Lecture note for
ASLS
accelerator School

SR monitor lecture 1, 2 and 3

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KEK,

Graduated school for advanced study

Lecture 1 Introduction for the beam instrumentation based on Synchrotron Radiation

Lecture 2 Geometrical optics of focusing system

Lecture 3 Wave optics of focusing system

Lecture 4 Coherence theory of light

Lecture 5 Theory of synchrotron radiation

Lecture 6 Interferometry

Lecture 1

Introduction for the
Beam instrumentation based on
Synchrotron Radiation

Classical Optics is disappearing from curriculum of Universities now!

Some geometrical optics can find in high school physics……..

You can buy or find not kind textbooks.

I try to introduce classical optics useful for beam instrumentation based on Synchrotron radiation .

1. **Introduction**

2. **Beam instrumentation based on visible SR**

3. **Popular equipments**

1. Introduction

What is the beam instrumentation based on SR ?

What is the beam instrumentation based on SR ?

Measure the fundamental parameters of Accelerator through the transverse and longitudinal profile, size, etc. statistically, or dynamically using *Optical technique* with the synchrotron radiation.

Light source is normally SR emitted from the bending magnet of accelerator.

Sometimes have dedicated magnet for light source.

Visible SR 400nm-800nm.
X-ray SR 0.05-0.3nm.

VUV region is not used because of difficulty in actual handling.

Based on Visible Synchrotron radiation

1. Transverse

Transverse beam profile or size diagnostics

geometrical optics

Imaging

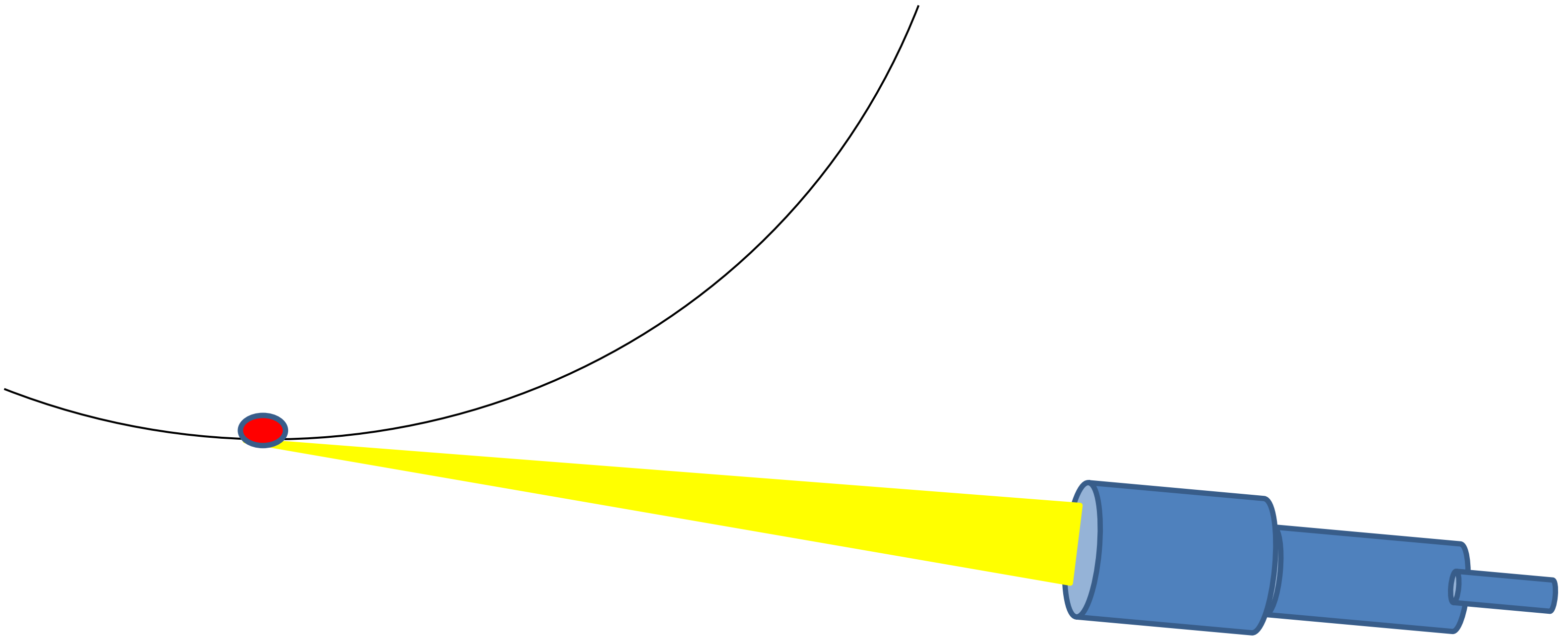
Wave optics

1st and 2ed order spatial coherence

Interferometry

Lecture on tomorrow

Imaging is simply watch the beam in inside of accelerator with a telescope



The coherence theory of light is given in Lecture 4

Transverse beam profile or size diagnostics

light observed as photon

Imaging

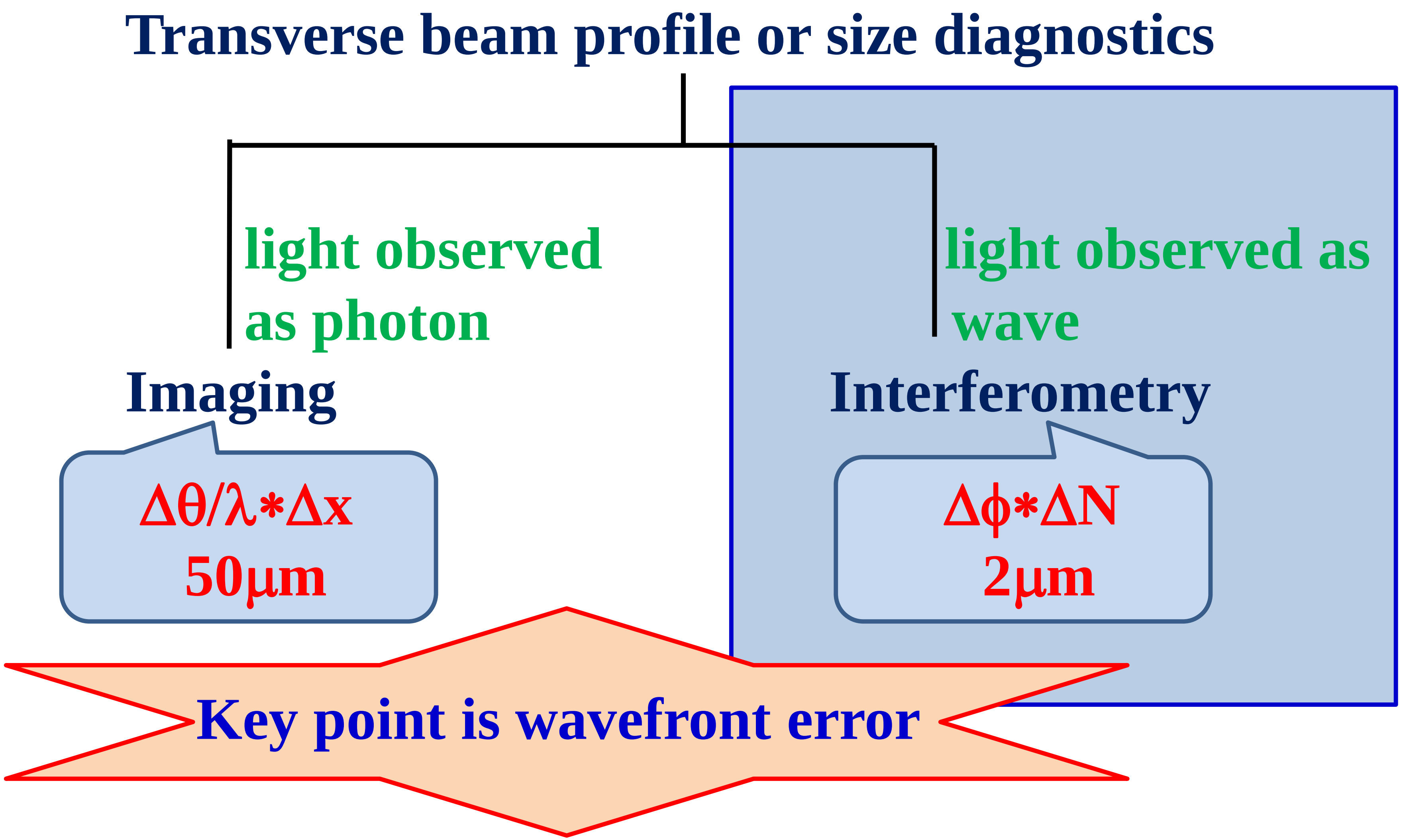
$\Delta\theta/\lambda * \Delta x \geq 1$

light observed as wave

Interferometry

$2\,\Delta\phi * \Delta N \geq 1$

Lecture 6


Transverse beam profile or size diagnostics
light observed as photon
Imaging
Δθ/λ*Δx
50μm
light observed as wave
Interferometry
Δϕ*ΔN
2μm
Key point is wavefront error

Longitudinal

Longitudinal beam profile or size diagnostics

imaging
temporal into spatial

Streak camera

2ed order temporal
coherence
(1st order coherence)

Intensity interferometry
Auto correlation
Cross correlation

Longitudinal beam profile or size diagnostics

imaging

2ed orde

Limited by glass dispersion

Streak

Typical streak camera

2-3ps

200fs

fs streak camera

Intensity interferometry <100fs

Auto correlation

Cross correlation

Influenced by timing jitter between two inputs

Based on X-ray synchrotron radiation for submicron beam size measurement

Transverse beam profile or size diagnostics

Imaging

Pinhole camera

Fresnel zone plate

K-B mirror (Kirkpatrick-Batz mirror)

Interferometry

X-ray interferomter

Transverse beam profile or size diagnostics

Geometrical optics

1^{st} and 2ed order spatial coherence

Imaging

Pinhole camera

Fresnel zone plate

K-B mirror

Few μm

Interferometry

X-ray interferomter

Sub μm

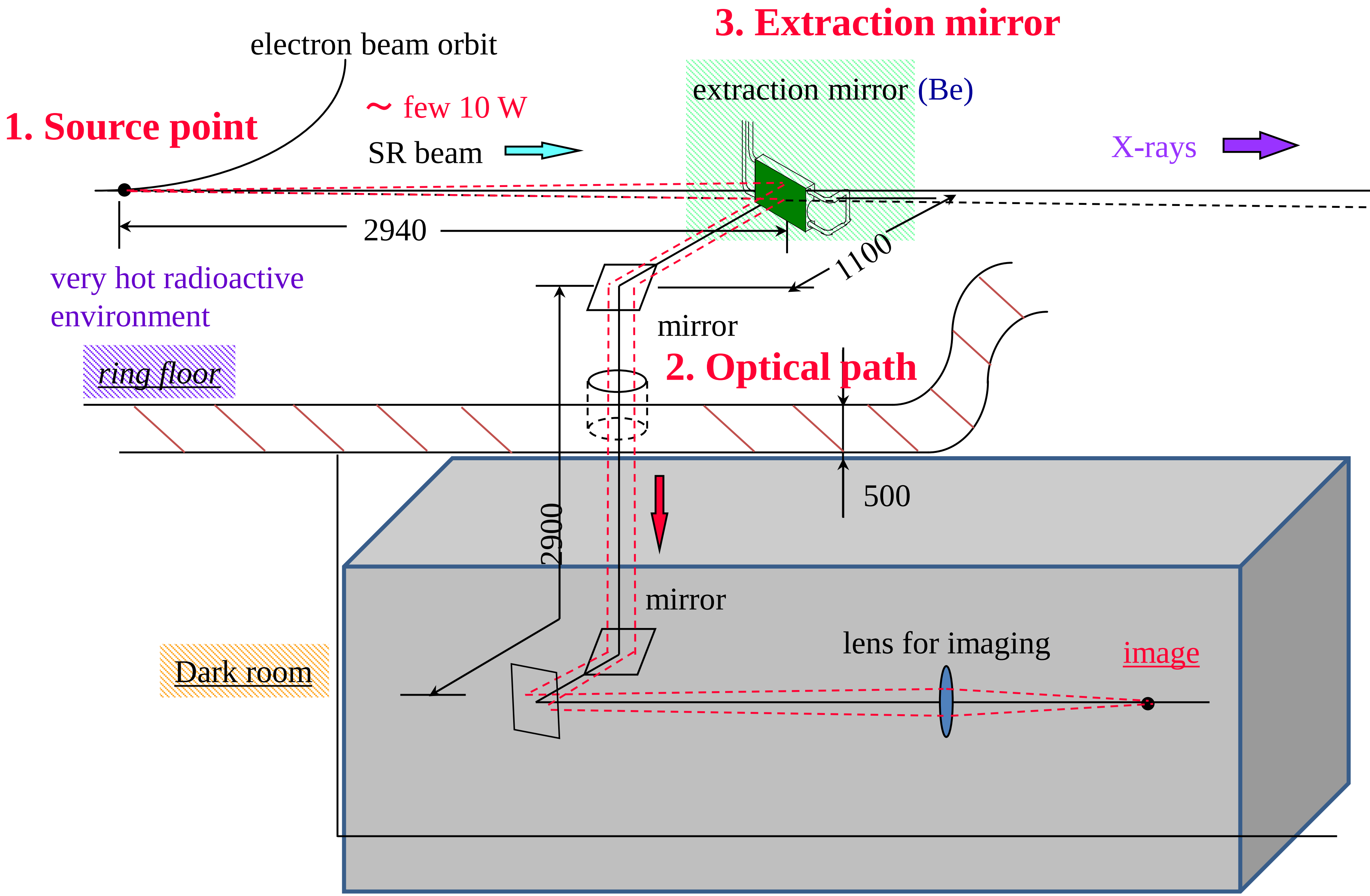
Key point is wavefront error

Transverse beam profile or size diagnostics

Geometrical optics	**1st and 2ed order spatial coherence**
Imaging	**Interferometry**
Fresnel zone plate	**X-ray interferomter**
K-B mirror (Kirkpatrick-Batz mirror)	
Few μm	**Sub μm**

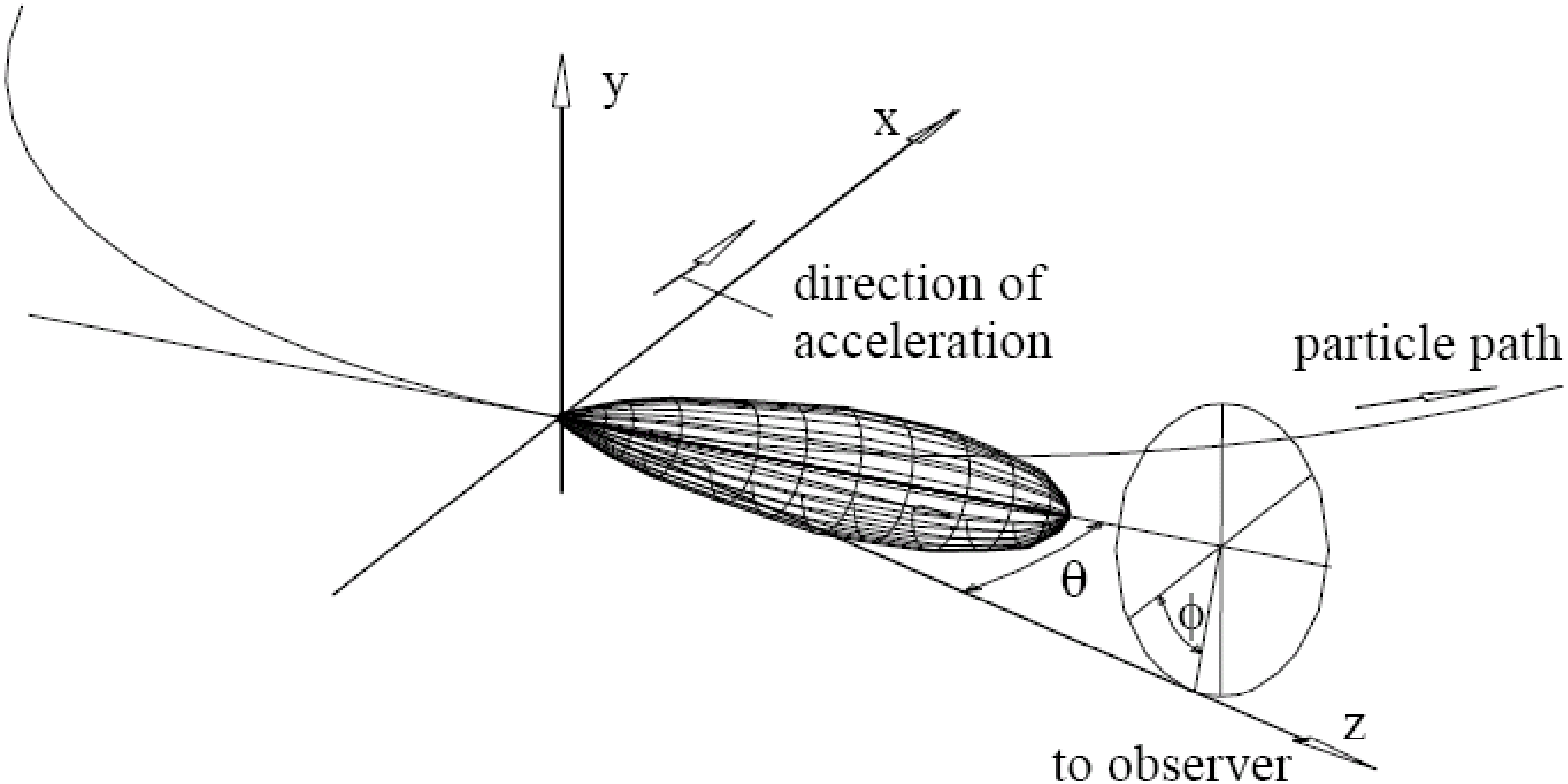
2. SR monitor based on visible SR

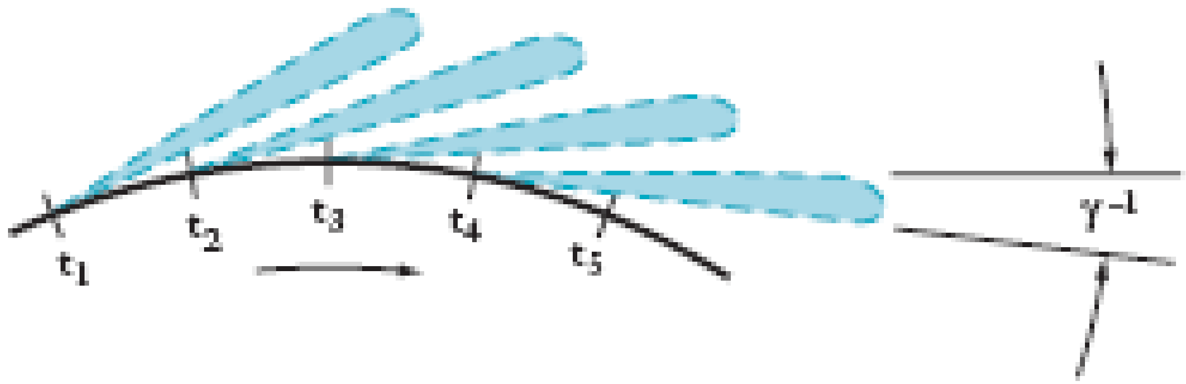
Set up of SR monitor



Synchrotron radiation
Optical path to dark room
Key components and it’s design

Synchrotron radiation from Bending Magnet





bending magnet - a "sweeping searchlight"

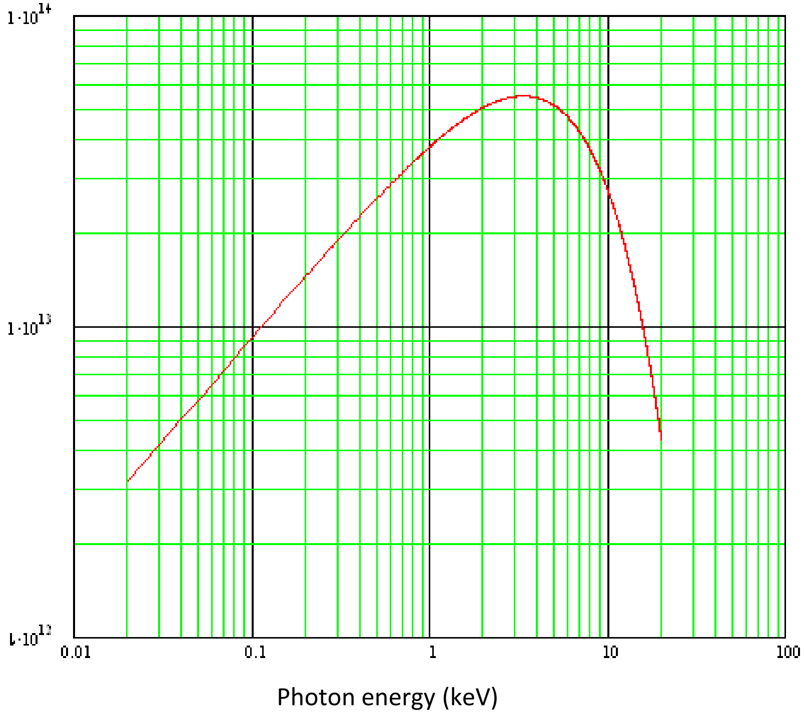
from ensemble of independent electrons

$$L \approx \frac{\rho}{\gamma^3} \quad range\,of\ \mu m$$

$$\tau \approx \frac{\rho}{c\gamma^3} \quad range\,of\ fs$$

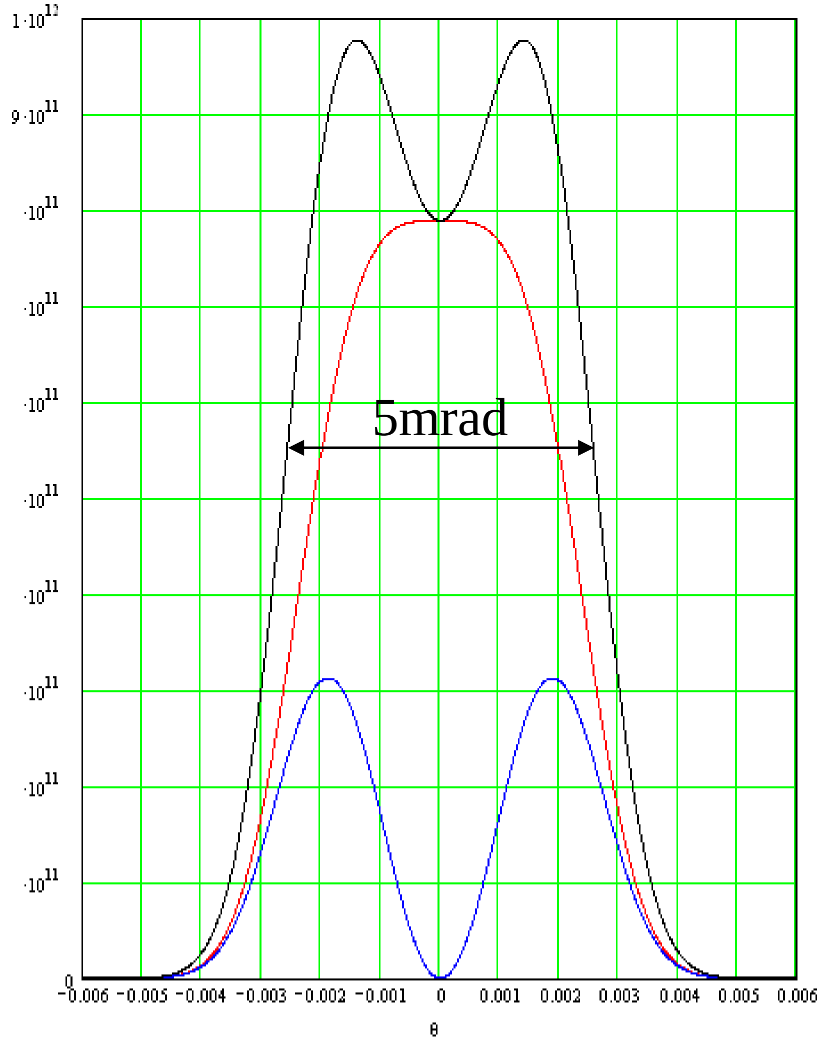
$$\omega_c \approx \frac{c}{L} \approx \left(\frac{c}{L}\right) \cdot \gamma^3 \quad > 10^{12}\ Hz$$

