U. Tietze · Ch. Schenk

# Electronic Circuits
## Design and Applications

With the Assistance of
E. Schmid

With 1168 Figures

Springer-Verlag Berlin Heidelberg New York
London Paris Tokyo Hong Kong Barcelona

# 2 Passive *RC* and *LRC* networks

*RC* networks are of fundamental importance to circuit design. As their effect is the same in all circuits, their operation will be described in some detail.

## 2.1 The lowpass filter

A lowpass filter is a circuit which passes low-frequency signals unchanged and attenuates at high frequencies, introducing a phase lag. Figure 2.1 shows the simplest type of *RC* lowpass filter circuit.

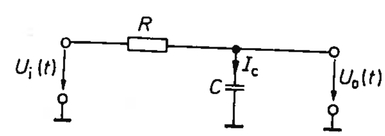



Fig. 2.1 Simple lowpass filter

### 2.1.1 Frequency-domain analysis

To calculate the frequency response of the circuit, we use the voltage divider formula, written in complex notation as:

$$\underline{A}(j\omega) = \frac{\underline{U}_o}{\underline{U}_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} \,. \tag{2.1}$$

Factoring according to

$$\underline{A} = |\underline{A}|e^{j\varphi}$$

we obtain the frequency response of the absolute value or magnitude and of the phase shift:

$$|\underline{A}| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \,, \qquad \varphi = -\arctan \omega RC \,. \tag{2.2}$$

The two curves are shown in Fig. 2.2.

To calculate the 3 dB cutoff frequency $f_c$, we substitute

$$|\underline{A}| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}}$$

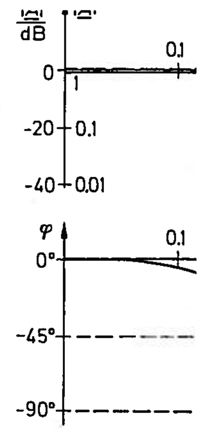



Fig. 2.:

into Eq. (2.2), which gives

From Eq. (2.2), the phase shif

As we can see from Fig. 2.

can be easily constructed usir

1) At low frequencies $f \ll$

2) At high frequencies $f \gg$

inversely proportional t

by a factor of 10, the gai

20 dB/decade or 6 dB/(

3) At $f = f_c$, $|\underline{A}| = 1/\sqrt{2}$ ≘

### 2.1.2

In order to analyze the circ

voltage to the input, as shown

apply Kirchhoff's current law

ance with Fig. 2.1

With $I_C = C\dot{U}_o$, we obtain the

$RC\dot{U}_o + U_o =$

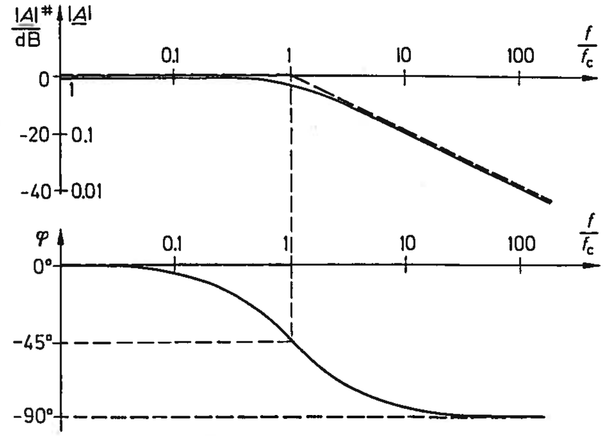



**Fig. 2.2** Bode plot of a lowpass filter

into Eq. (2.2), which gives

$$\boxed{f_c = \frac{1}{2\pi}\,\omega_c = \frac{1}{2\pi RC}}\ . \tag{2.3}$$

From Eq. (2.2), the phase shift at this frequency is $\varphi = -45°$.

As we can see from Fig. 2.2, the amplitude-frequency response $|\underline{A}| = \hat{U}_o/\hat{U}_i$ can be easily constructed using the two asymptotes:

1) At low frequencies $f \ll f_c$, $|\underline{A}| = 1 \mathrel{\hat{=}} 0$ dB.
2) At high frequencies $f \gg f_c$, from Eq. (2.2) $|\underline{A}| \approx 1/\omega RC$, i.e. the gain is inversely proportional to the frequency. When the frequency is increased by a factor of 10, the gain is reduced by the same factor, i.e. it decreases by 20 dB/decade or 6 dB/octave.
3) At $f = f_c$, $|\underline{A}| = 1/\sqrt{2} \mathrel{\hat{=}} -3$ dB.

