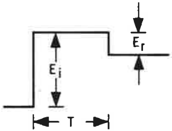


Figure 5. Oscilloscope display when $E_r \neq 0$

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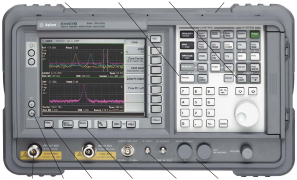
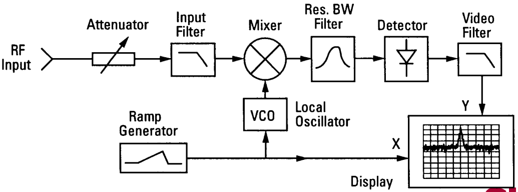
Common Control Room Instrumentation

Spectrum Analyzer and Frequency domain

- Tuned radio Receiver
  - analog heterodyned receiver with multiple IF stages
  - Dynamic Range up to 120dB or more
  - Bandwidth to 50 + GHz Common (gets expensive)
  - Requires periodic waveform
  - Can be set zero span, triggered sweep
  - Intrinsic relationship between resolution bandwidth, sweep time





Common Control Room Instrumentation

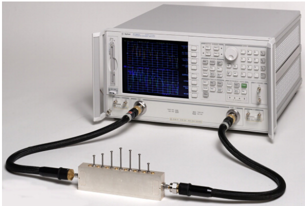
Spectrum Analyzer and Frequency domain

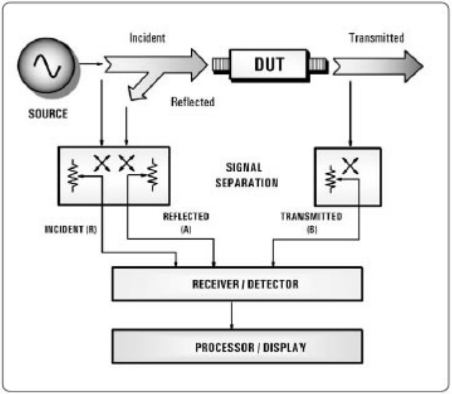
- FFT (Fast Fourier Transform) spectrum analyzers
  - Time domain sampling data acquisition
  - Bandwidth to 10’s MHz Common (gets expensive)
  - FFT Band can be heterodyned from higher band
  - Numeric computation of DFT
  - Intrinsic relationship between resolution, sampling rate, length of sequence bandwidth
  - Make pretty waterfall displays

Common Control Room Instrumentation

Network Analyzer - tool of frequency domain measurement

- Swept excitation, swept complex receiver
- Measures ratio of incident and reflected, incident and transmitted
- S parameter (Scattering Matrix) representation





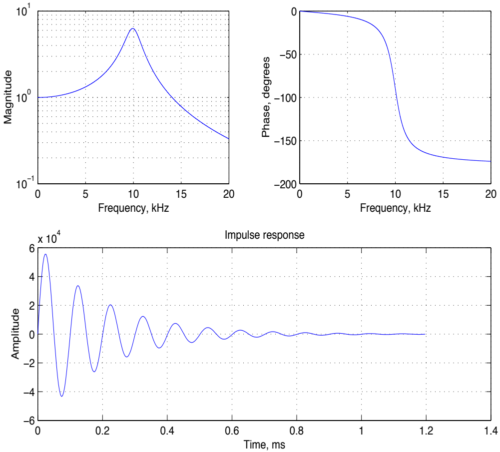
Acknowledgements

It has been a pleasure to think about these issues of Signals from Beams with colleagues from many labs, who have all contributed to this talk in their expertise. I particularly want to thank J-L Pellegrin, F. Pedersen, M. Serio, D. Teytelman, and M. Tobiyama for their contributions to the field and for so many fun hours in the control room doing interesting measurements and so freely sharing their expertise with young scientists.

Harmonic Oscillators, a review

Equation of motion $\ddot{x} + \gamma \dot{x} + \omega_0^2 = f(t)$ where $\omega_0 = \sqrt{\frac{k}{m}}$

Damping term γ proportional to $\dot{x}$



Parallel RLC Circuit

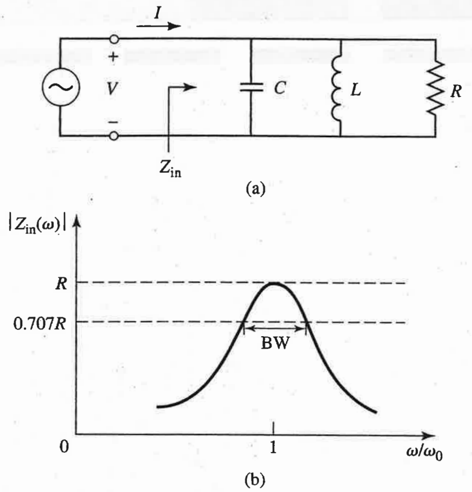
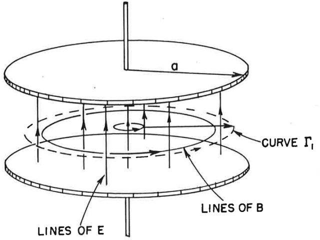
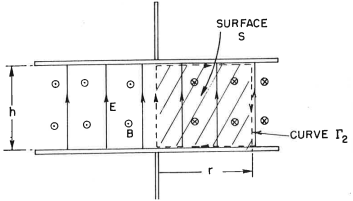


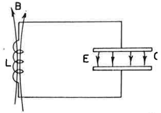
FIGURE 6.2 A parallel RLC resonator and its response. (a) The parallel RLC circuit. (b) The input impedance magnitude versus frequency.

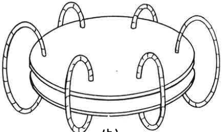
Fields in a capacitor



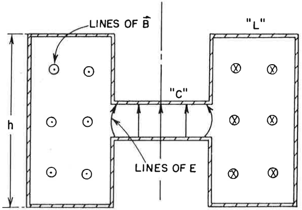


turn an RLC into a cavity

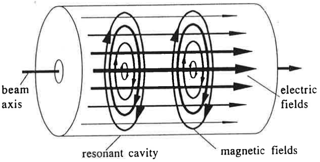




(b)



Use RF cavities in a LINAC



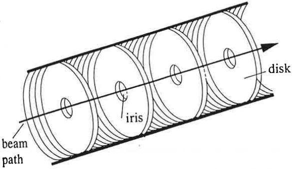


Fig. 2.8. Disk loaded accelerating structure for an electron linear accelerator (schematic)

- A time varying Electric (Magnetic) field
- Why? at resonance build up very large voltage
- Fields oscillate at resonance frequency
- To accelerate - particles must arrive at proper time
- How do you get the RF in?