Very wide range in spectrum



Visible light to X-ray

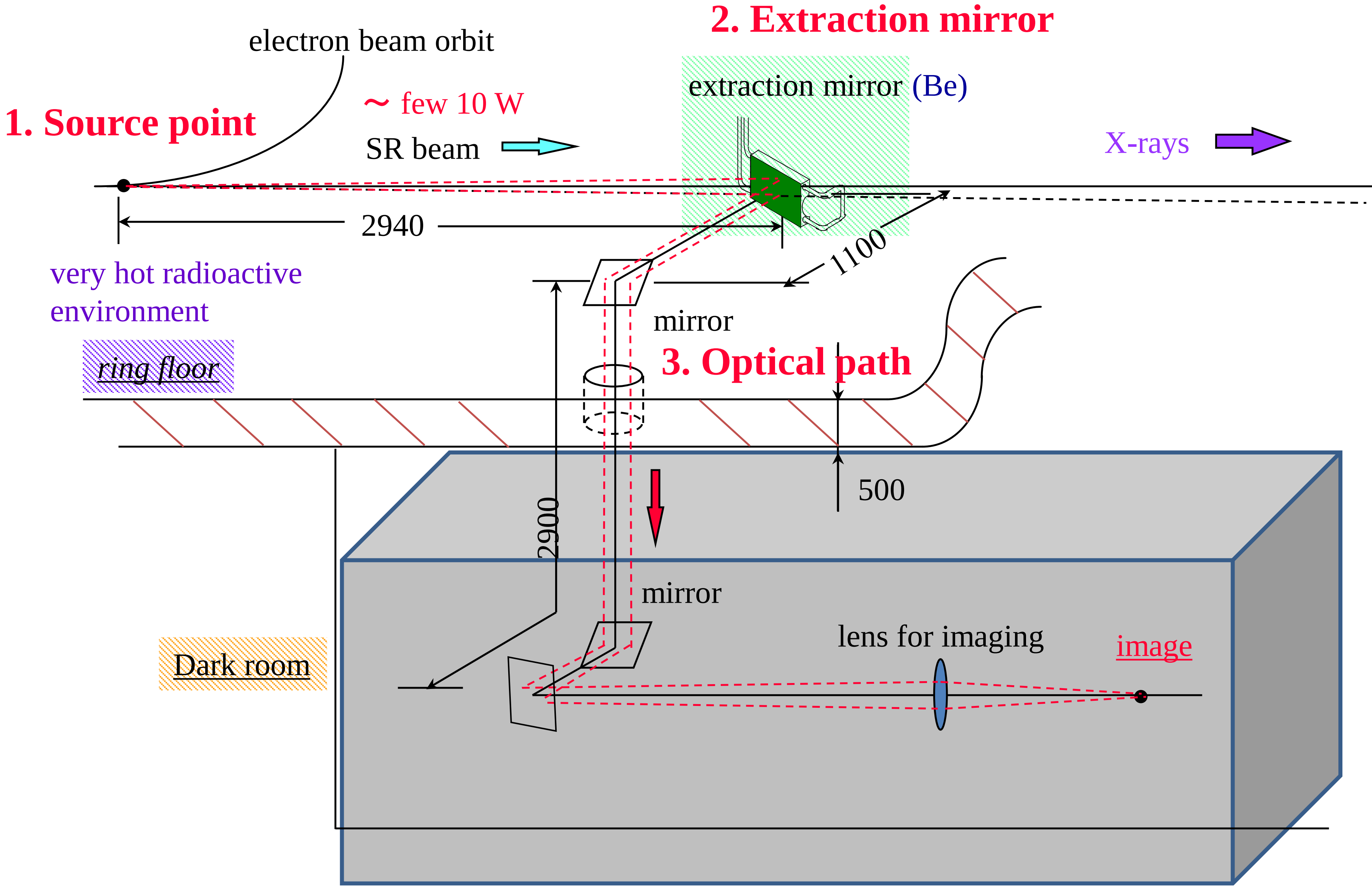
Narrow opening angle



Detail of the synchrotron radiation is in Lecture 5

Components in optical beamline

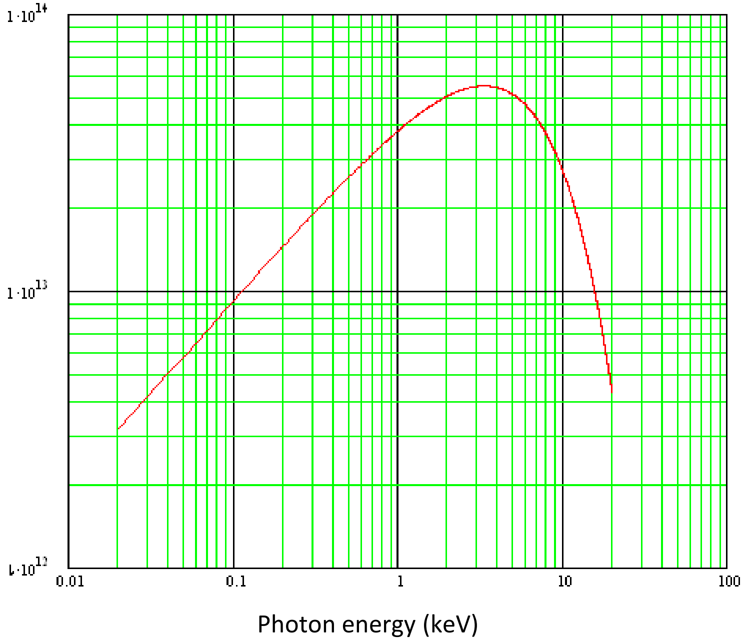
Set up of SR monitor

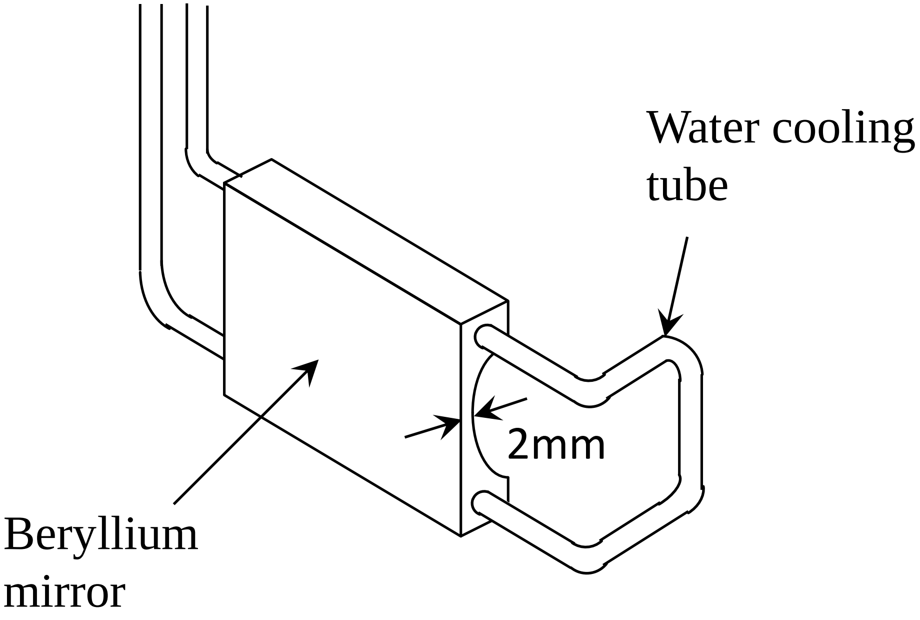


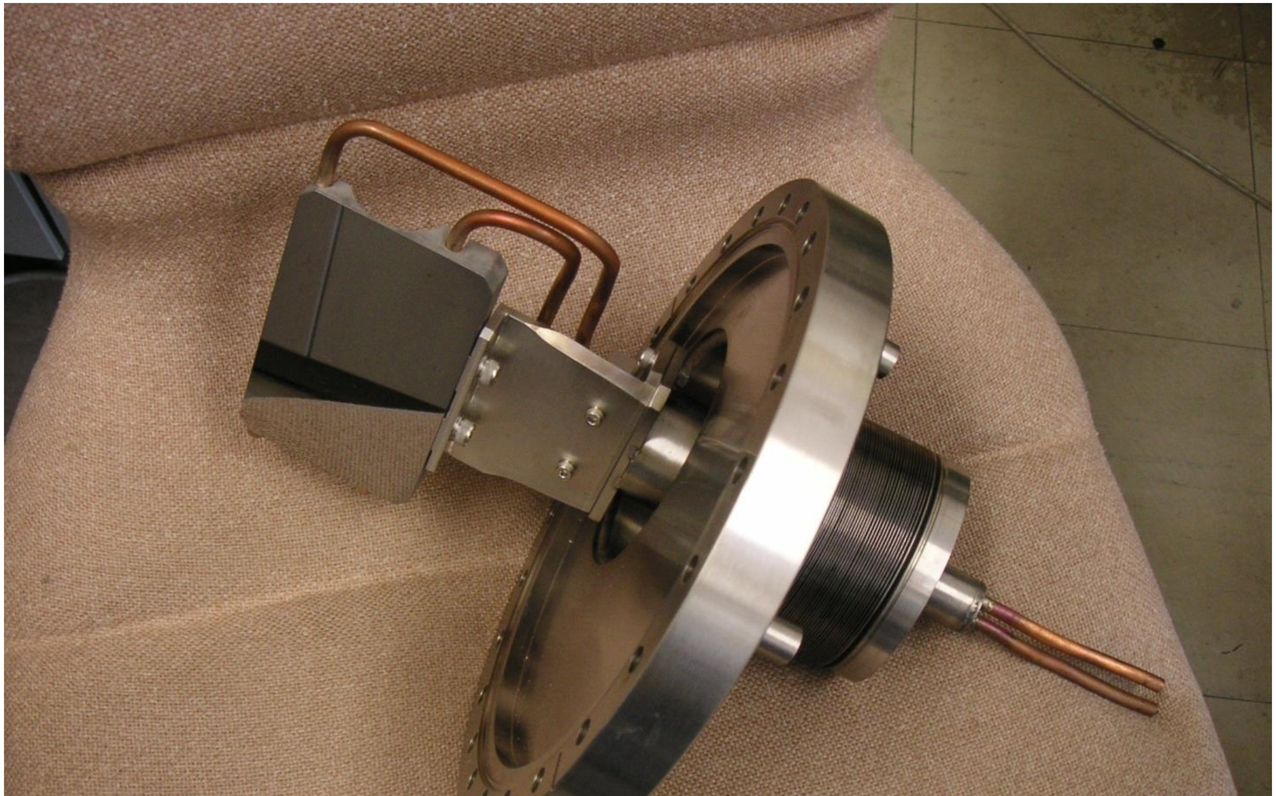
1. Extraction mirror

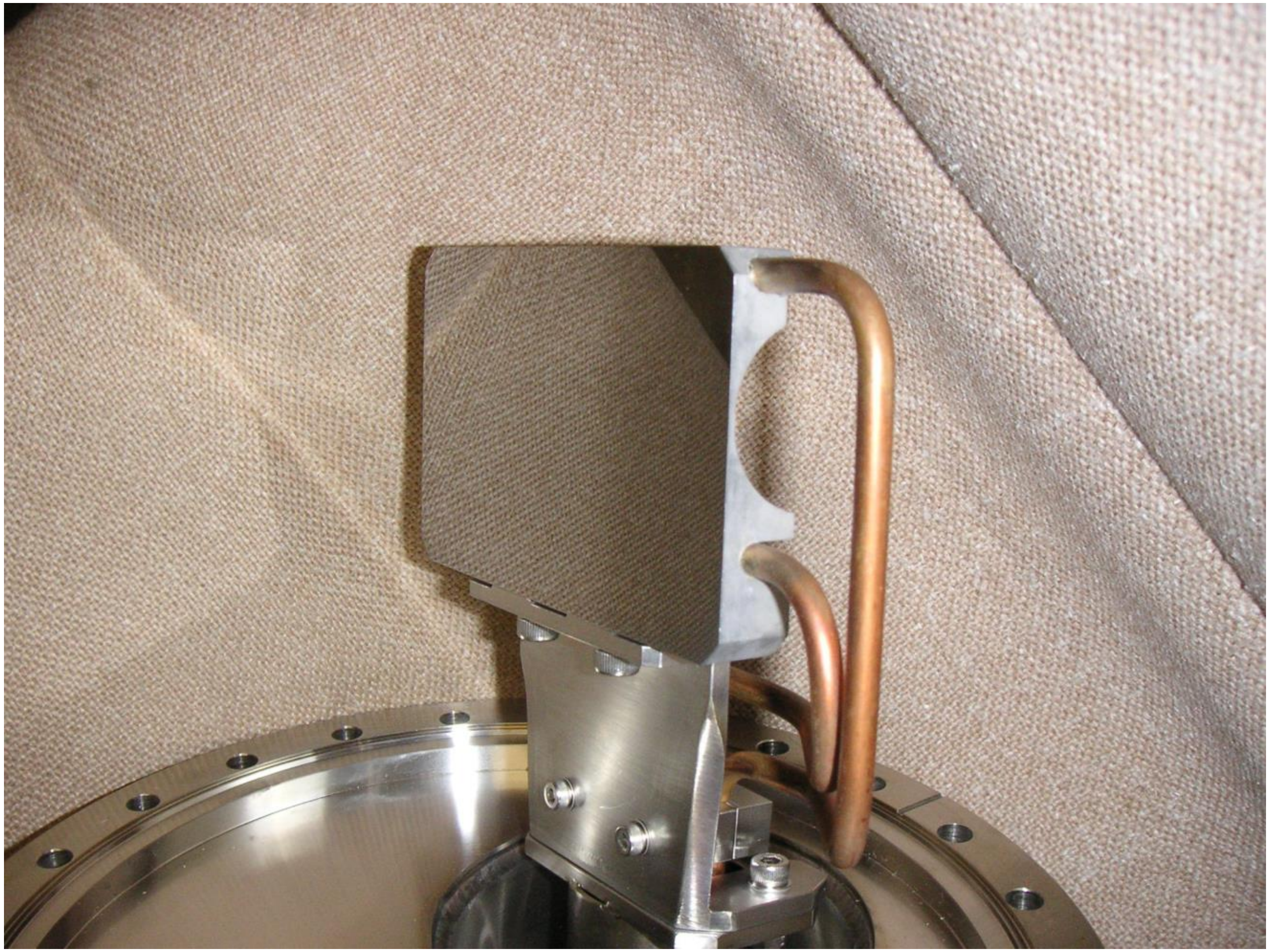
Beryllium extraction mirror

Photon Factory E=2.5GeV, ρ=8.66m

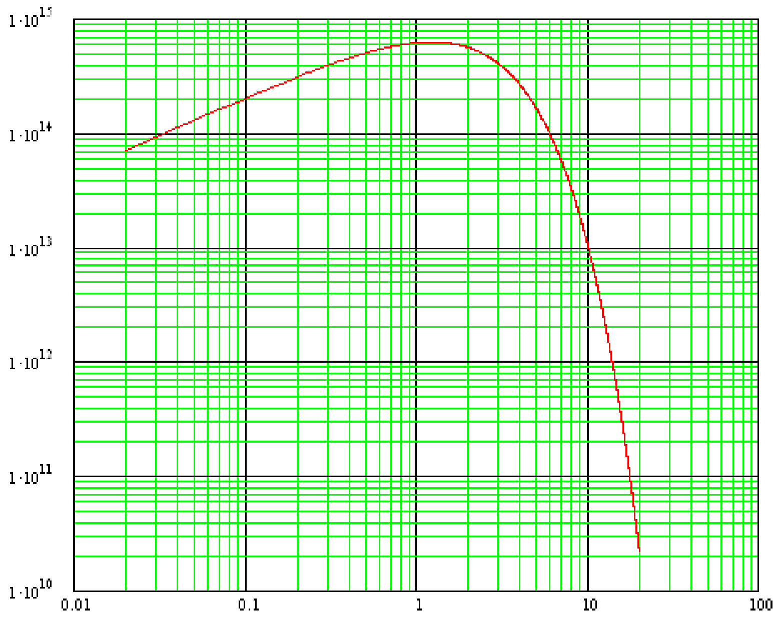




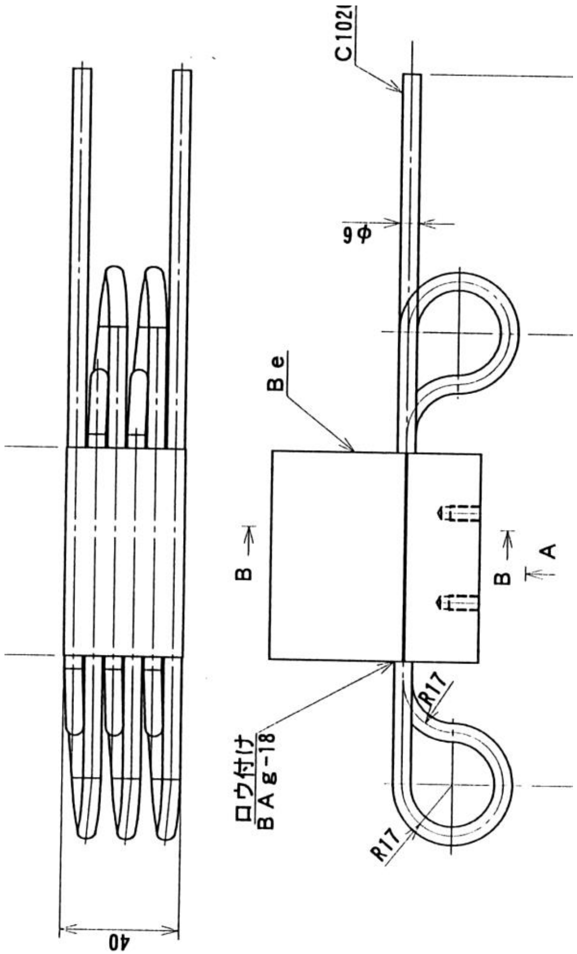




Beryllium extraction mirror for the B-factory
E=3.5GeV, ρ=65m



Photon energy (keV)