### 2.1.2 Time-domain analysis

In order to analyze the circuit in the time domain, we apply a step function of voltage to the input, as shown in Fig. 2.3. To calculate the output voltage, we apply Kirchhoff's current law to the (unloaded) output and obtain in accordance with Fig. 2.1

$$\frac{U_i - U_o}{R} - I_C = 0\ .$$

With $I_C = C\dot{U}_o$, we obtain the differential equation

$$RC\dot{U}_o + U_o = U_i = \begin{cases} U_r & \text{for } t > 0 \text{ in Case a}, \\ 0 & \text{for } t > 0 \text{ in Case b}. \end{cases} \tag{2.4}$$

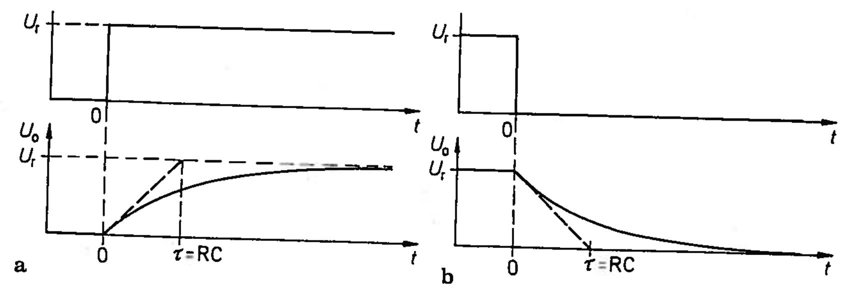



Fig. 2.3 a and b Step-response of a lowpass filter

It has the following solutions:

Case a: Case b:

$$U_o(t) = U_r(1 - e^{-\frac{t}{RC}}) \; ; \qquad U_o(t) = U_r e^{-\frac{t}{RC}} \; . \qquad (2.5)$$

This curve is also plotted in Fig. 2.3. We can see that the steady-state values $U_o = U_r$ or $U_o = 0$ are only attained asymptotically. As a measure of the response time, a *time constant* $\tau$ is therefore defined. This indicates how long it takes for the deviation from the steady-state value to equal $1/e$ times the step

| Response accuracy | 37% | 10% | 1% | 0.1% |
|---|---|---|---|---|
| Response time | $\tau$ | $2.3\,\tau$ | $4.6\,\tau$ | $6.9\,\tau$ |

Fig. 2.4 Response time of a lowpass filter

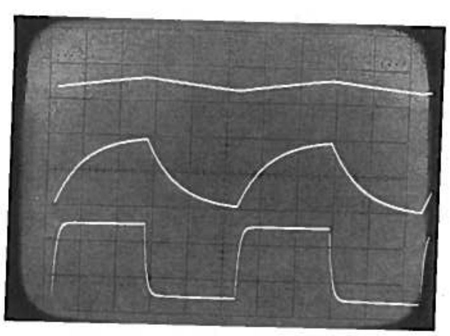


Fig. 2.5 Square-wave response of a lowpass filter for various frequencies

*Upper curve:* $f_i = 10 f_c$
*Middle curve:* $f_i = f_c$
*Lower curve:* $f_i = \frac{1}{10} f_c$

The response time for sma
Figure 2.4 lists a number o
If a square-wave volta
e-function is truncated afte
is obtained at the output de
constant $\tau$. This characteris

*Lowpa*

In the previous section
compared with the input
lowpass filter operates the
inferred directly from differ
that

*Lowpa*

For unsymmetrical alte
satisfied. The Fourier expa
the *arithmetic mean*

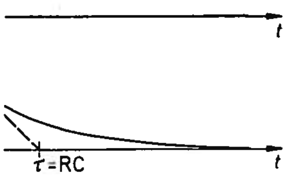
where $T$ is the period of th
Fourier series are combine
corresponds to that of the i
that its arithmetic mean is z
the form

For voltage $U_i'(t)$, the conc
the DC component is trans

$U_o$

If the time constant $\tau = R$

·ks




wpass filter

Case b:

$t) = U_r e^{-\frac{t}{RC}}$ . (2.5)

that the steady-state values
cally. As a measure of the
d. This indicates how long it
e to equal 1/e times the step

| | |
|---|---|
| % | 0.1% |
| ·.6 τ | 6.9 τ |

ıss filter

for various frequencies



magnitude. From Eq. (2.5) the time constant is

$$\boxed{\tau = RC} \ . \tag{2.6}$$

The response time for smaller deviations can also be derived from Eq. (2.5). Figure 2.4 lists a number of important parameters.