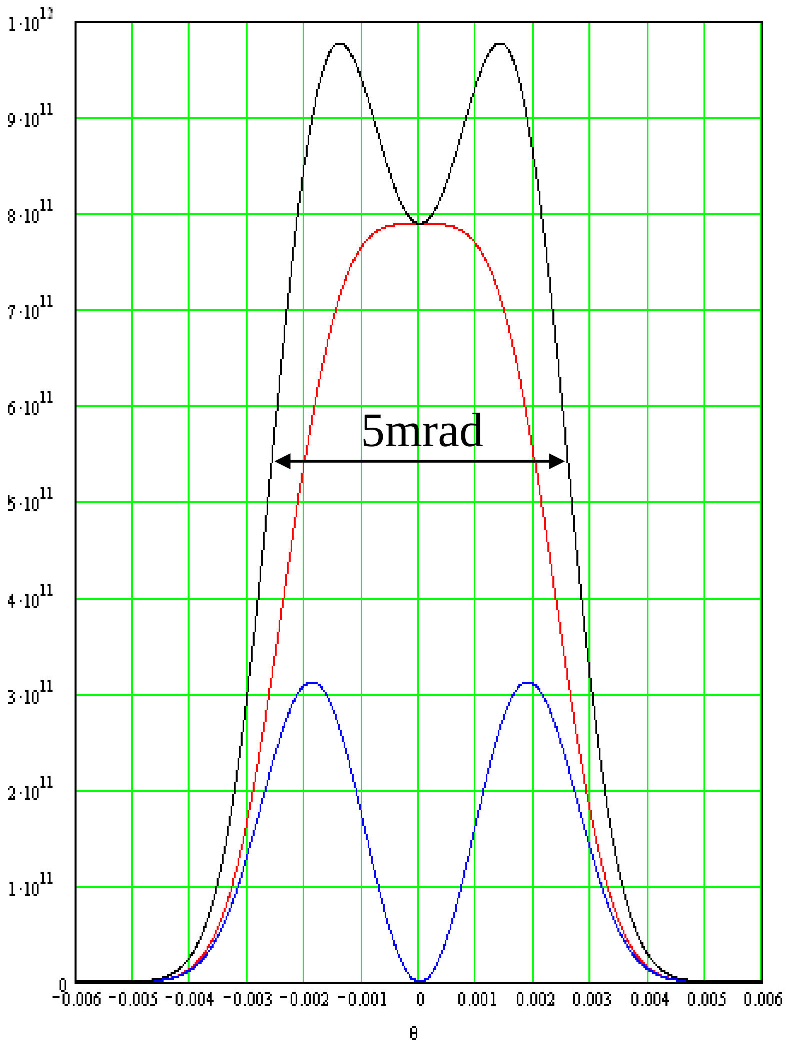



'98 9 27

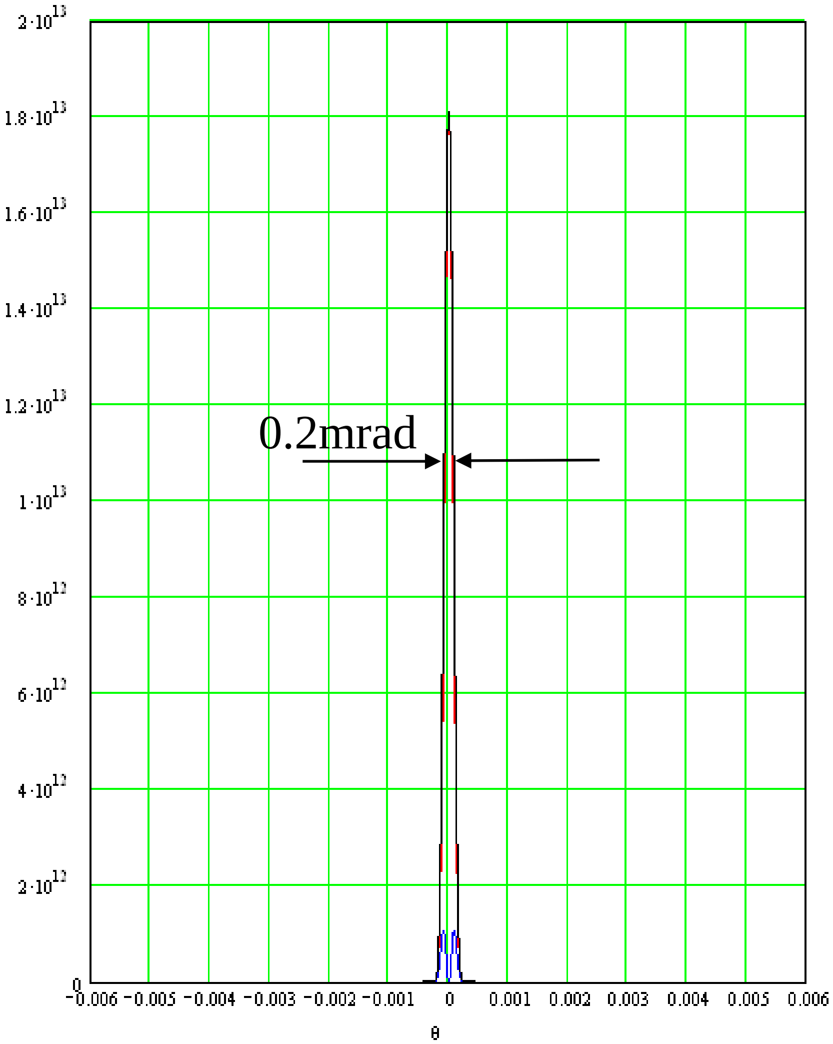

'98 7 20

Deformation of extraction mirror in the case of X-rays in center

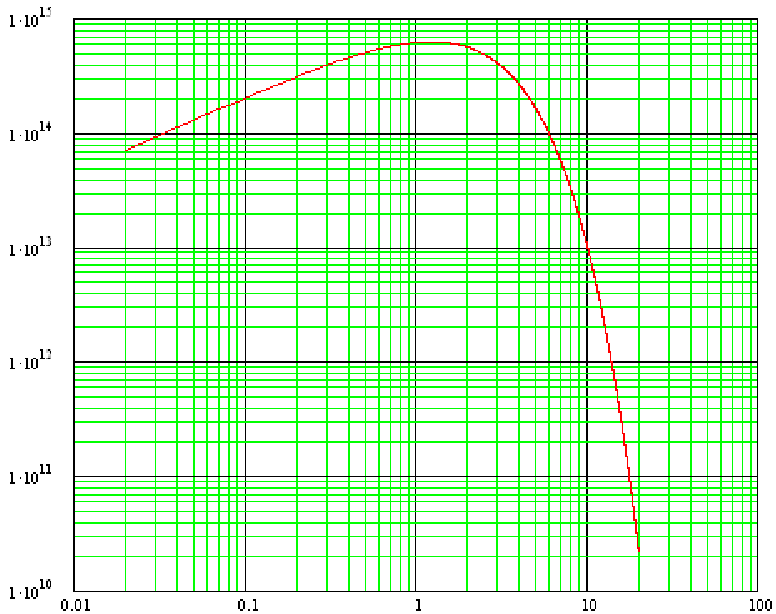
Blue-Green500nm



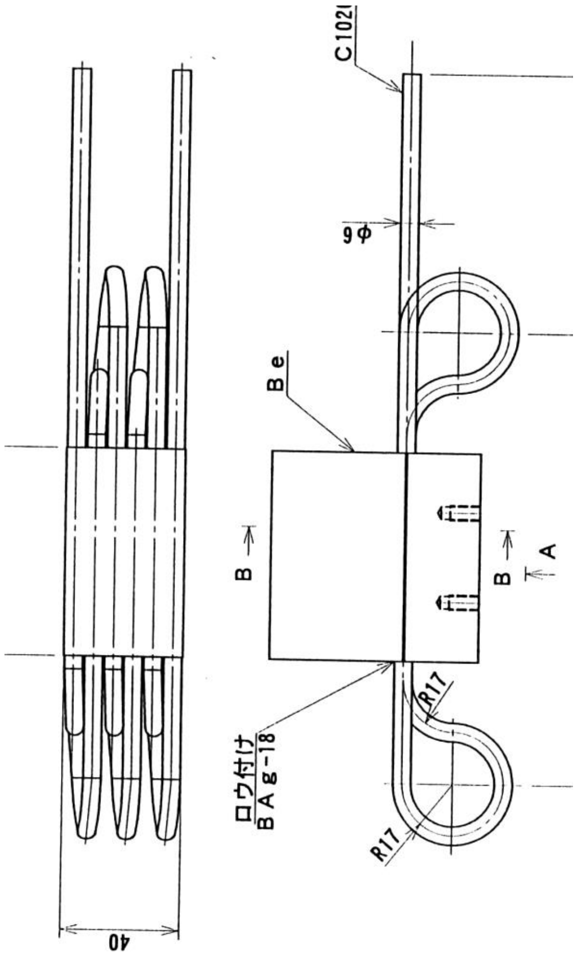
X-Ray0.1nm



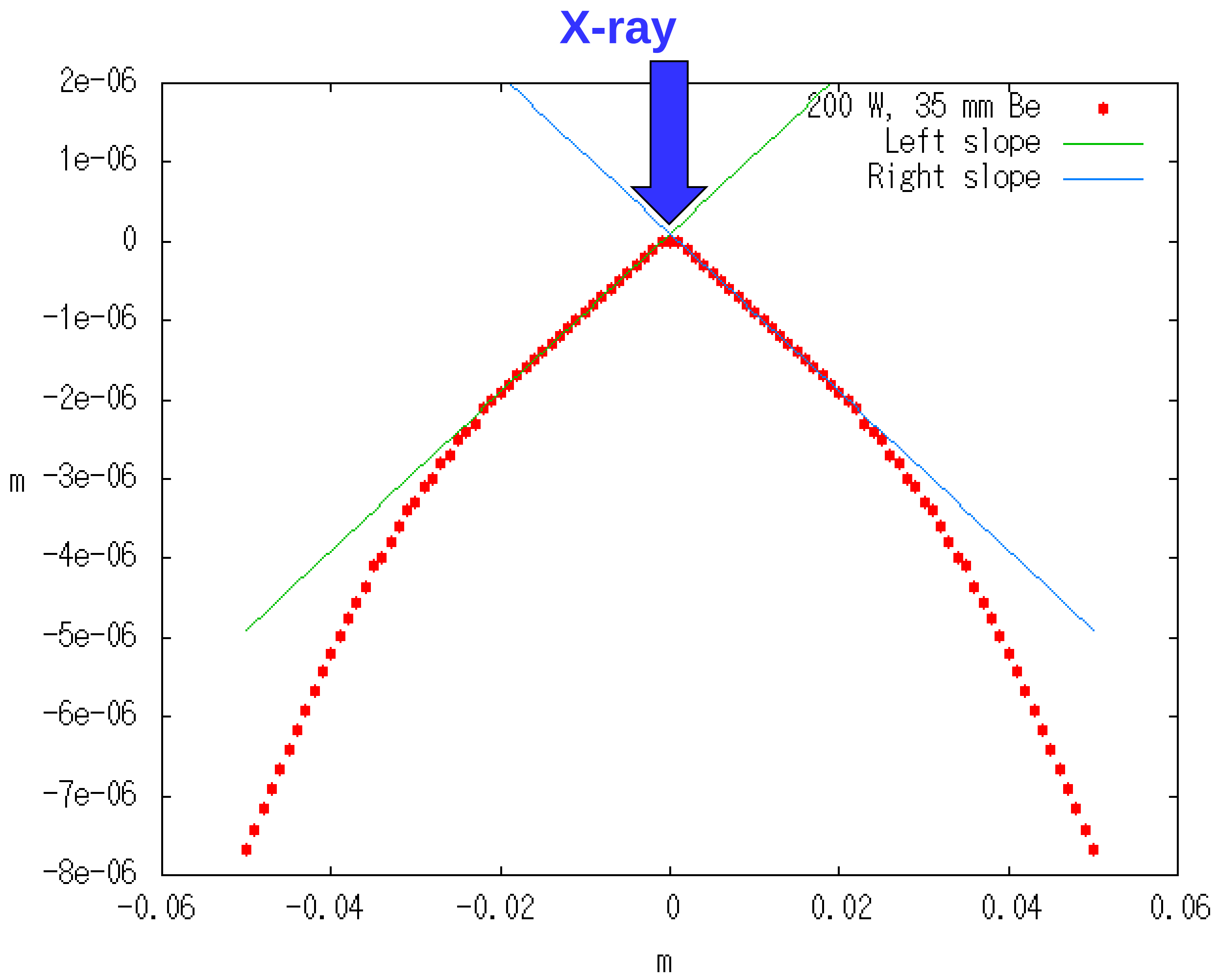
Beryllium extraction mirror for the B-factory
E=3.5GeV, ρ=65m

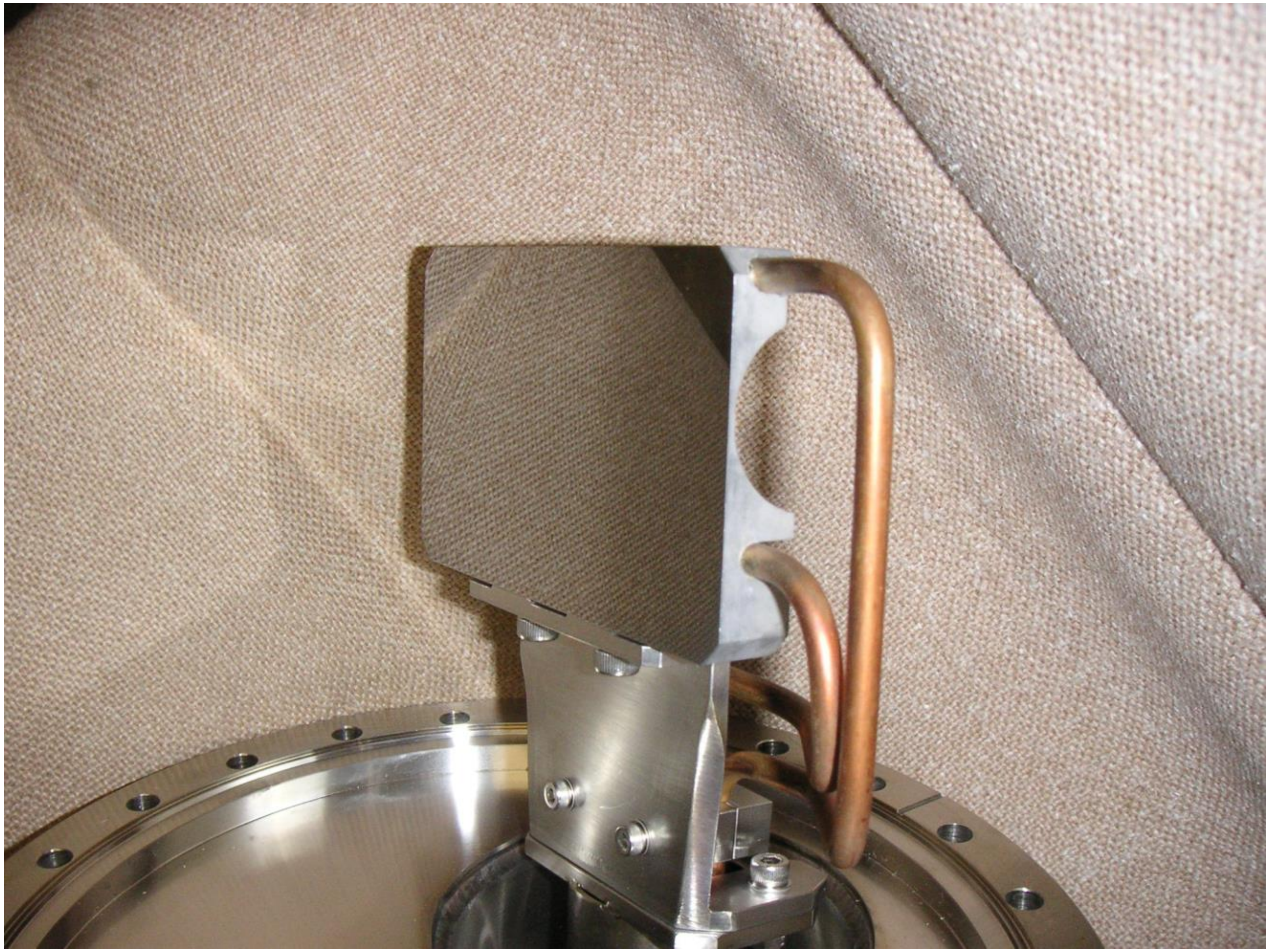


Photon energy (keV)

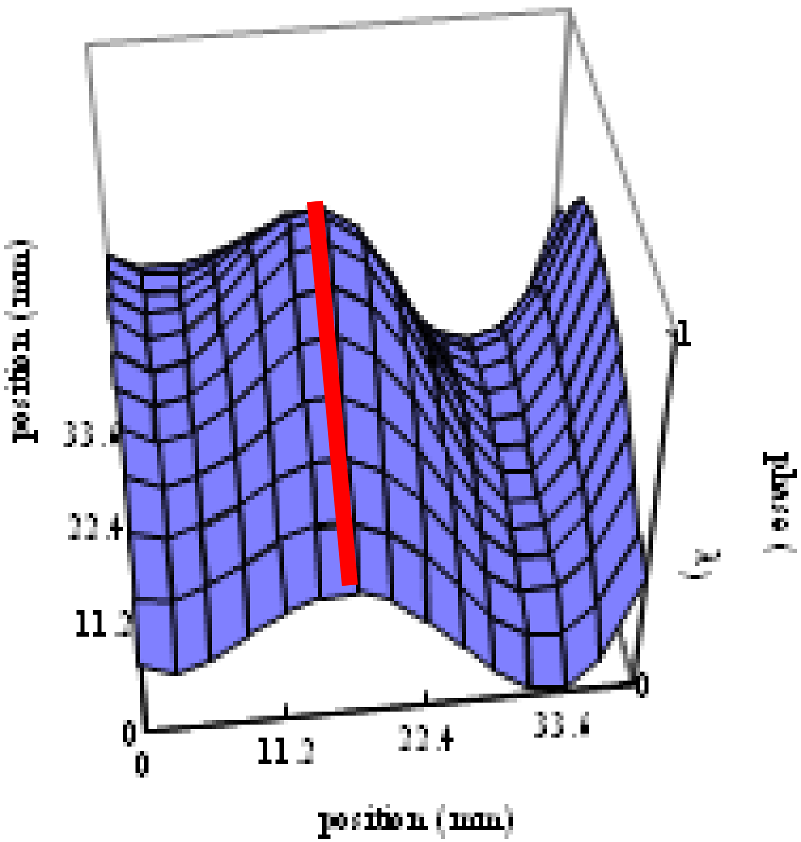


Surface deformation for Be mirror of type used at KEKB
200W (ten times intense) beam will come in Supper KEKB.





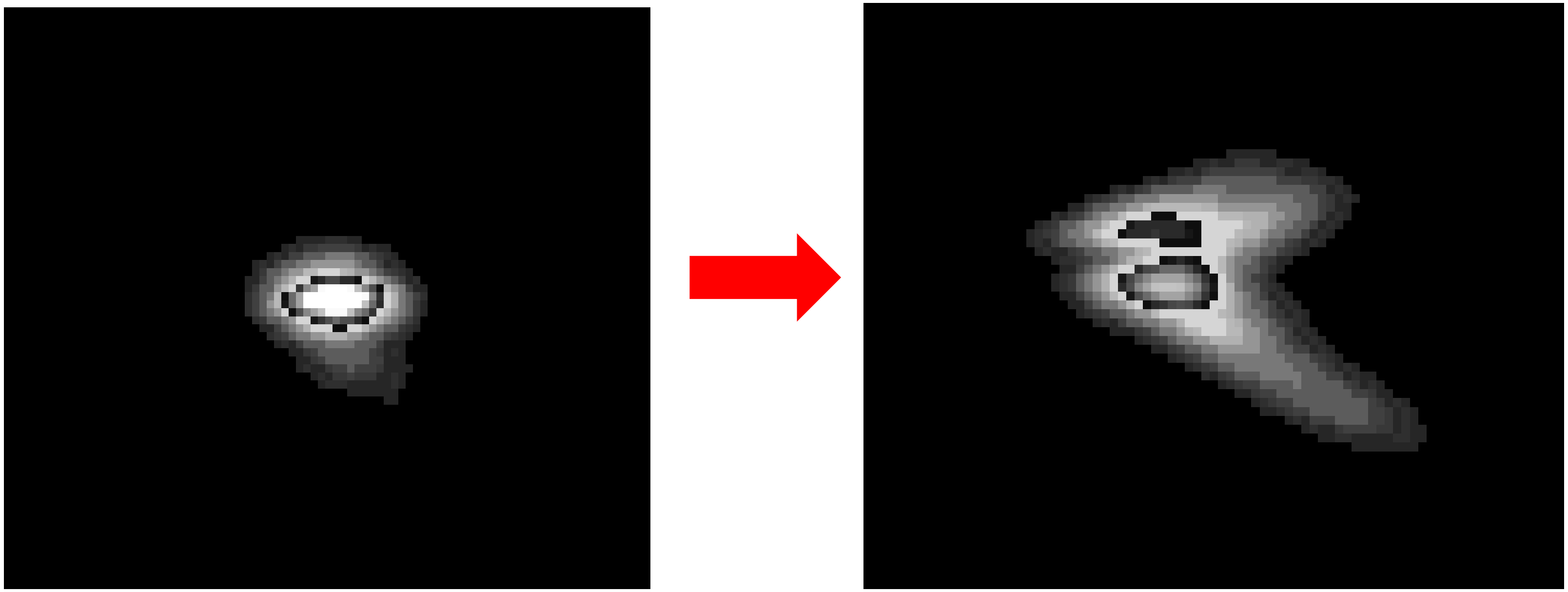
Example of mirror surface deformation by X-ray irradiation



p-v : 0.91l

rms. : 0.21l

Distortion of the beam image due to deformation of the extraction mirror



Wavefront error due to deformation of the extraction mirror is key point in the SR monitor!!

How to identify wavefront error Due to deformation of extraction mirror

1. Fieau interferometer

2. Schack-Hartmann method

3. Ray tracing using Hartmann mask

1. **Fieau interferometer**

   1st order coherence

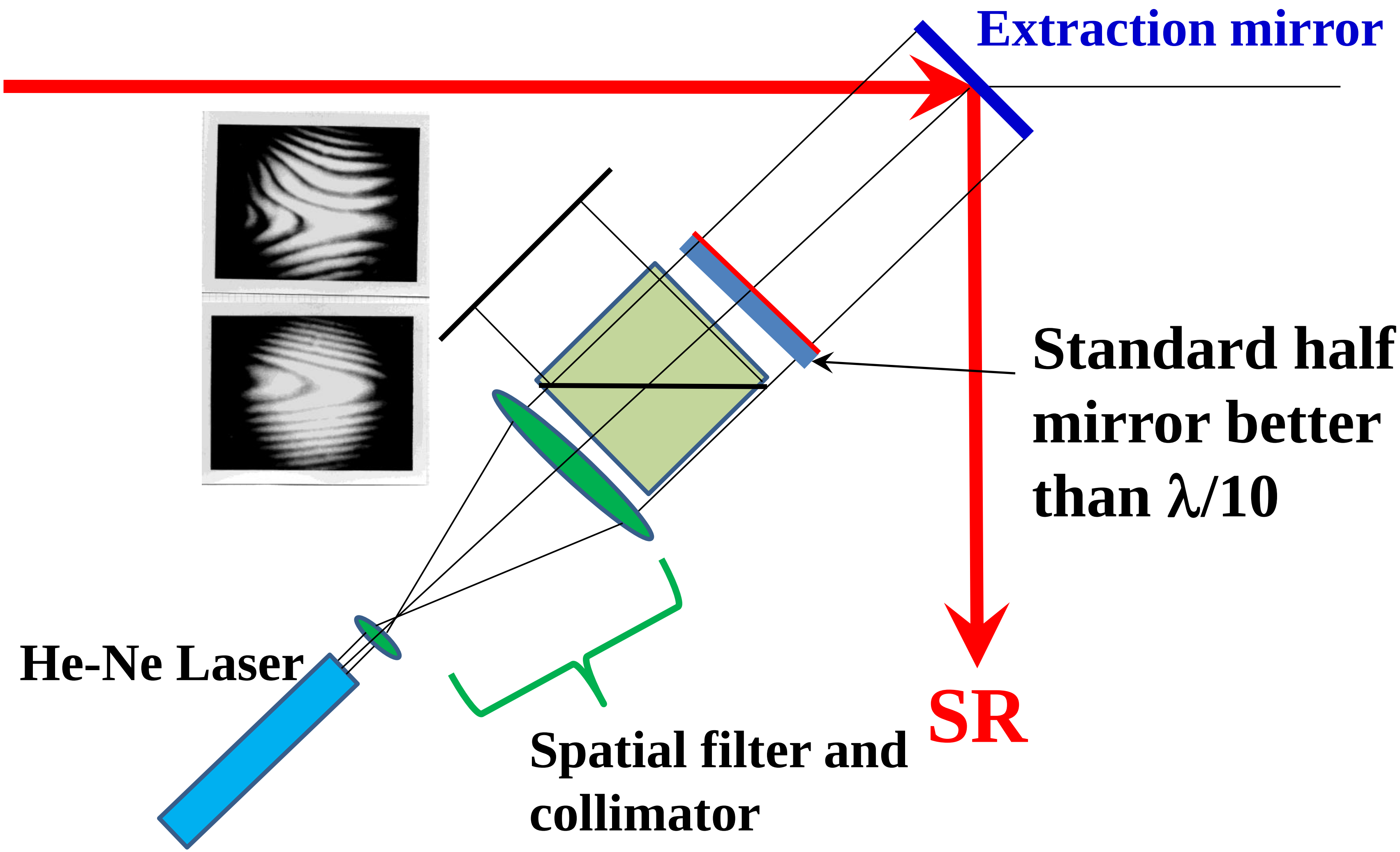
2. **Schack-Hartmann method**

   Geometrical optics

3. **Ray tracing using Hartmann mask**

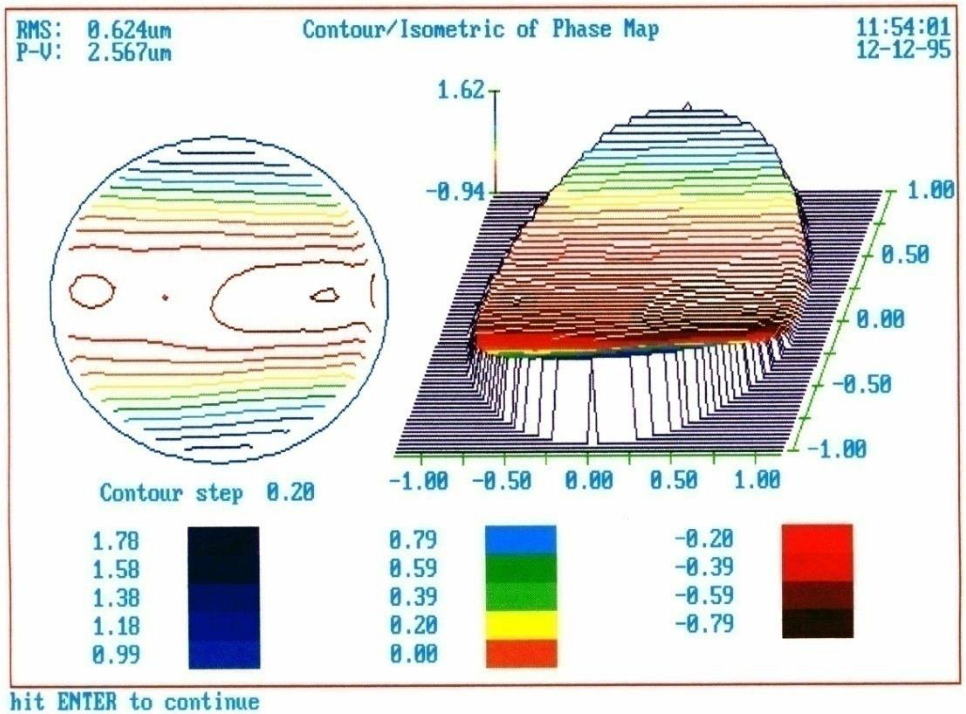
   Geometrical optics

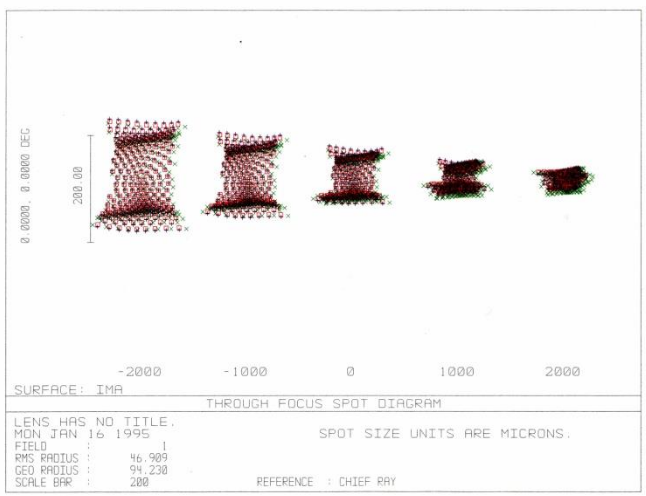
1. Fizeau interferometer



Surface of Be mirror

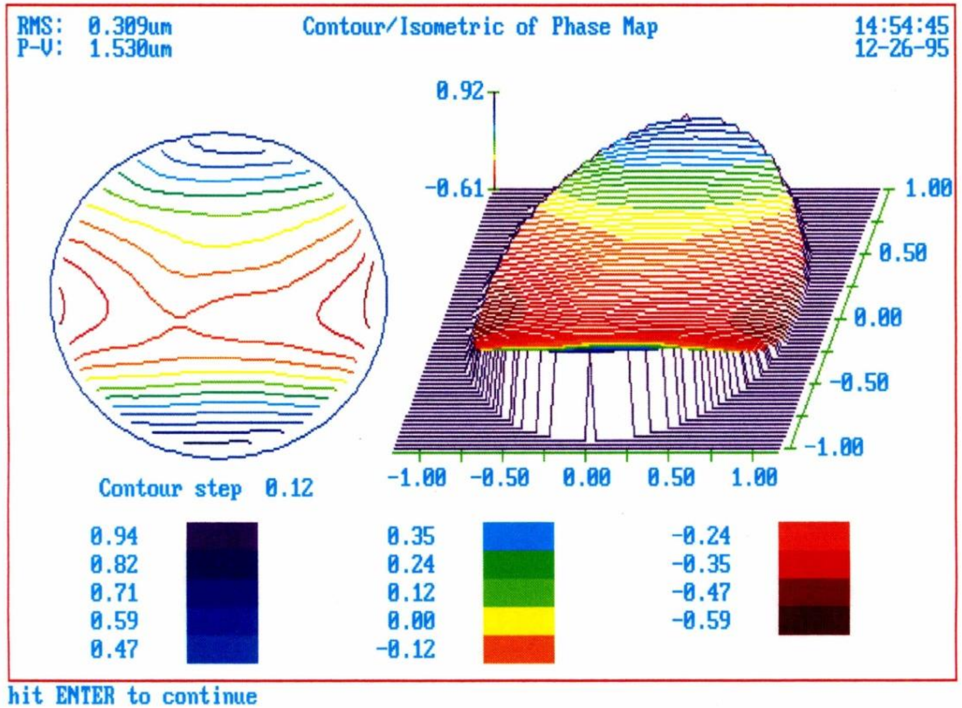
without beam



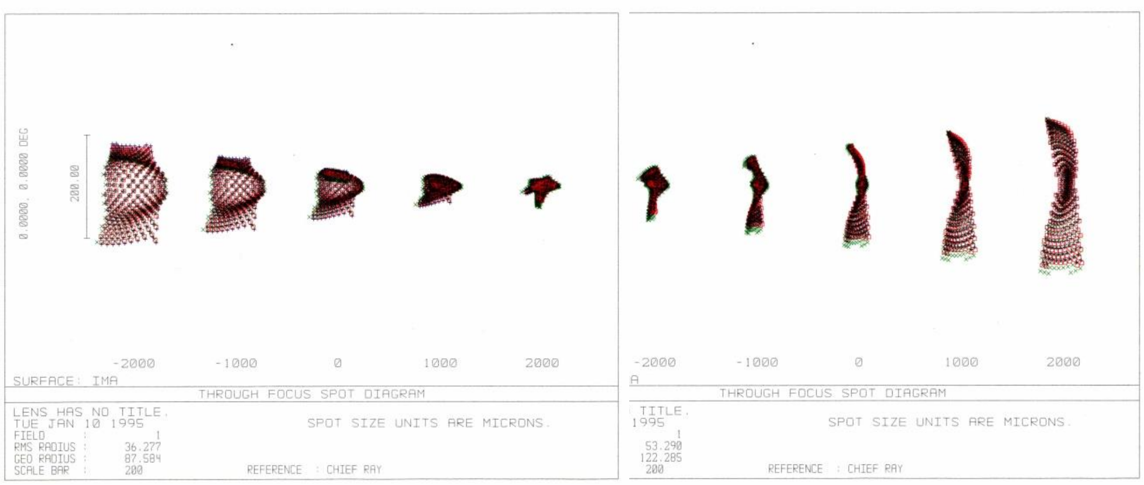


Surface of Be mirror

50mA

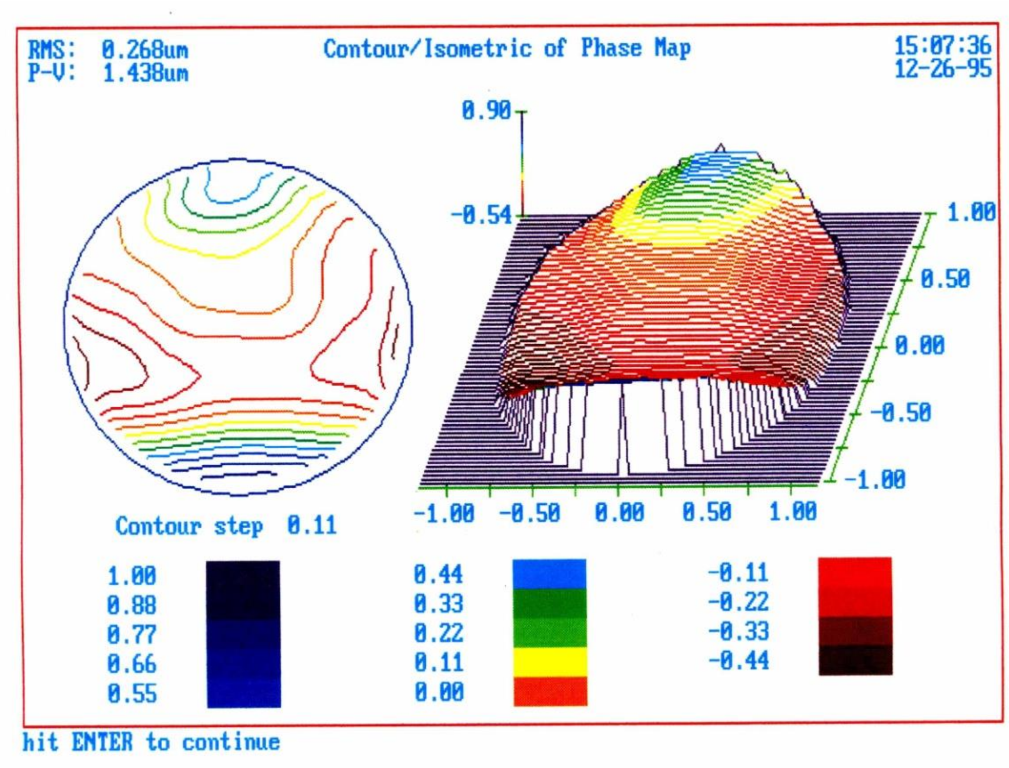


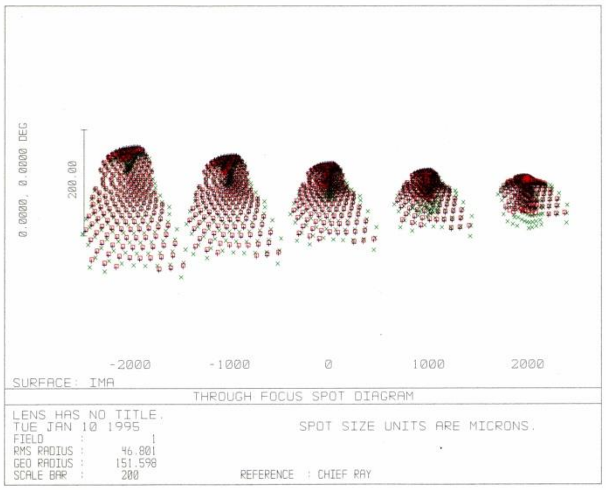
Geometrical ray tracing Spot diagram

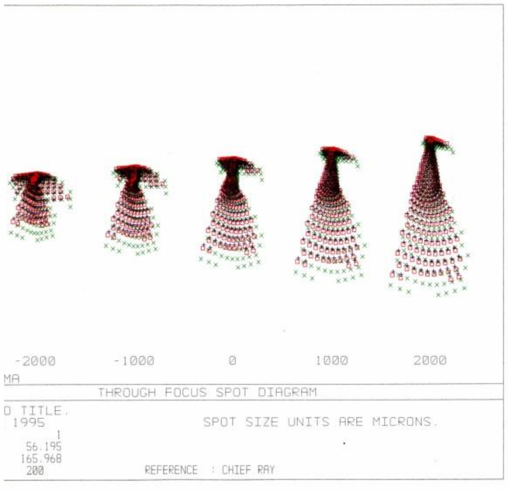


Surface of Be mirror

75mA

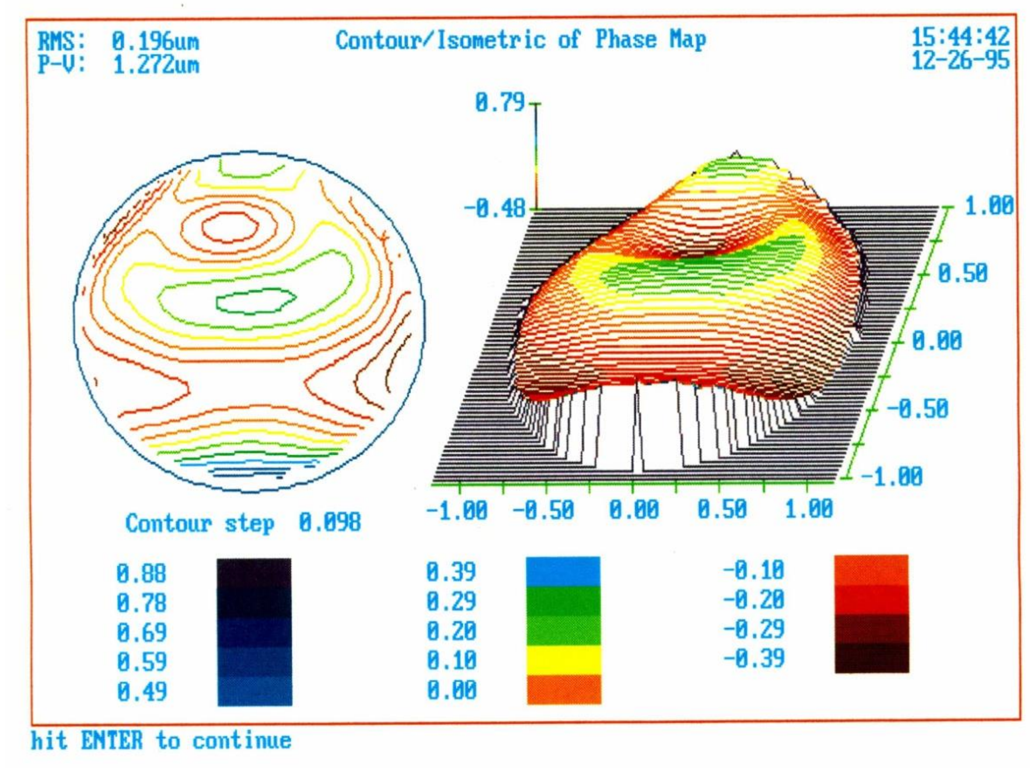


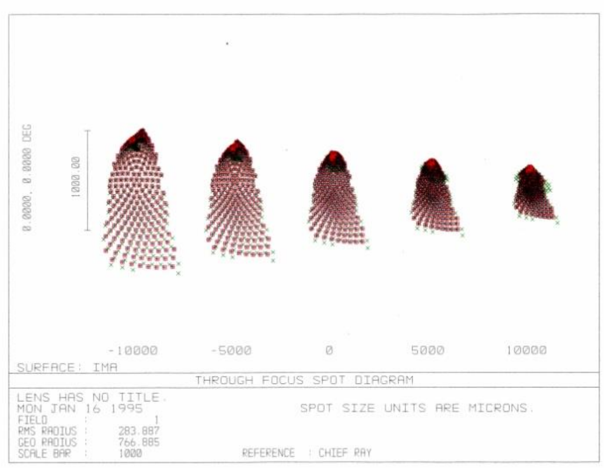


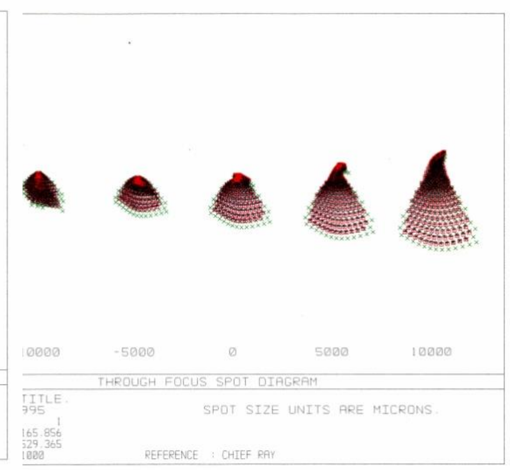


Surface of Be mirror

125mA

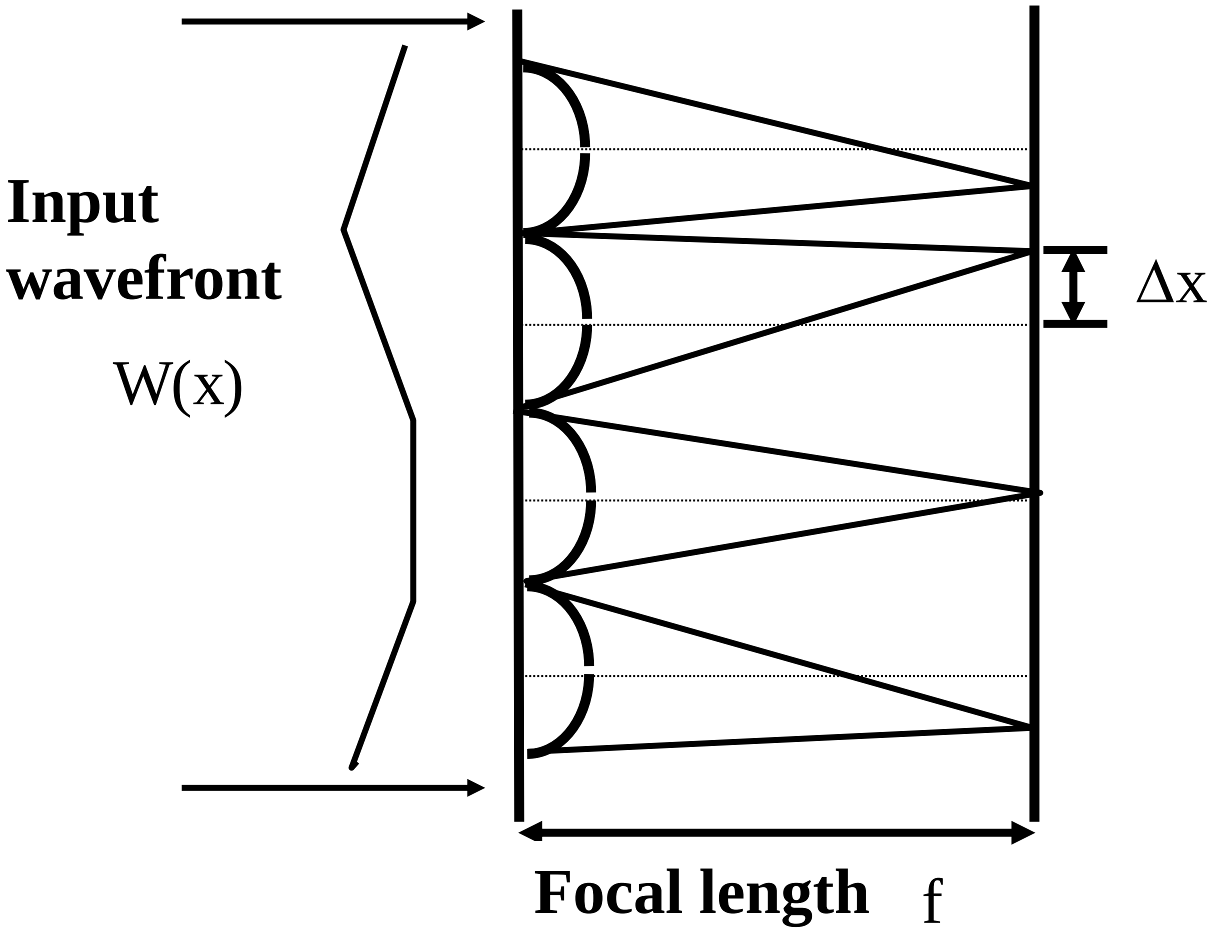




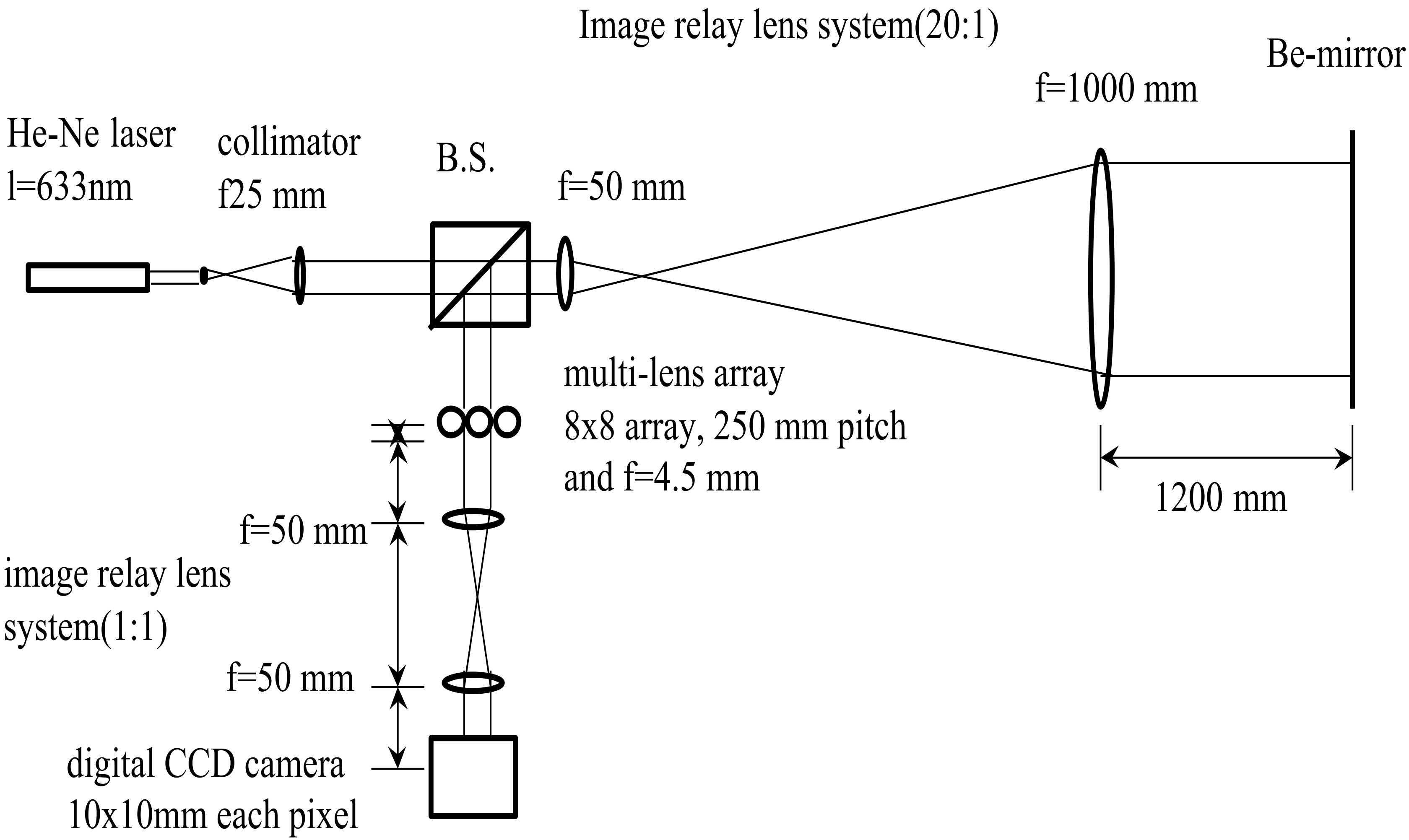


2. Schack-Hartmann method

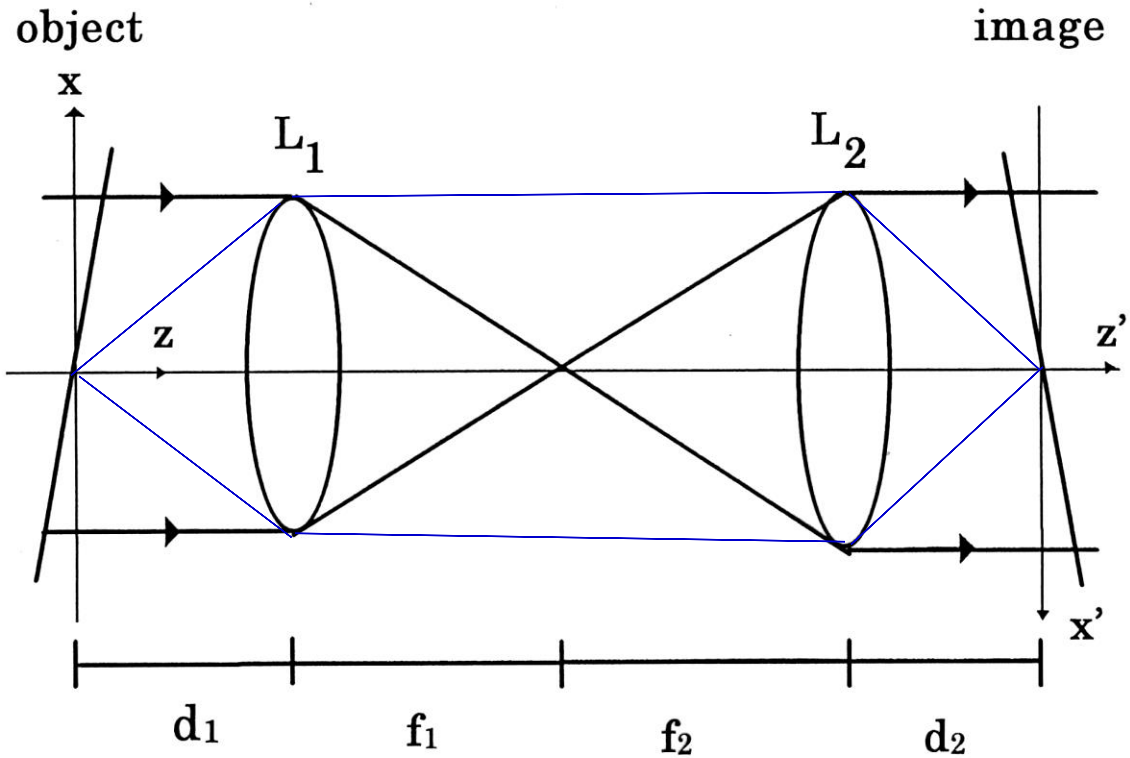
$$\frac{dW(x)}{dx} = -\frac{\Delta x}{f}$$



Practical set up of Schack-Hartmann method



Projection optics

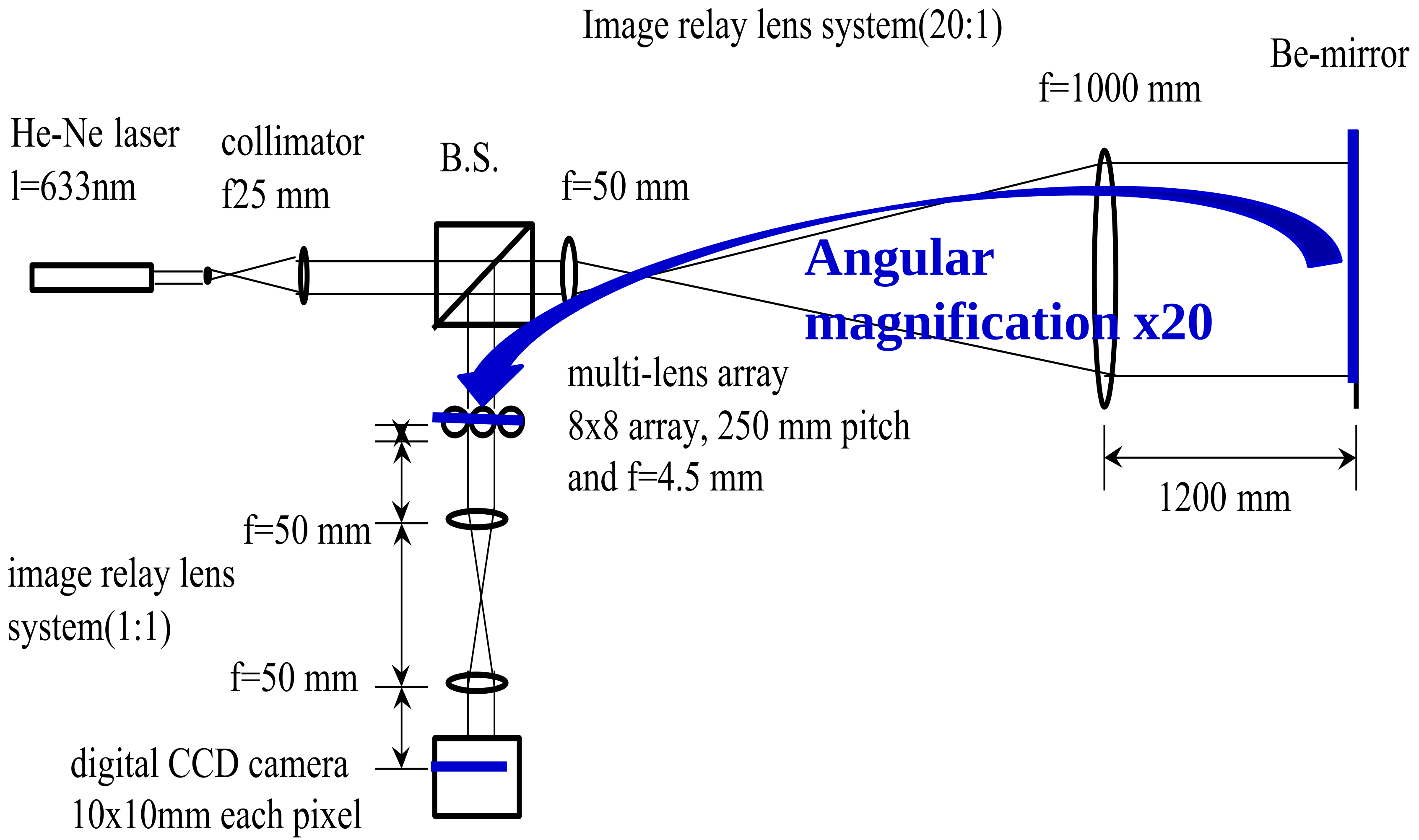


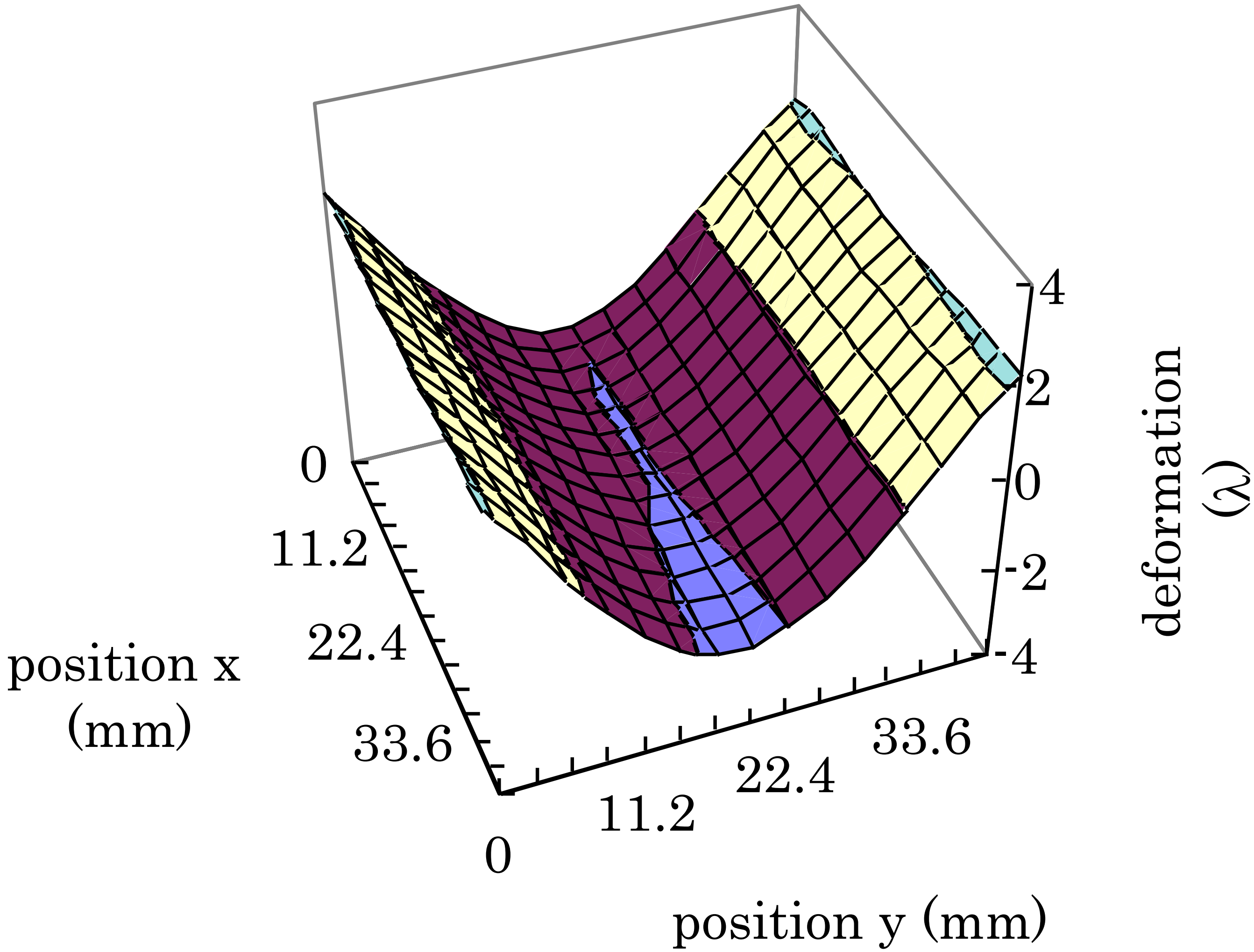
$$d_2 = (f_1 + f_2)\left(\frac{f_2}{f_1}\right) - d_1\left(\frac{f_2}{f_1}\right)^2$$