If a square-wave voltage of period $T$ is applied as the input signal, the e-function is truncated after time $T/2$ by the subsequent step. Which final value is obtained at the output depends on the ratio between the time $T/2$ and the time constant $\tau$. This characteristic is clearly illustrated by the oscillogram in Fig. 2.5.

*Lowpass filter as an integrating circuit*

In the previous section we saw that the alternating output voltage is small compared with the input voltage if a signal frequency $f \gg f_c$ is selected. The lowpass filter operates then as an integrating circuit. This property can be inferred directly from differential Eq. (2.4). Assuming that $|U_o| \ll |U_i|$, it follows that

$$RC\dot{U}_o = U_i \ ,$$

$$U_o = \frac{1}{RC}\int_0^t U_i(\tilde{t})\,d\tilde{t} + U_o(0) \ .$$

*Lowpass filter as an averaging circuit*

For unsymmetrical alternating voltages, the above condition $f \gg f_c$ is not satisfied. The Fourier expansion in fact contains a constant which is identical to the *arithmetic mean*

$$\boxed{\bar{U}_i = \frac{1}{T}\int_0^T U_i(t)\,dt}$$

where $T$ is the period of the input voltage. If all the higher-order terms of the Fourier series are combined, a voltage $U_i'(t)$ is obtained whose characteristic corresponds to that of the input voltage, but which is displaced from zero such that its arithmetic mean is zero. The input voltage may therefore be expressed in the form

$$U_i(t) = \bar{U}_i + U_i'(t) \ .$$

For voltage $U_i'(t)$, the condition $f \gg f_c$ can be satisfied; it is integrated, whereas the DC component is transferred linearly. The output voltage therefore becomes

$$U_o = \underbrace{\frac{1}{RC}\int_0^t U_i'(\tilde{t})\,d\tilde{t}}_{\text{residual ripple}} + \underbrace{\bar{U}_i}_{\text{mean value}} \ . \tag{2.7}$$

If the time constant $\tau = RC$ is made sufficiently large, the ripple is insignificant

$$U_o \approx \bar{U}_i \ . \tag{2.8}$$

### 2.1.3 Rise time and cutoff frequency

Another parameter for characterizing lowpass filters is the rise time $t_r$. This denotes the time taken for the output voltage to rise from 10 to 90% of the final value when a step is applied to the input. From the e-function in Eq. (2.5) we obtain

$$t_r = t_{90\%} - t_{10\%} = \tau(\ln 0.9 - \ln 0.1) = \tau \ln 9 \approx 2.2\tau \ .$$

Consequently, with $f_c = 1/2\pi\tau$

$$\boxed{t_r \approx \frac{1}{3f_c}} \ . \tag{2.9}$$

In approximation this relation is also true for higher-order lowpass filters.

If a number of lowpass filters with various rise times $t_{ri}$ are connected in series, the resultant rise time is

$$t_r \approx \sqrt{\sum_i t_{ri}^2} \ , \tag{2.10}$$

and the cutoff frequency is

$$f_c \approx \left(\sum_i f_{ci}^{-2}\right)^{-\frac{1}{2}} \ .$$

Hence, for $n$ lowpass filters having the same cutoff frequency

$$\boxed{f_c \approx \frac{f_{ci}}{\sqrt{n}}} \ . \tag{2.11}$$

## 2.2 The highpass filter

A highpass filter is a circuit which passes high-frequency signals unchanged and attenuates at low frequencies, introducing a phase lead. Figure 2.6 shows the simplest form of $RC$ highpass filter circuit. The frequency response of the gain and phase shift is again obtained from the voltage divider formula:

$$\underline{A}(j\omega) = \frac{\underline{U}_o}{\underline{U}_i} = \frac{R}{R + 1/j\omega C} = \frac{1}{1 + 1/j\omega RC} \ . \tag{2.12}$$