Magnification $$\frac{1}{m} = \frac{x'}{x} = \frac{f_2}{f_1}$$

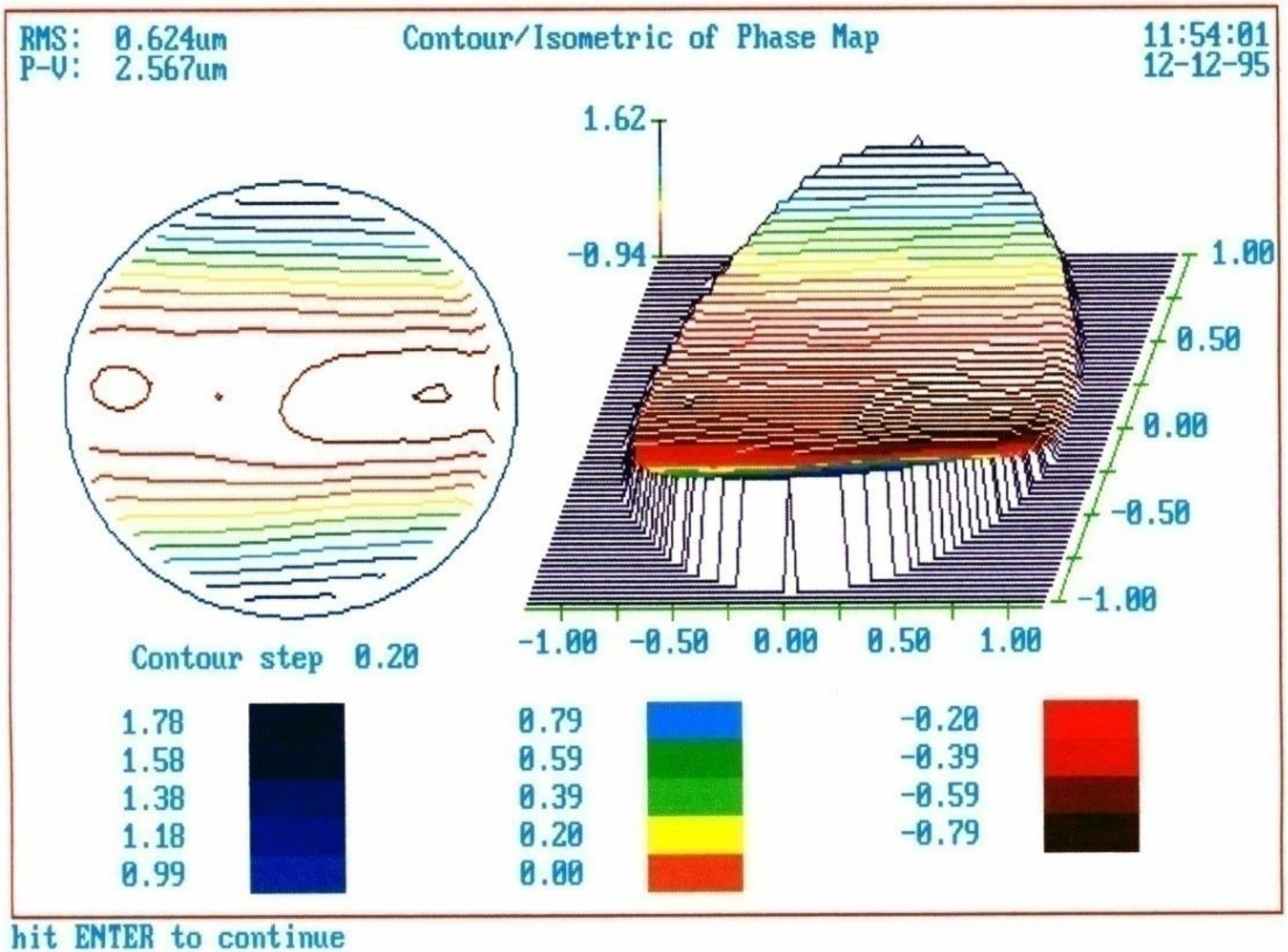
Gradient on image $$\frac{dz'}{dx'} = m\frac{dz}{dx}$$

Local gradients on mirror are magnified by angular magnification of relay system

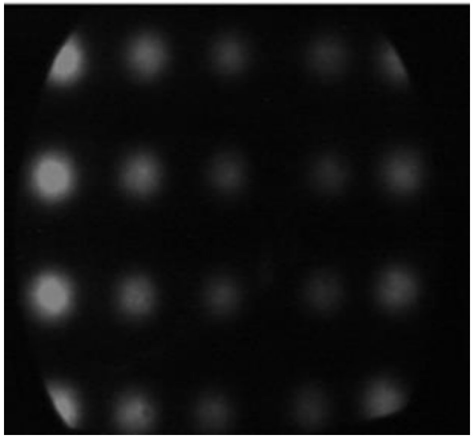




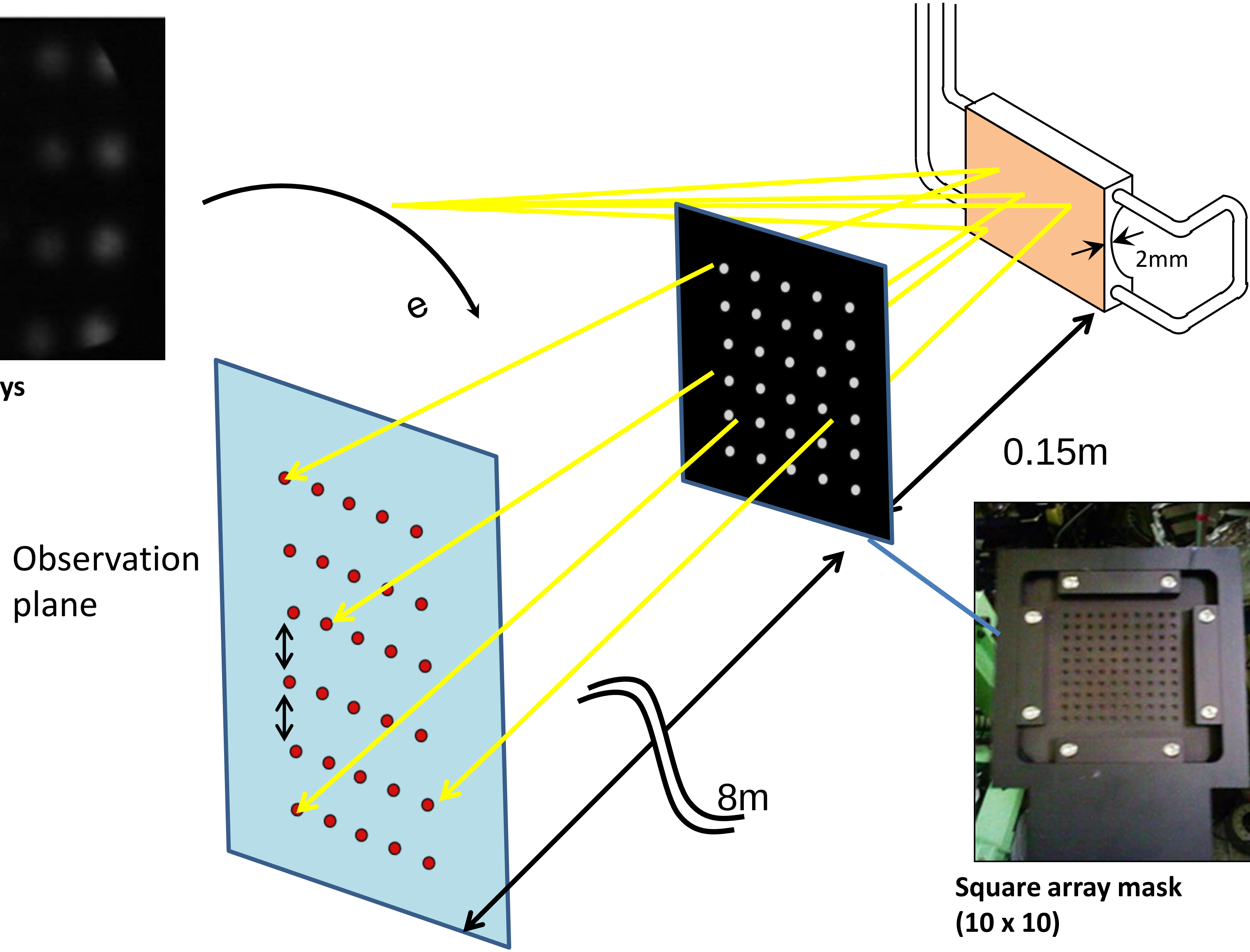
4
2
0
-2
-4
deformation
(λ)
0
11.2
22.4
33.6
position x
(mm)
0
11.2
22.4
33.6
position y (mm)


RMS: 0.624um
P-V: 2.567um
Contour/Isometric of Phase Map
11:54:01
12-12-95
1.62
-0.94
1.00
0.50
0.00
-0.50
-1.00
-1.00 -0.50 0.00 0.50 1.00
Contour step 0.20
1.78
1.58
1.38
1.18
0.99
0.79
0.59
0.39
0.20
0.00
-0.20
-0.39
-0.59
-0.79
hit ENTER to continue

3. Ray tracing by Hartmann mask



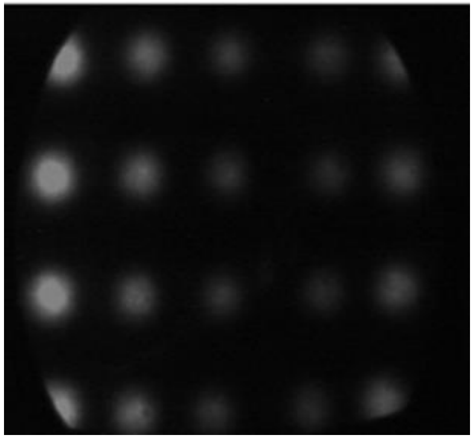
Projection of rays



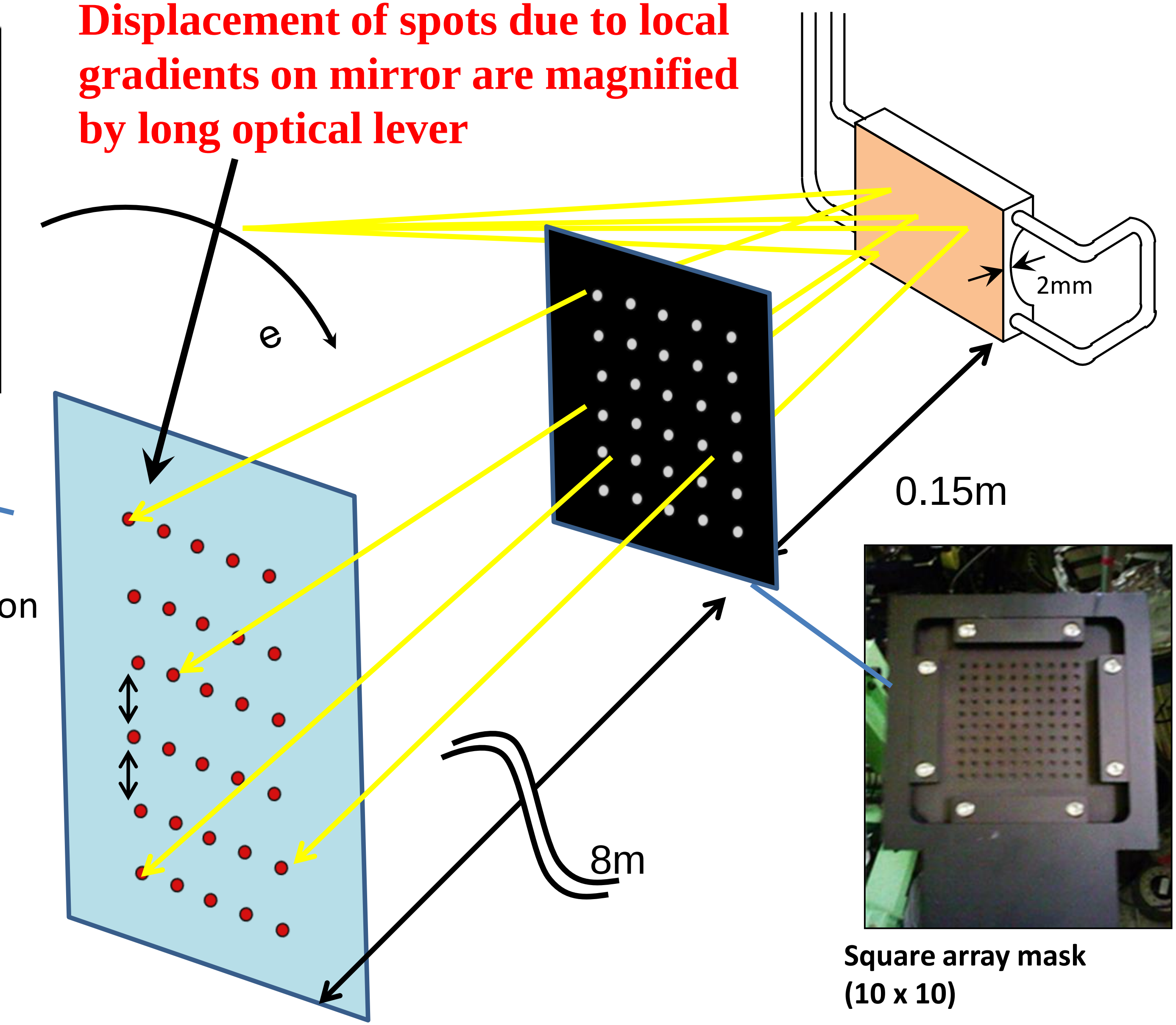
Square array mask
(10 x 10)

Ray tracing by Hartmann mask

Displacement of spots due to local gradients on mirror are magnified by long optical lever



Projection of rays

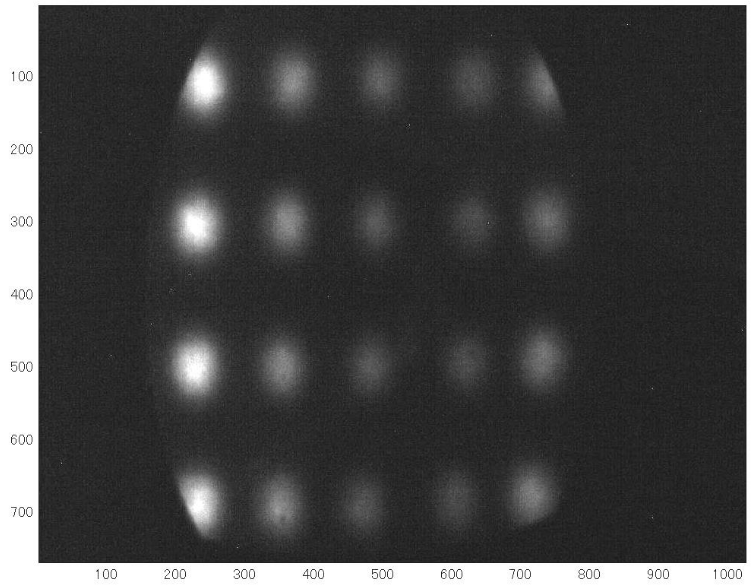


Square array mask
(10 x 10)

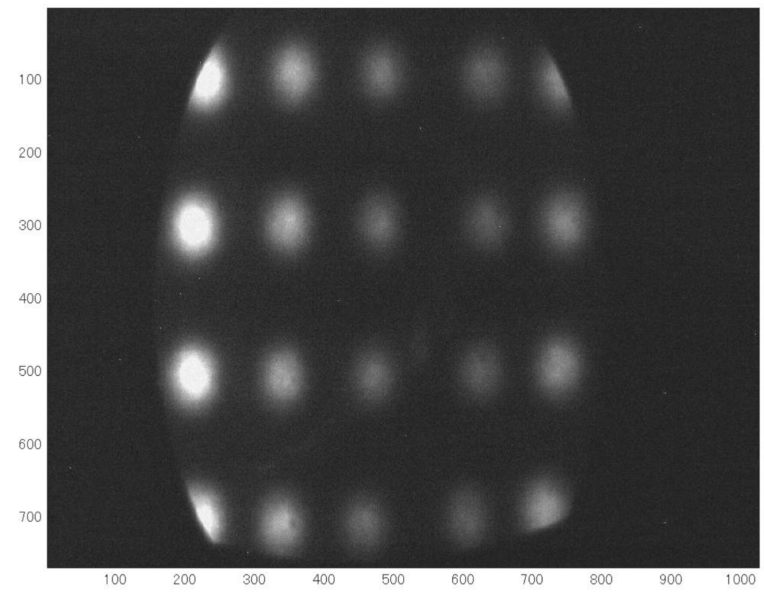
Characterization of mirror deformation due to SR irradiation by the use of Hartmann screen

Hartmann square screen
Diameter of hole 1mm
10x10, 5mm spacing

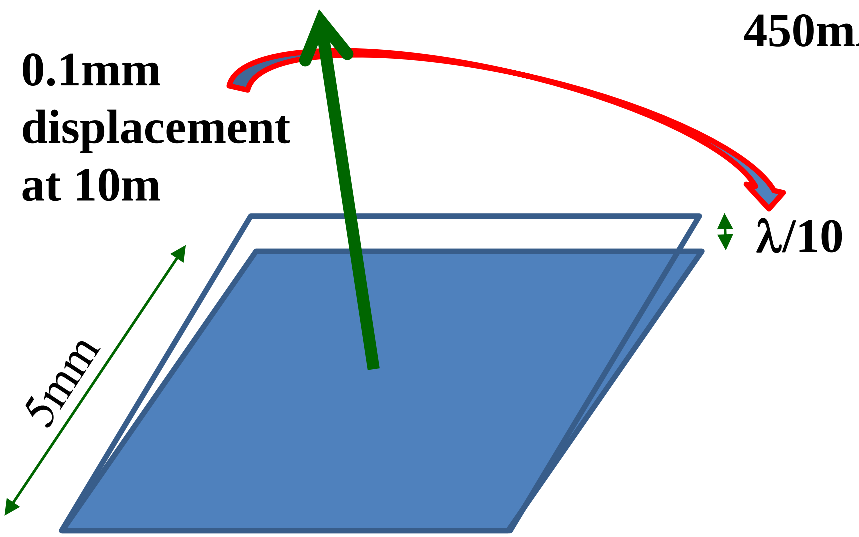
Spots pattern on observation plane by square hole array



280mA

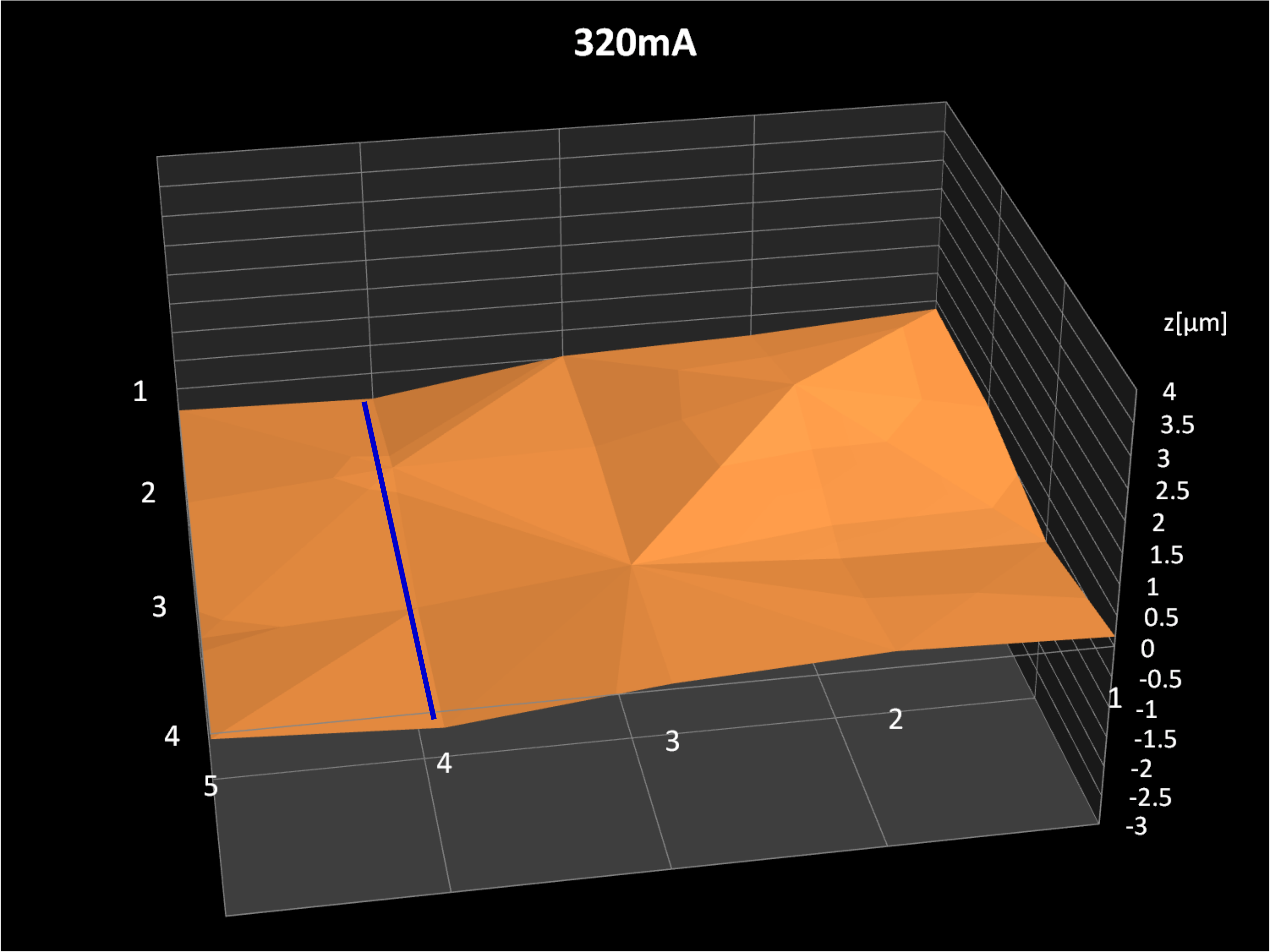


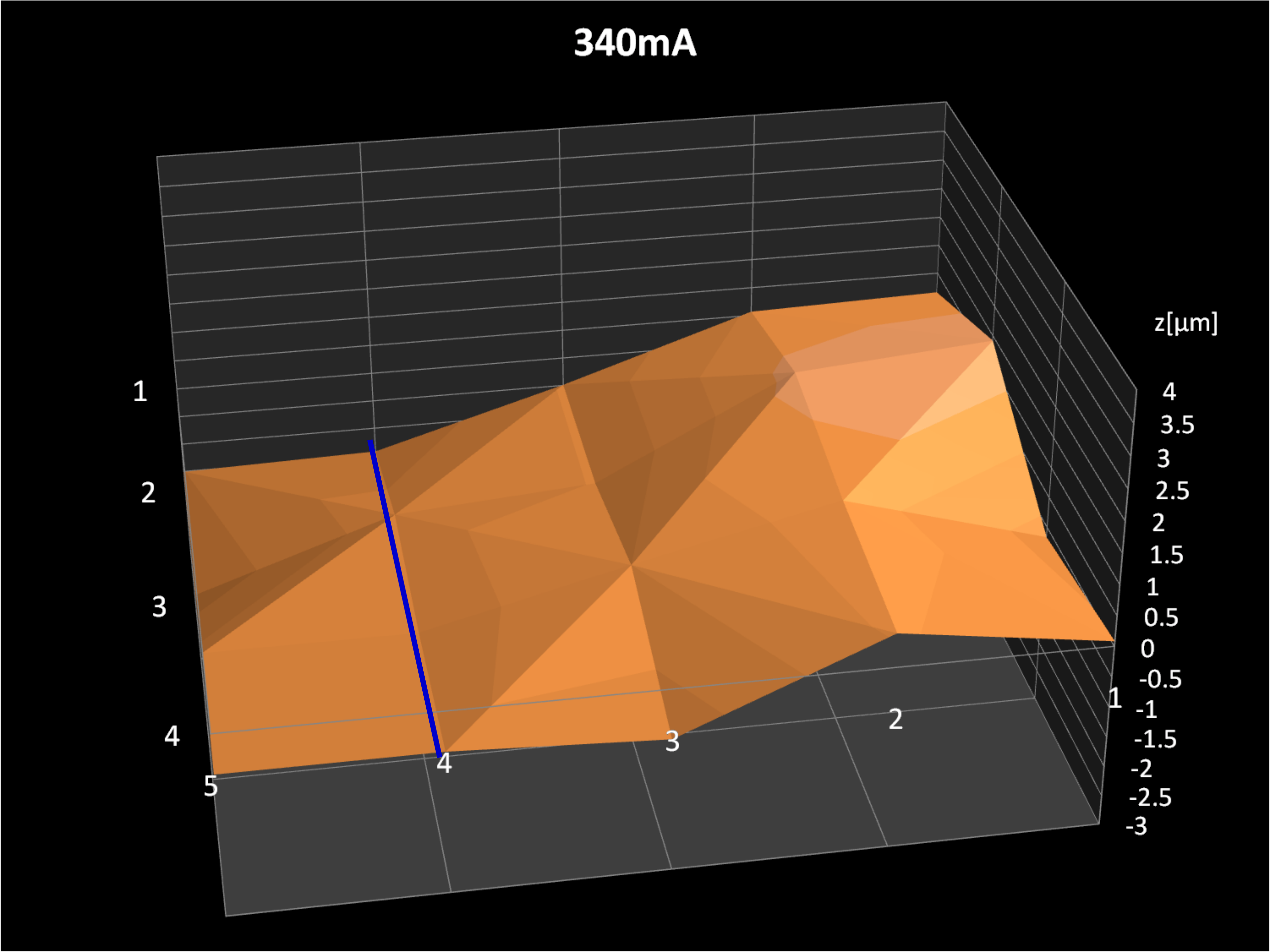
450mA

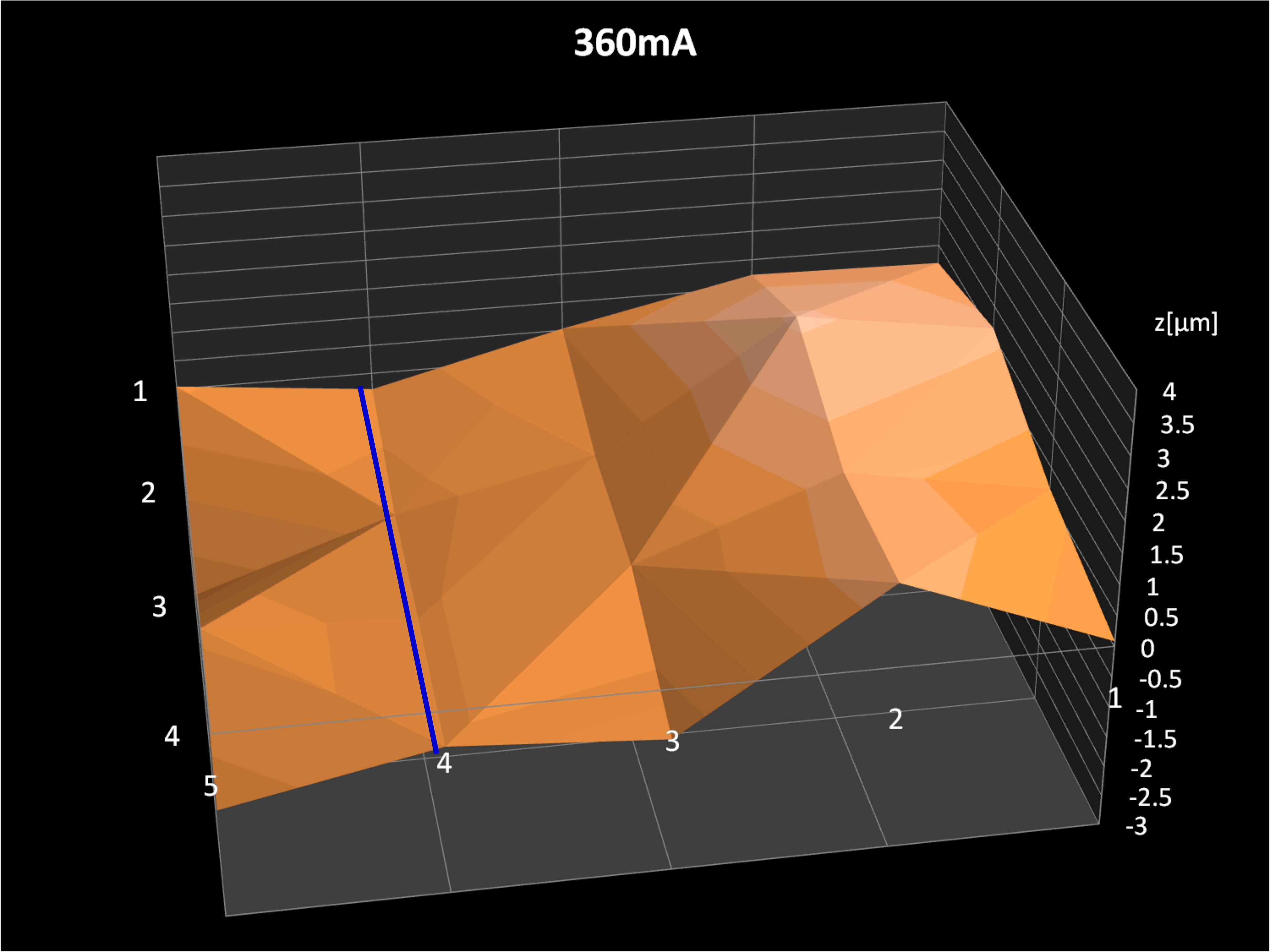


Wavefront error due to thermal deformation of extraction mirror From 300mA to 450mA at the Photon Factory, KEK

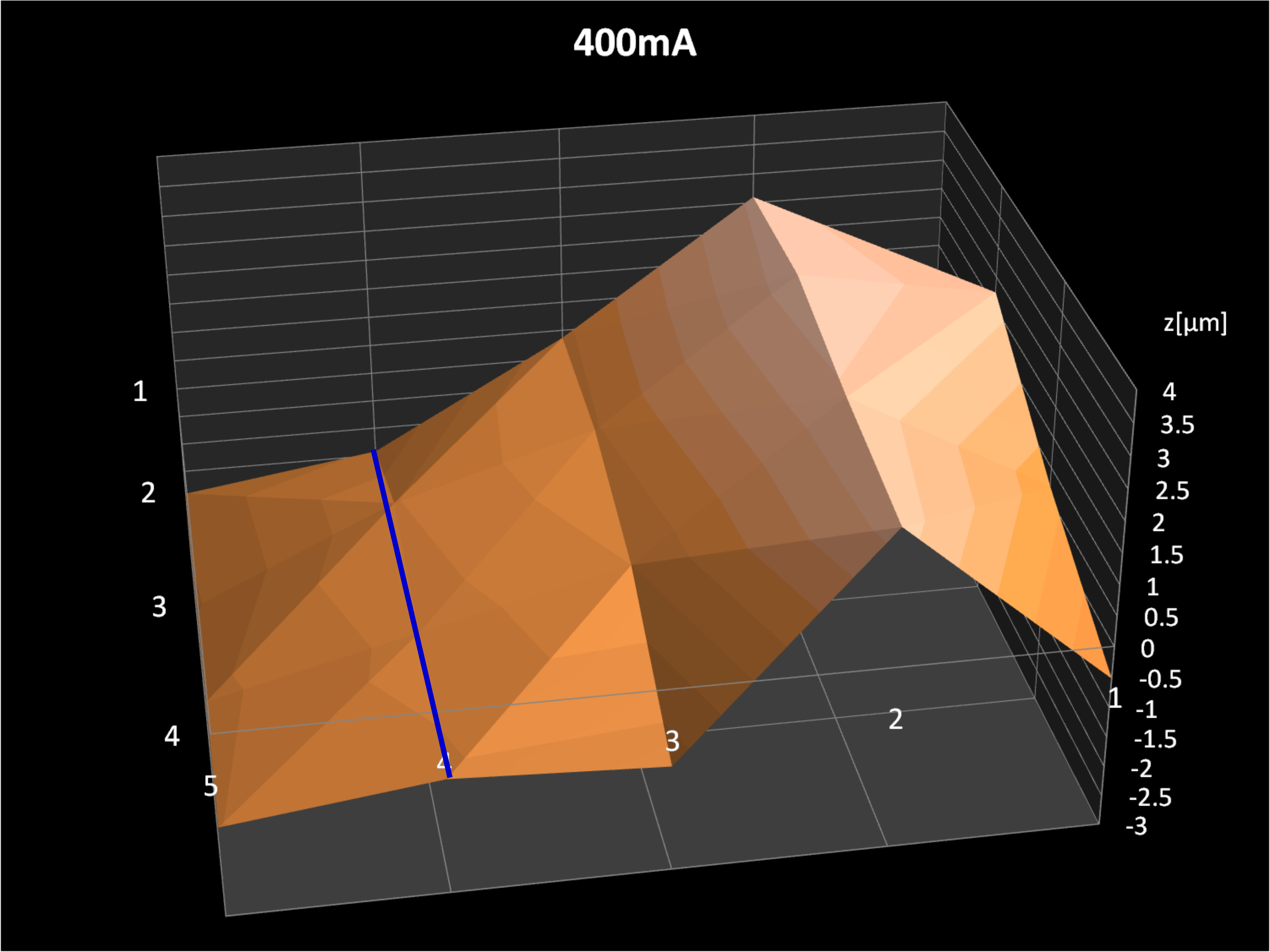
Let’s put wavefront at 300mA into null, and observe wavefront distortion

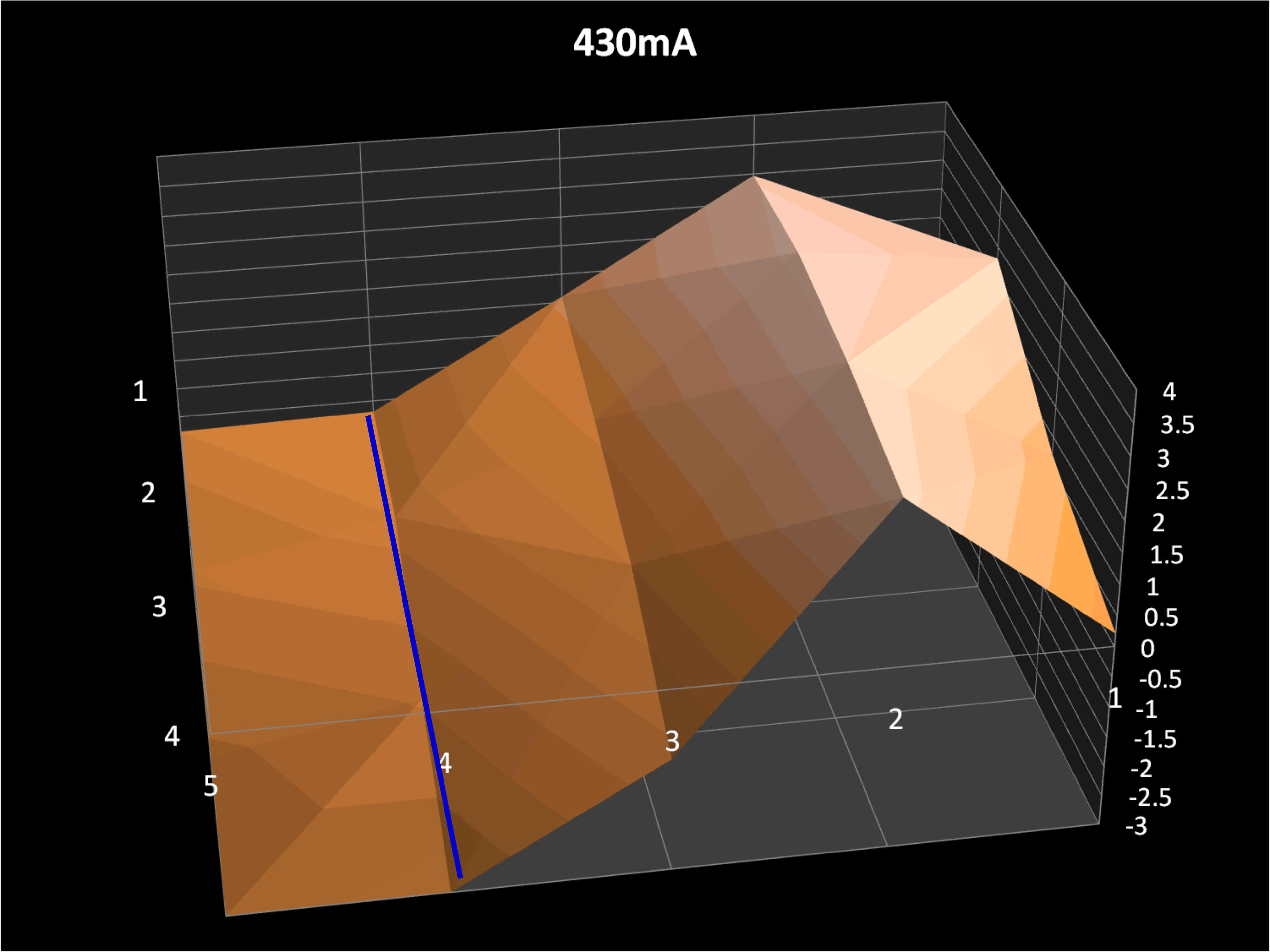

320mA
z[µm]
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
1
2
3
4
5
4
3
2
1

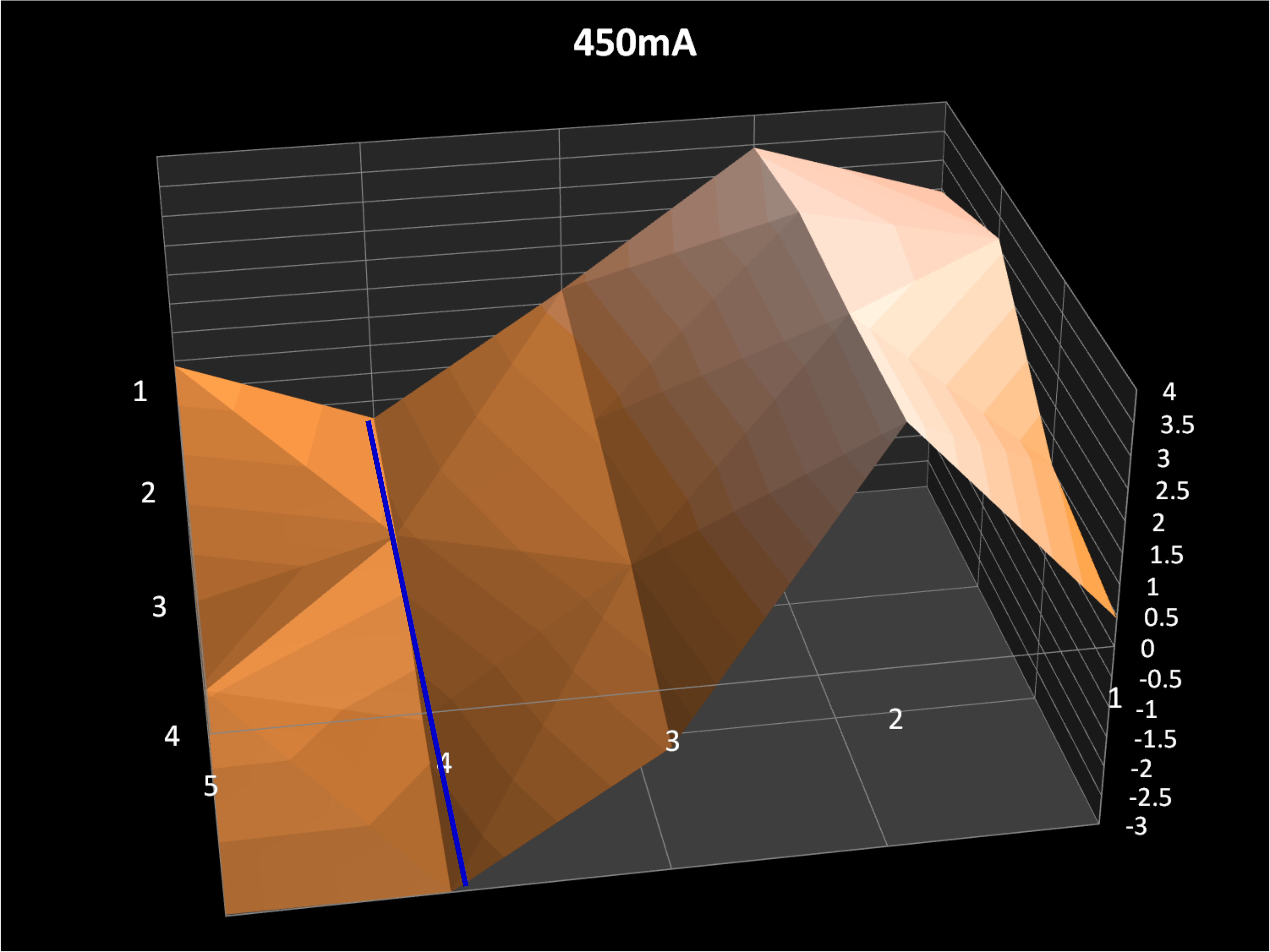

340mA
z[µm]
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
1
2
3
4
5
4
3
2
1

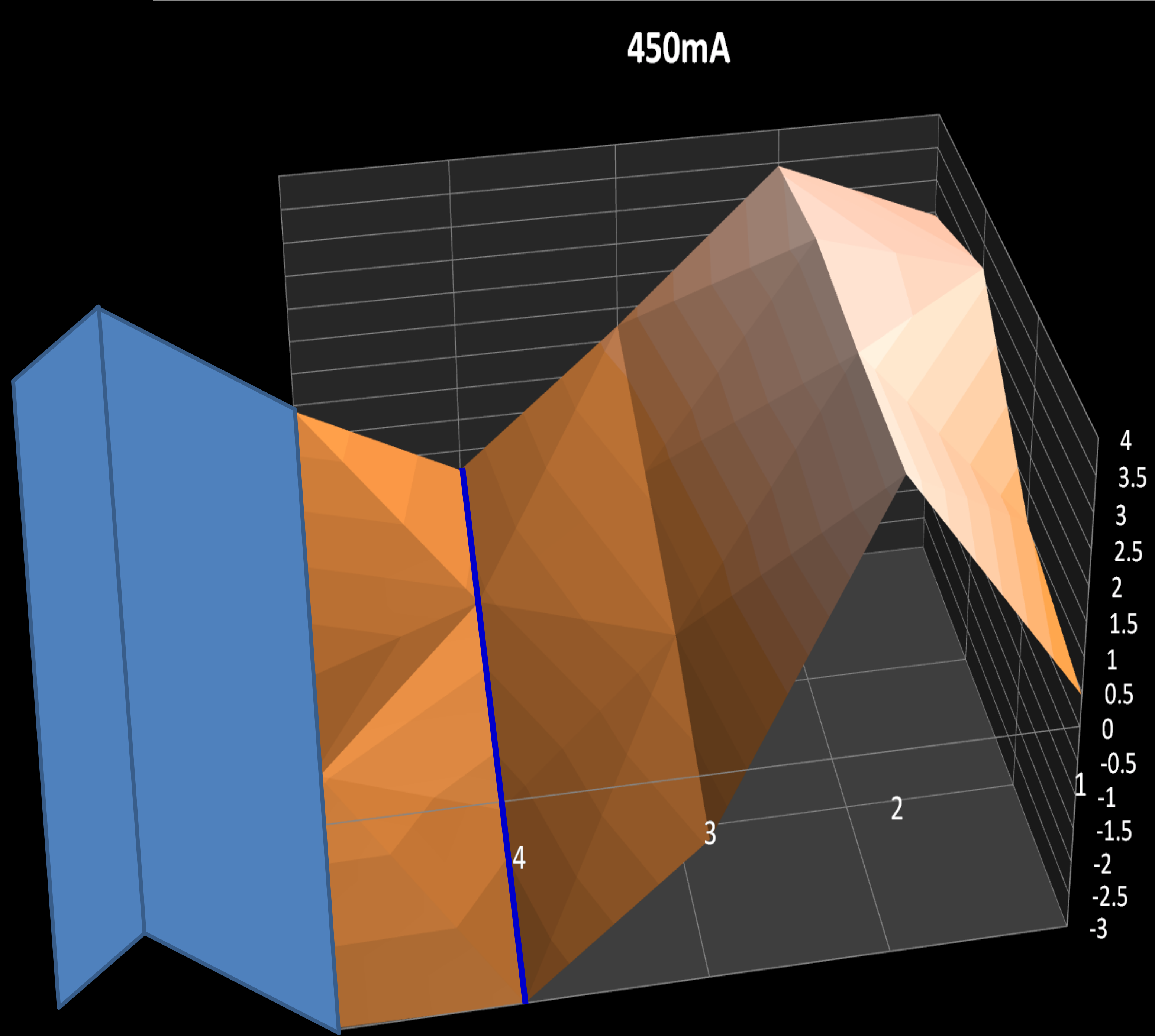


360mA
z[µm]
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
1
2
3
4
5
1
2
3
4


400mA
z[µm]
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
1
2
3
4
5
1
2
3
4


430mA
1
2
3
4
5
4
3
2
1
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3


450mA
1
2
3
4
5
4
3
2
1
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3


450mA
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
1
2
3
4

Comparison of these methods to identify the wavefront distortion

1. Fizeau interferometer

- **Coherent method:** very week for floor vibrations
- **Sensitivity:** $\lambda/5$- $\lambda/10$(depend on reference plate)
- **Device location:** In front of mirror
- **Optical path:** Not included
- **Other:** Non destructive, Expensive

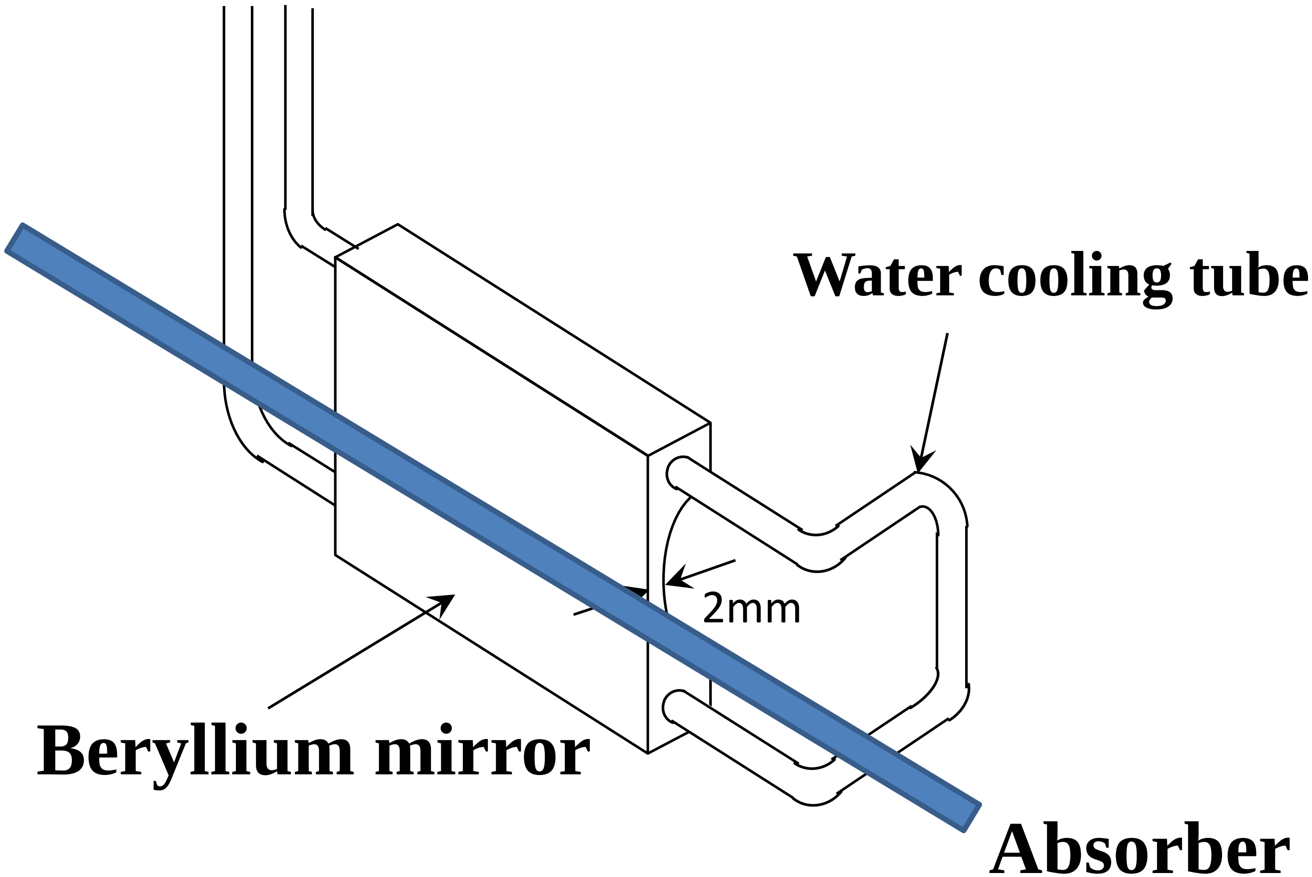
2.Shack-Hartmann

- **Incoherent method:** strong for floor vibrations
- **Sensitivity:** $\lambda/5$- $\lambda/10$ (depends on angular magnification)
- **Device location:** In front of mirror
- **Optical path:** Not included
- **Other:** Non destructive, Expensive