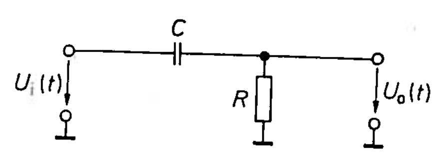



Fig. 2.6 Simple highpass filter

This yields

$$|\underline{A}| = \frac{1}{\sqrt{1 + 1/\omega^2 R^2 C^2}} \quad \text{and} \quad \varphi = \arctan \frac{1}{\omega RC} \; . \tag{2.13}$$

The two curves are shown in Fig. 2.7. For the cutoff frequency, we obtain as with the lowpass filter:

$$f_c = \frac{1}{2\pi RC} \; . \tag{2.14}$$

At this frequency the phase shift is $+45°$.

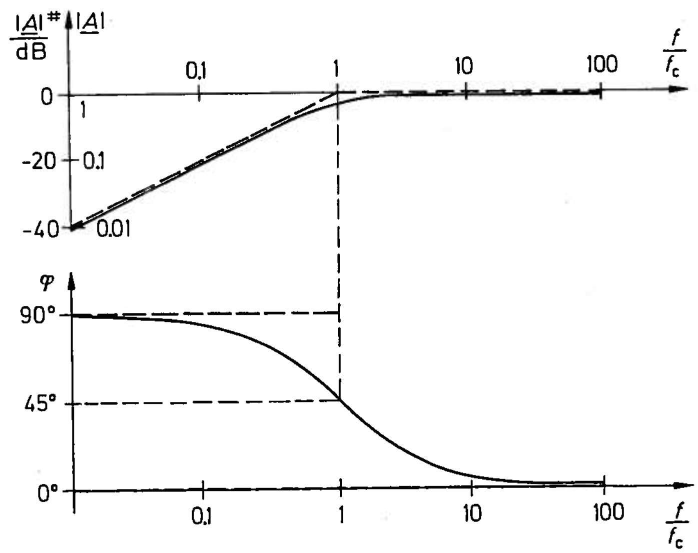



Fig. 2.7 Bode plot of a highpass filter

As in the case of the lowpass filter, the amplitude-frequency response can be easily plotted on a double-logarithmic scale using the asymptotes:

1) At high frequencies $f \gg f_c$, $|\underline{A}| = 1 \triangleq 0$ dB.
2) At low frequencies $f \ll f_c$, from Eq. (2.13) $|\underline{A}| \approx \omega RC$, i.e. the gain is proportional to the frequency. The slope of the asymptote is therefore +20 dB/decade or +6 dB/octave.
3) For $f = f_c$, $|\underline{A}| = 1/\sqrt{2} \triangleq -3$ dB, as with the lowpass filter.

To calculate the step response, we apply Kirchhoff's current law to the (unloaded) output:

$$C \cdot \frac{d}{dt}(U_i - U_o) - \frac{U_o}{R} = 0 \; . \tag{2.15}$$

With $\dot{U}_i = 0$, this yields the differential equation

$$RC\dot{U}_o + U_o = 0 \tag{2.16}$$

$$U_o(t) = U_{oo} e^{-\frac{t}{RC}} \ . \tag{2.17}$$

The time constant is therefore $\tau = RC$, as in the case of the lowpass filter.

In order to determine the initial value $U_{oo} = U_o(t = 0)$, we have to consider that at the instant when the input voltage changes abruptly, the capacitor charge remains unchanged. The capacitor therefore acts as a voltage source of value $U = Q/C$. The output voltage accordingly shows the same step $\Delta U$ as the input voltage. If $U_i$ goes from zero to $U_r$, the output voltage likewise jumps from zero to $U_r$ (see Fig. 2.8a) then decays exponentially to zero again in accordance with Eq. (2.17).

If the input voltage now goes abruptly from $U_r$ to zero, $U_o$ jumps from zero to $-U_r$ (see Fig. 2.8b). Note that the output voltage assumes negative values even though the input voltage is always positive. This distinctive characteristic is frequently used in circuit design.