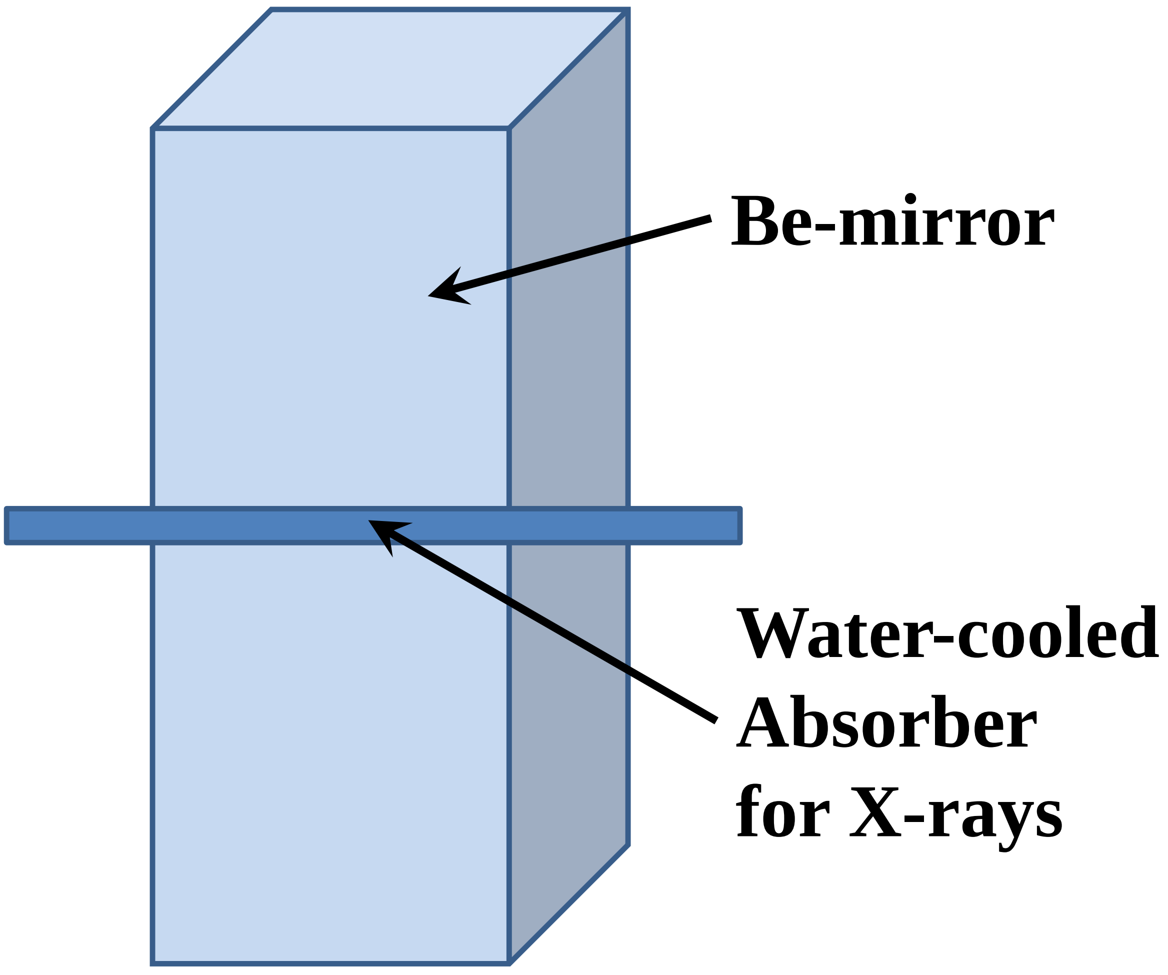
3.Hartmann screen

- **Incoherent method:** strong for floor vibrations
- **Sensitivity:** $\lambda/5$- $\lambda/10$ (depends on optical lever length)
- **Device location:** Mask locates in front of mirror
- **Optical path:** Included
- **Other:** Destructive, Cheep

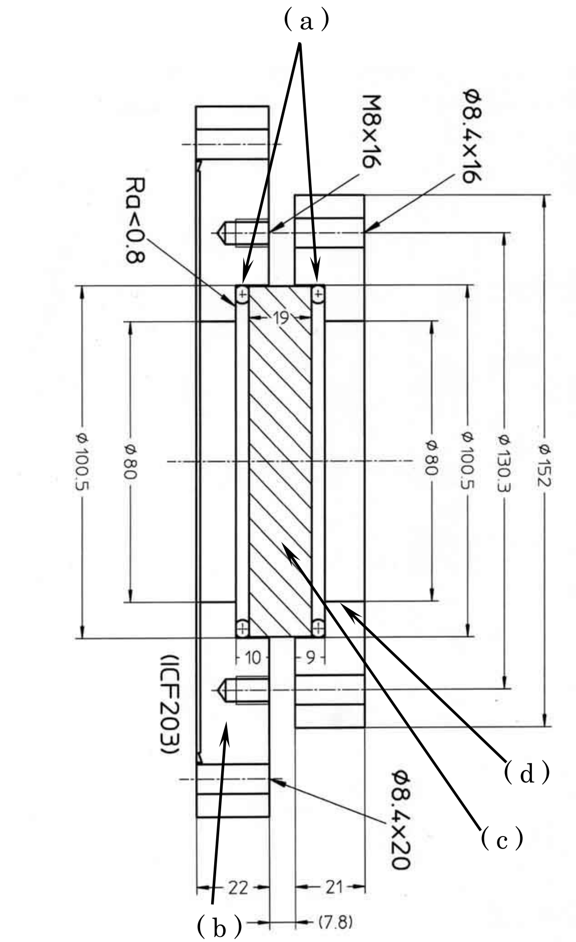
Idea for extraction mirror with X-ray absorber



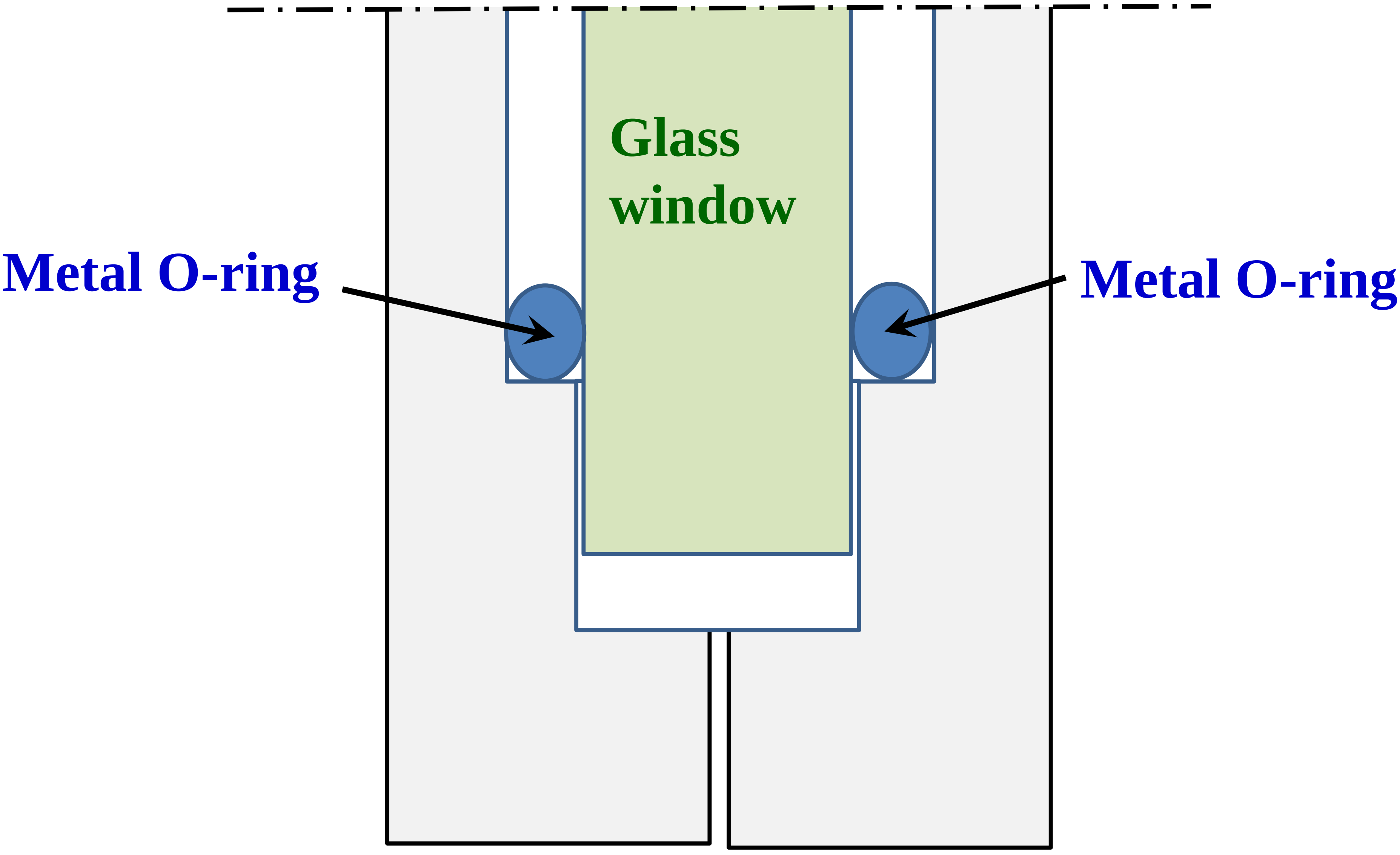
Extraction mirror with X-ray absorber



2. Glass window for the extraction of visible SR

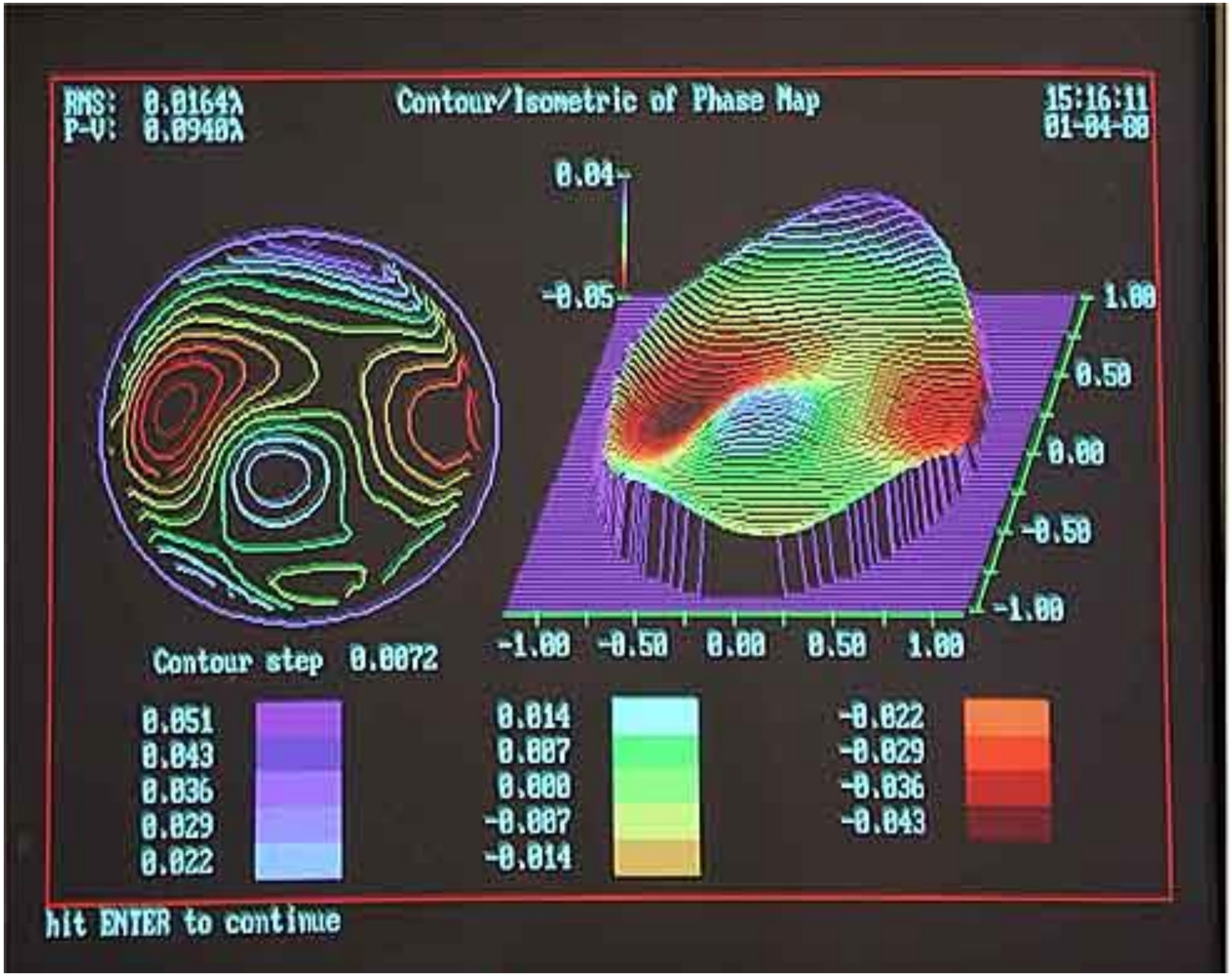


General design of the glass window. In this figure, (a): metal O-ring, (b): vacuum-side conflat flange, (c): optical glass flat, (d): air-side flange.


Glass window
Metal O-ring
Metal O-ring

Transmitted wavefront measured with interferometer

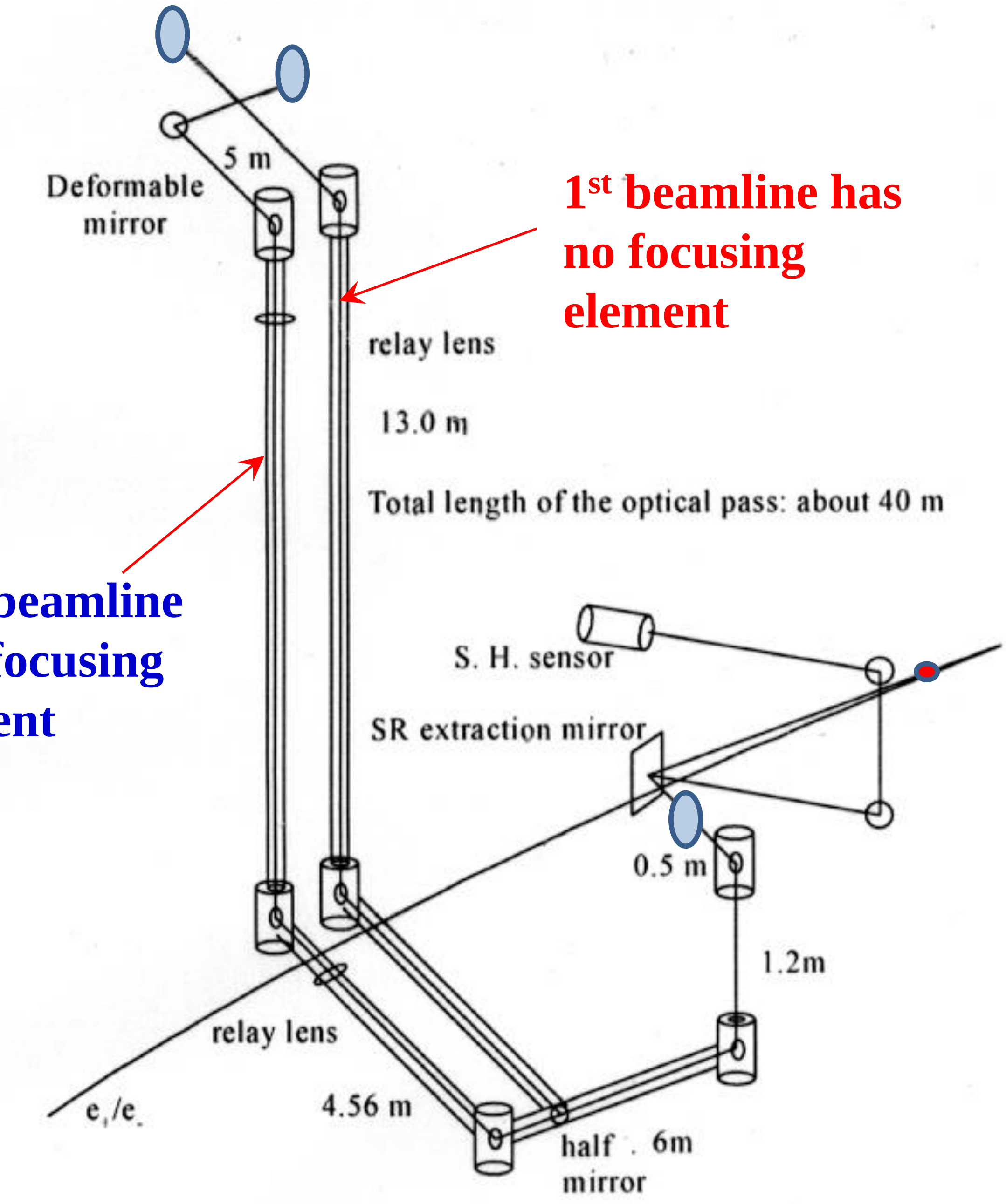
wavefront error is less than λ/20



3.Optical path layout

Normaly, 10-20m, but sometinmes very long!

For example, In the KEK B-factory, total length is about 40m.

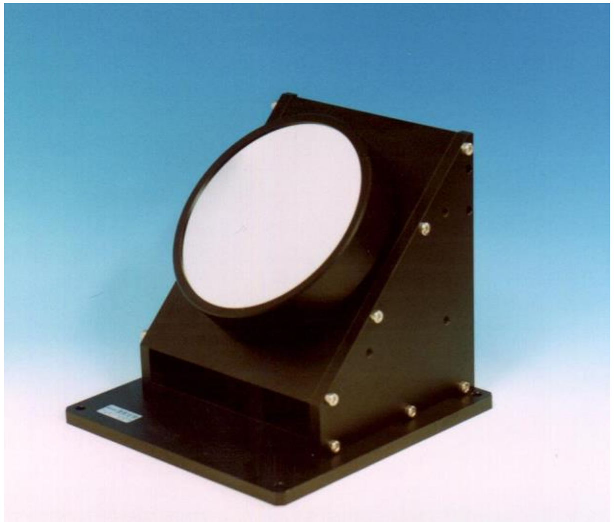
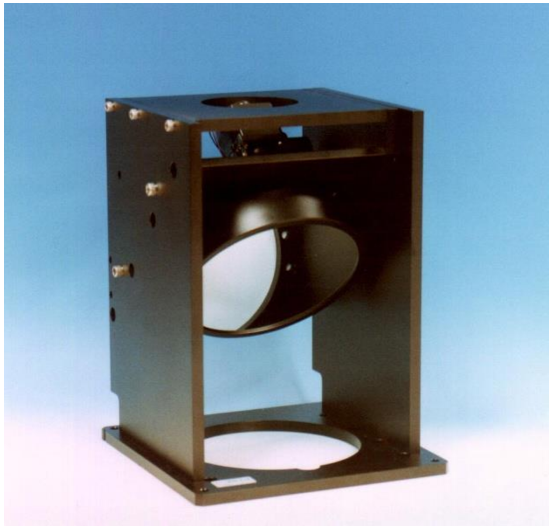
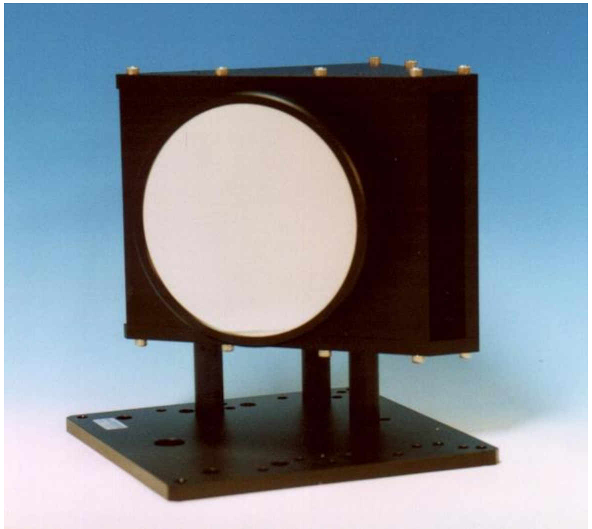


Important points

Air tight structures surrounding of optical line are very important to escape from air turbulence.

Both end of optical line must be closed by glass window to making it air tight.

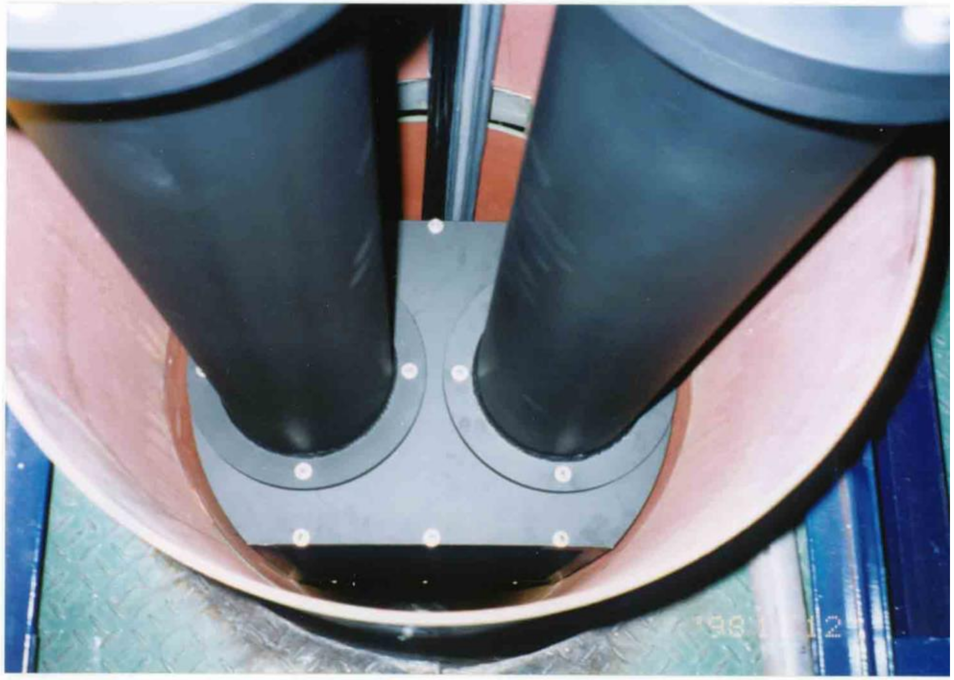
Mirror with its holder used for the optical path

Surface quality: $\lambda/10$

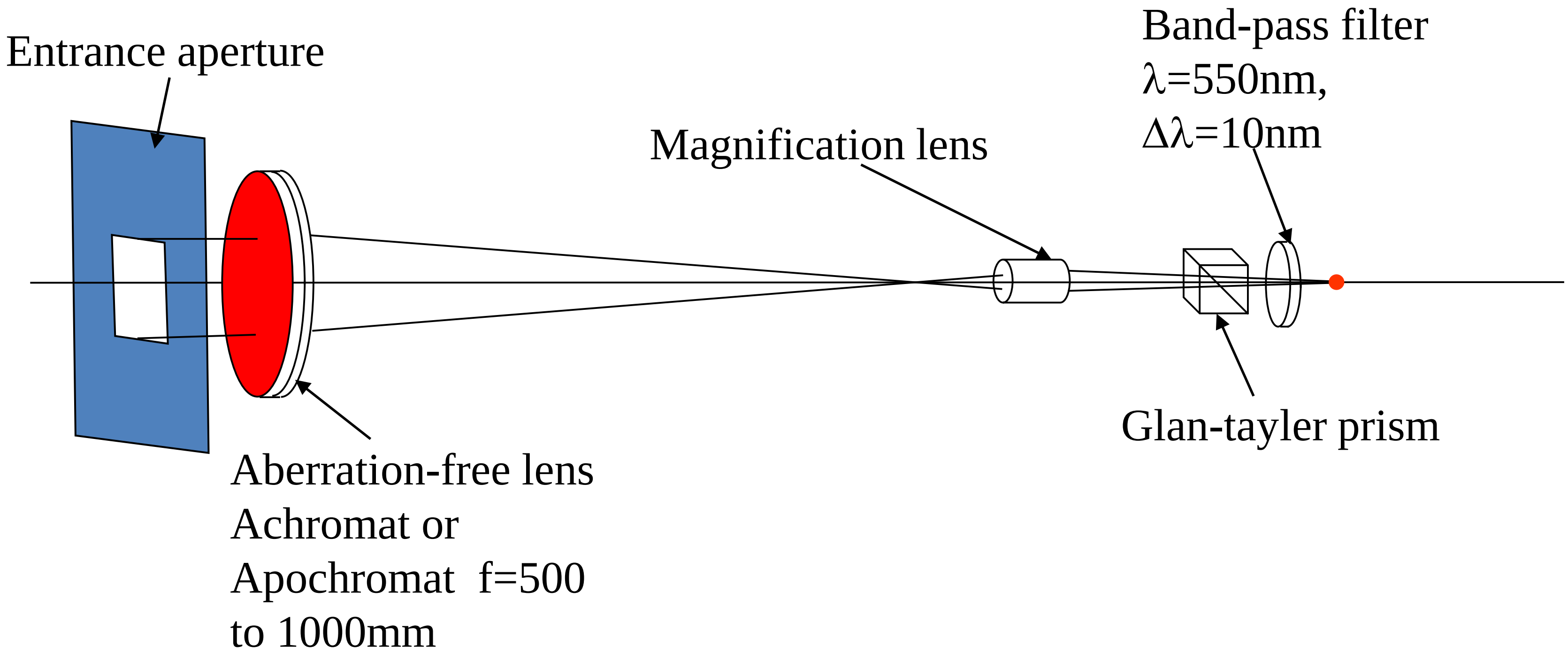
Installation of optical path ducts and boxes at the KEK B-factory