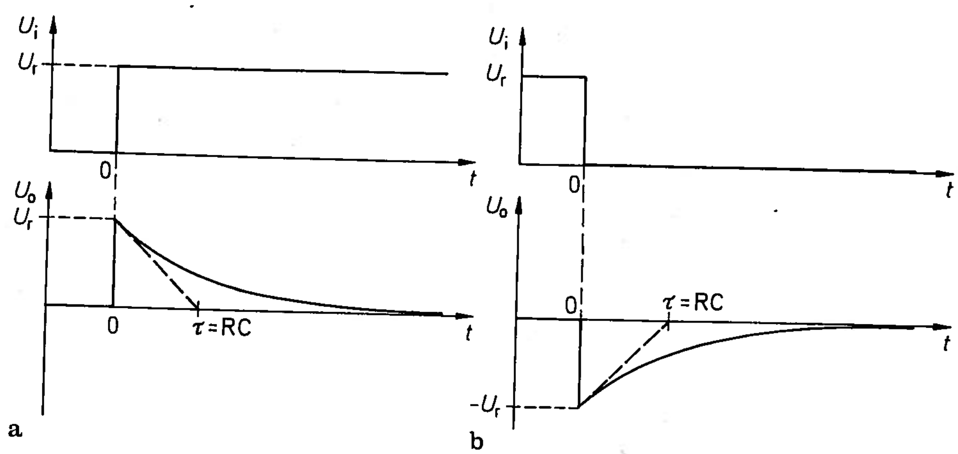



Fig. 2.8 a and b Step response of a highpass filter

### *Use as an RC coupling network*

If a square-wave voltage periodic in $T \ll \tau$ is applied to the input, the capacitor charge barely changes during one half-cycle; the output voltage is identical to the input voltage apart from an additive constant. As no direct current can flow via the capacitor, the arithmetic mean of the output voltage is zero. No DC component of the input voltage is therefore transferred. It is this property which enables a highpass filter to be used as an $RC$ coupling network.

### *Use as a differentiating circuit*

If input voltages with frequencies $f \ll f_c$ are applied, $|\underline{U}_o| \ll |\underline{U}_i|$. Consequently, from differential Eq. (2.15)

$$U_o = RC \frac{dU_i}{dt} \ .$$

Low-frequency input voltages are therefore differentiated.

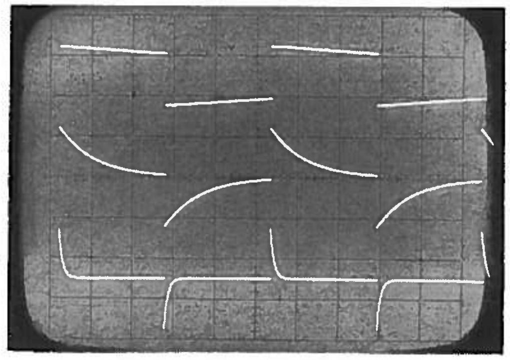


Fig. 2.9 Square-wave response of a highpass filter for various frequencies

*Upper curve:* $f_i = 10 f_c$
*Middle curve:* $f_i = f_c$
*Lower curve:* $f_i = \frac{1}{10} f_c$

The oscillograms in Fig. 2.9 summarize the transient response of a highpass filter.

*Series connection of several highpass filters*

If a number of highpass filters are connected in series, the resultant cutoff frequency is

$$f_c \approx \sqrt{\sum_i f_{ci}^2} \; . \tag{2.18}$$

Consequently, for $n$ highpass filters having identical cutoff frequencies

$$\boxed{f_c \approx f_{ci} \cdot \sqrt{n}} \; . \tag{2.19}$$

## 2.3 Compensated voltage divider

It is frequently the case that a resistive voltage divider is capacitively loaded, making it a lowpass filter. The lower the resistance selected for the voltage divider, the higher the cutoff frequency of the filter. However, limits are imposed in that the input resistance of the divider should not be reduced below a specified value.

Another possible way of raising the cutoff frequency is to use a highpass filter to compensate for the effect of the lowpass filter. This is the purpose of capacitor $C_k$ in Fig. 2.10. It is dimensioned such that the resultant parallel-connected capacitive voltage divider has the same division ratio as the resistive voltage divider. Consequently, the same voltage division is produced at high and low frequencies. This means that

$$\frac{C_k}{C_L} = \frac{R_2}{R_1} \; .$$