Relay lens installed in the optical path duct

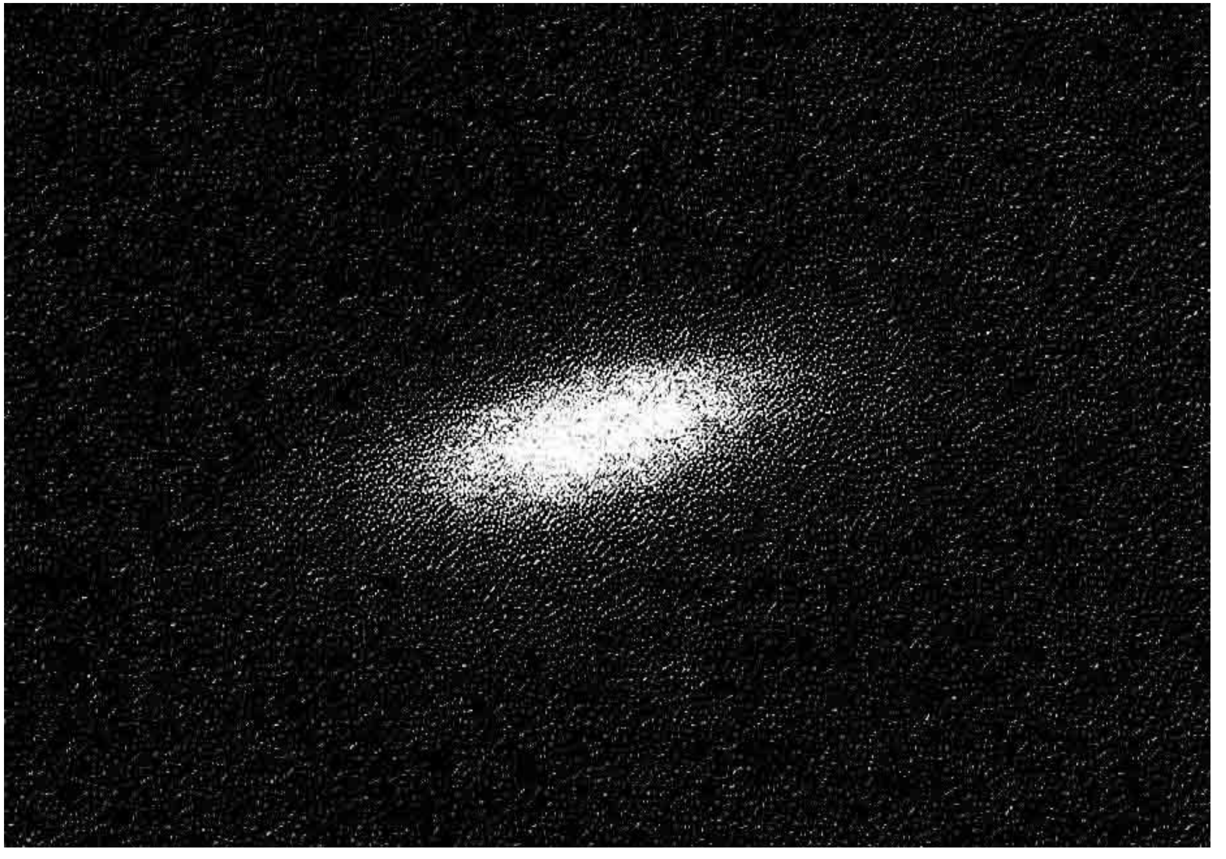


3. Popular equipments in down stream of optical path

Most fundamental equipment ;
focusing system to observe the beam image

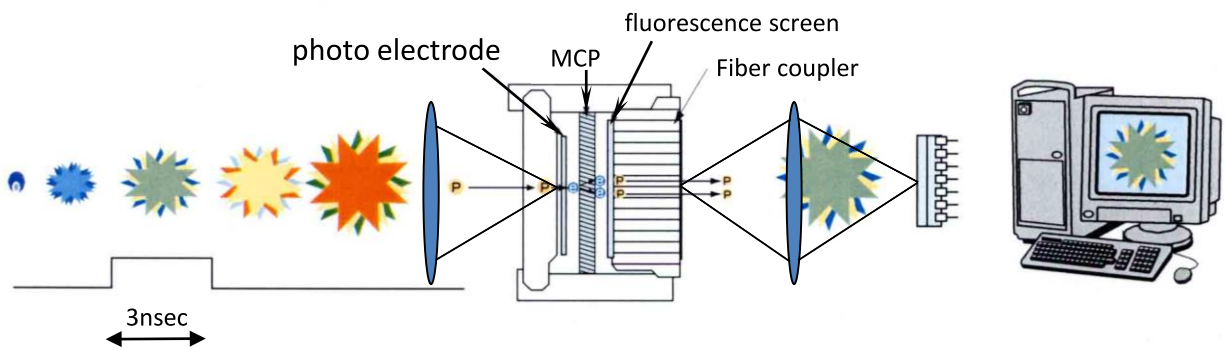


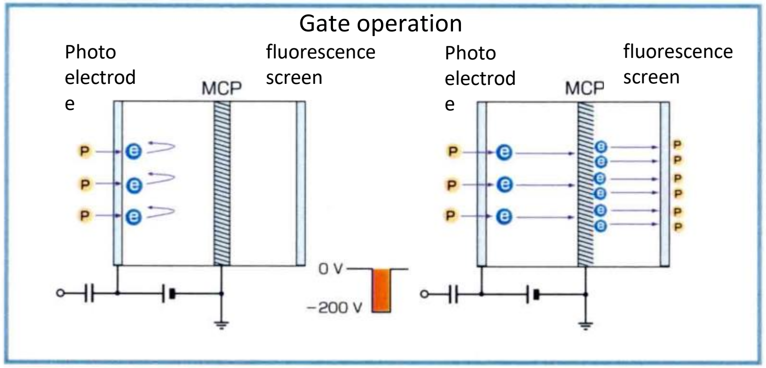
Typical image of the beam



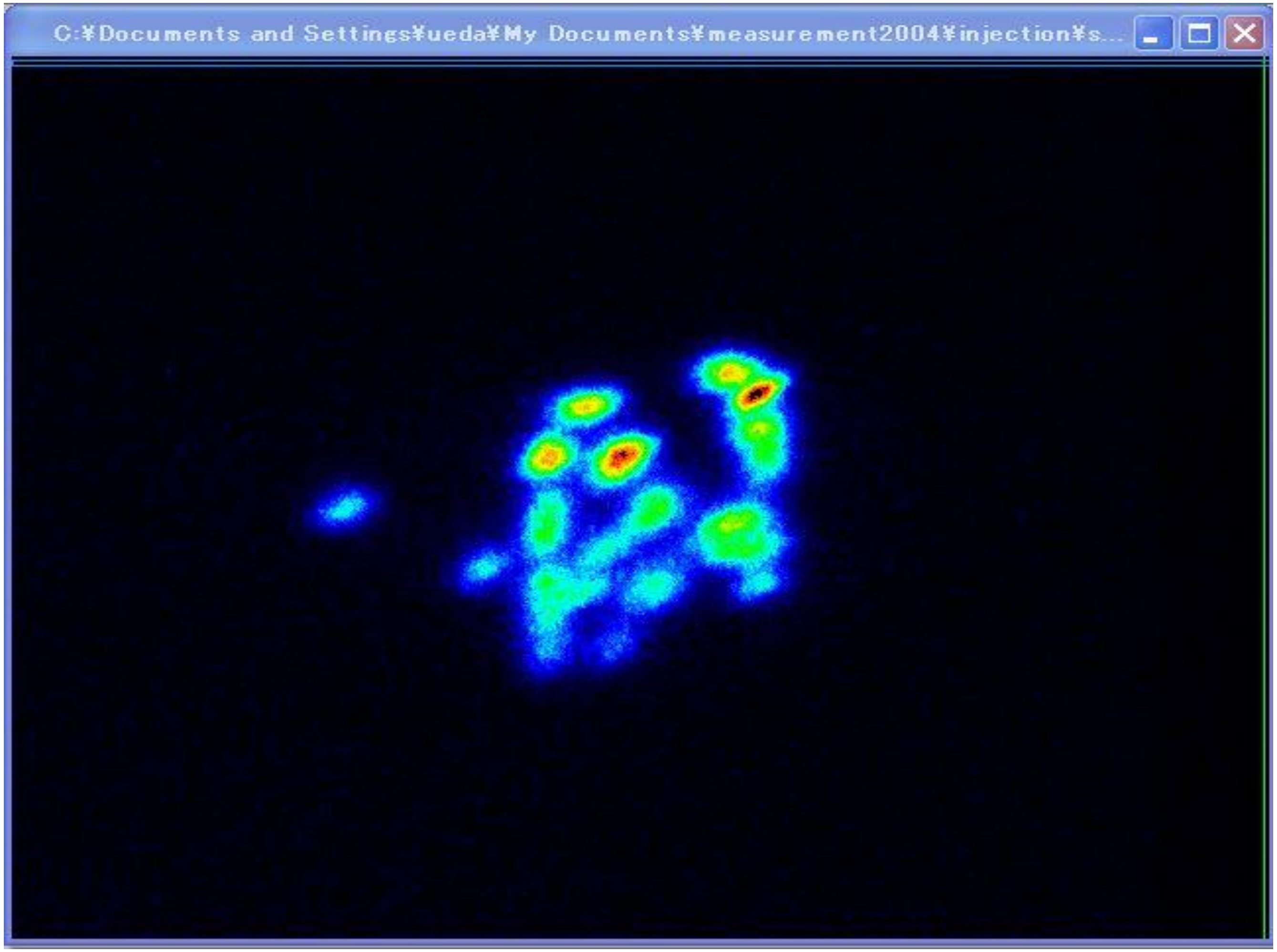
Dynamical observation of beam profile with high-speed gated camera

Function of high-speed gated camera

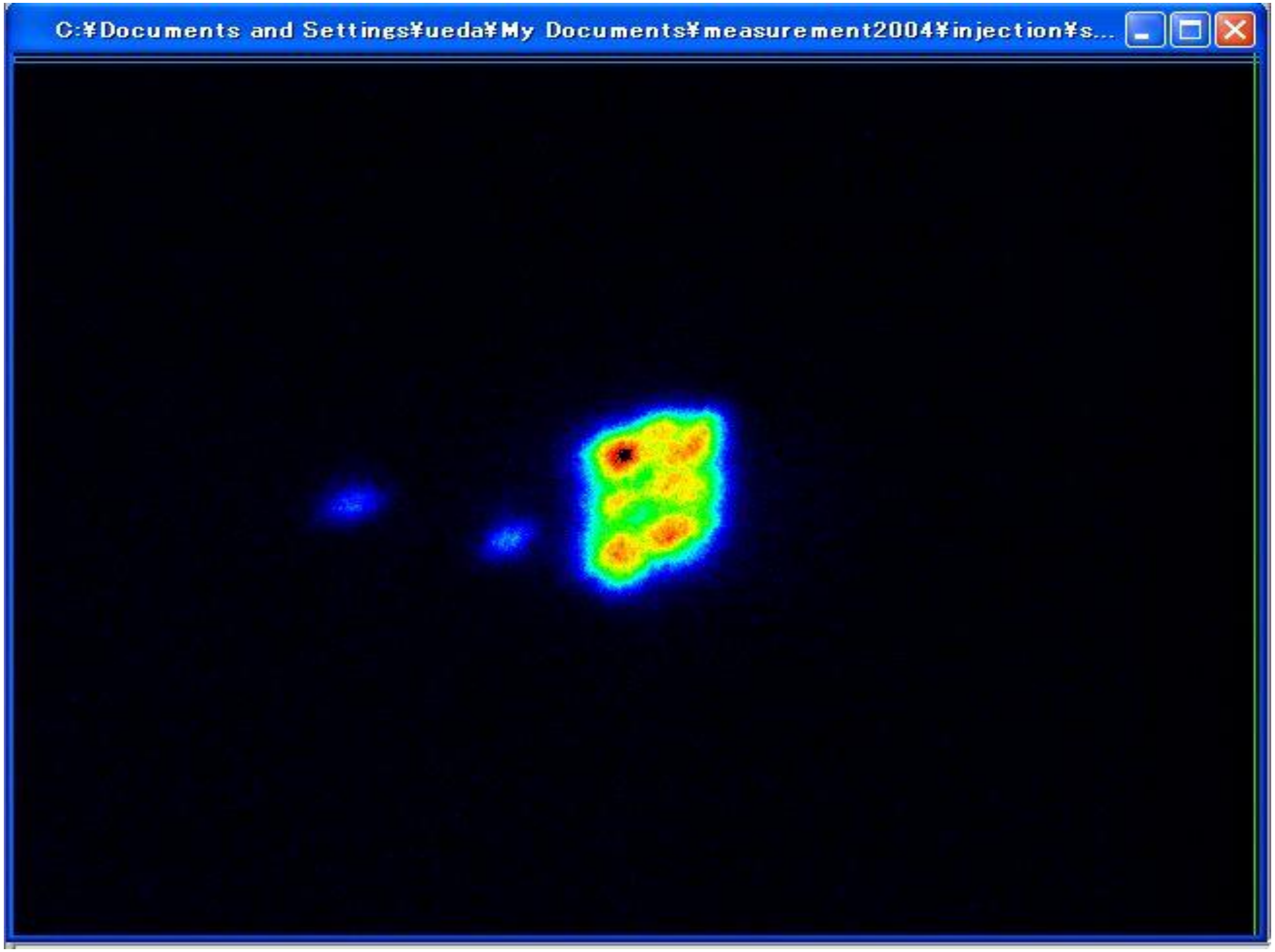




Turn by turn image of injected beam into storage ring

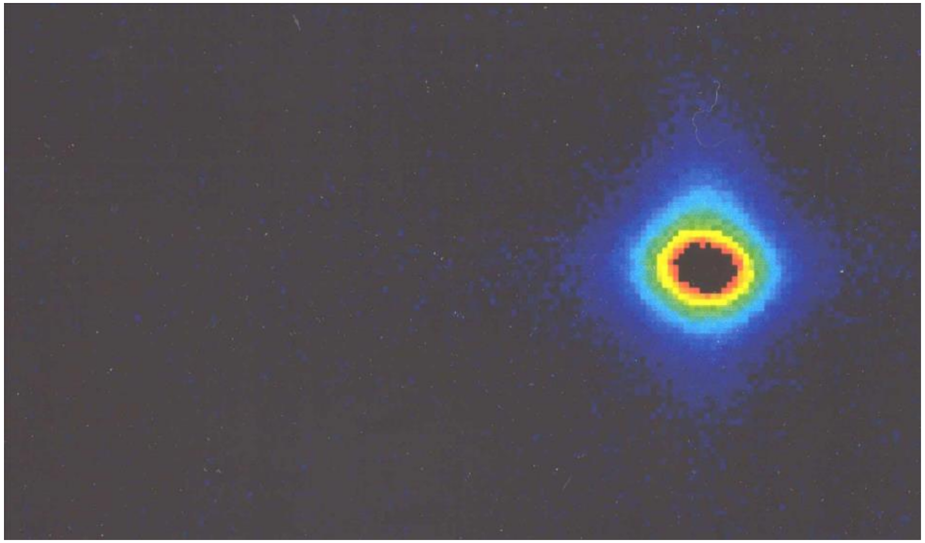


Optimization of injection

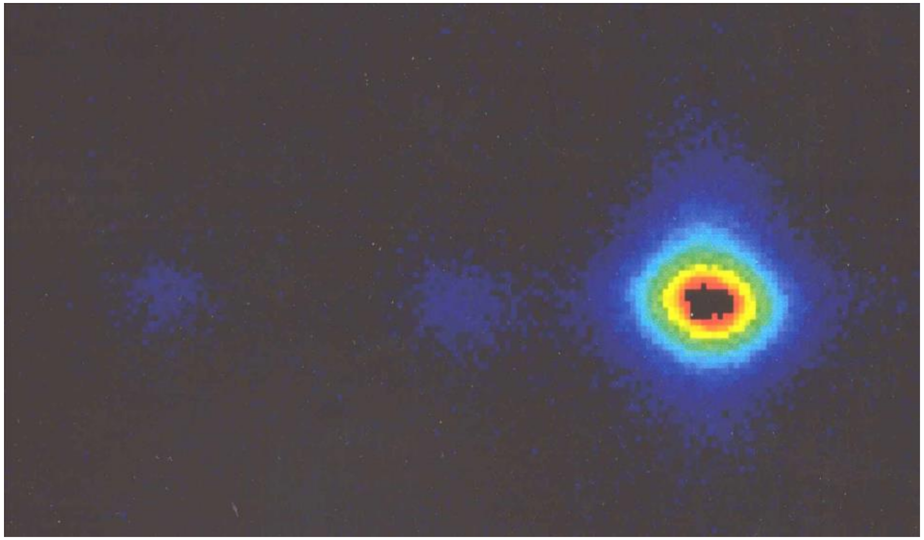


Optimization for Kicker bump

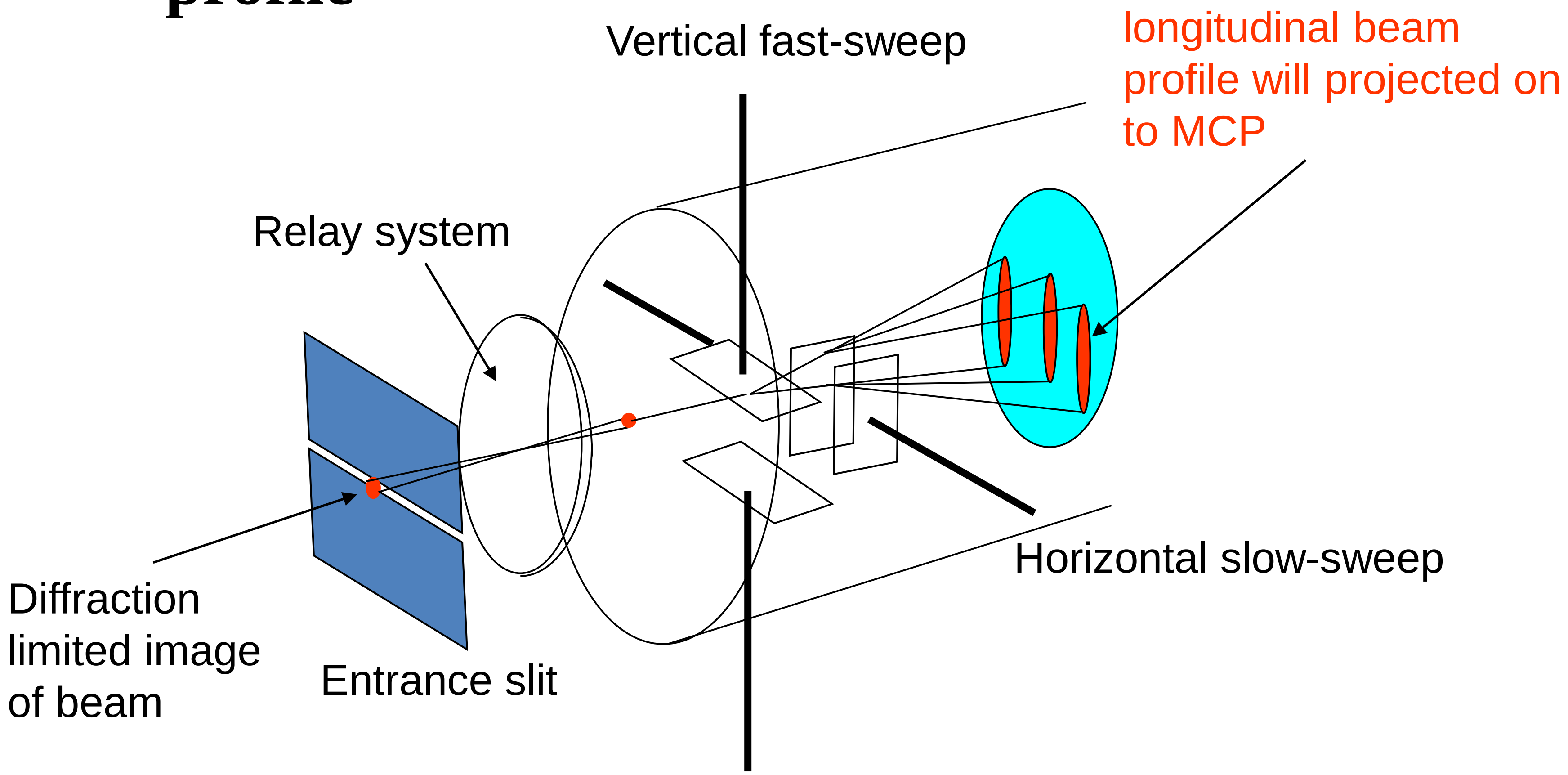
Beam profile without kicker magnet

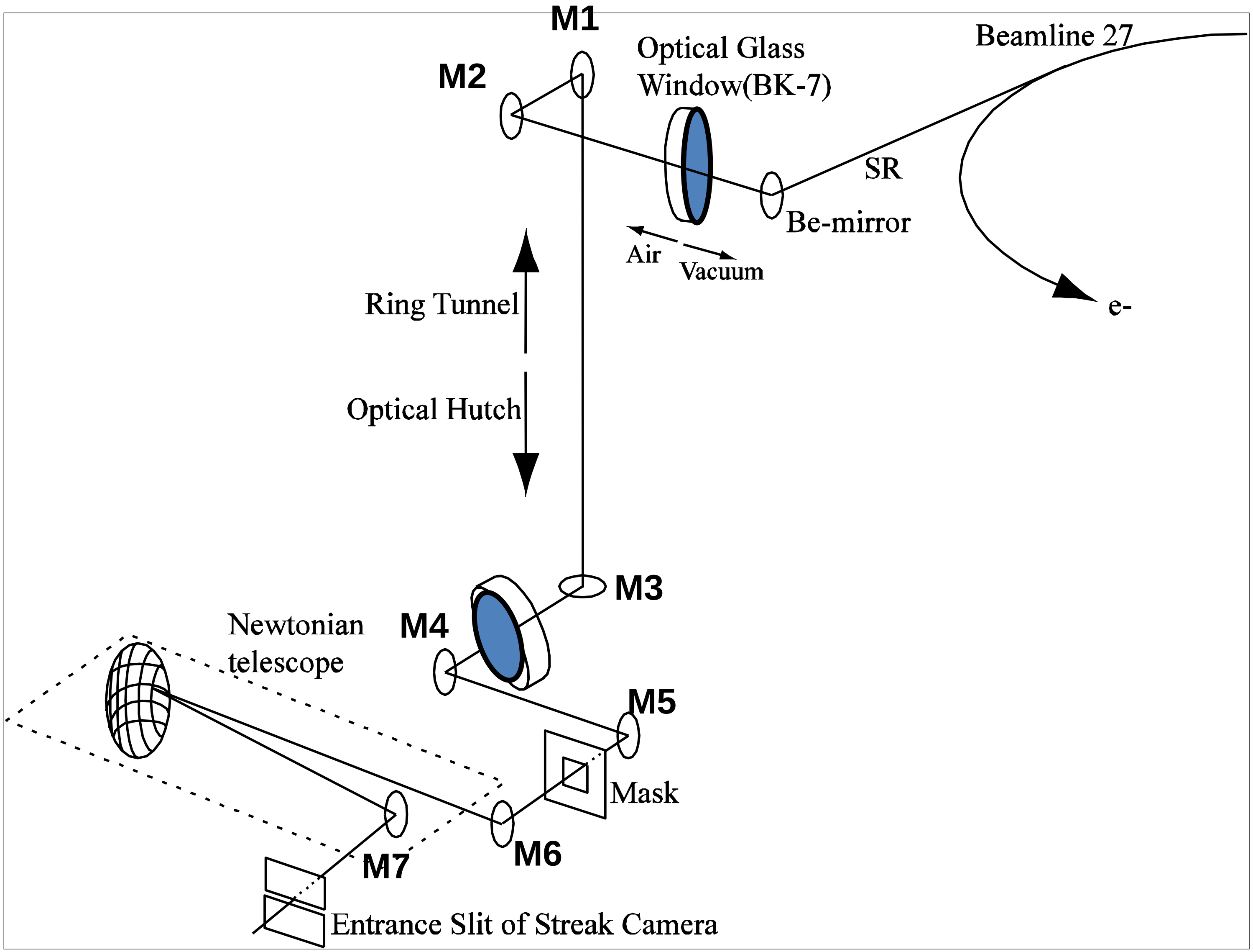


Beam profile with kicker magnet

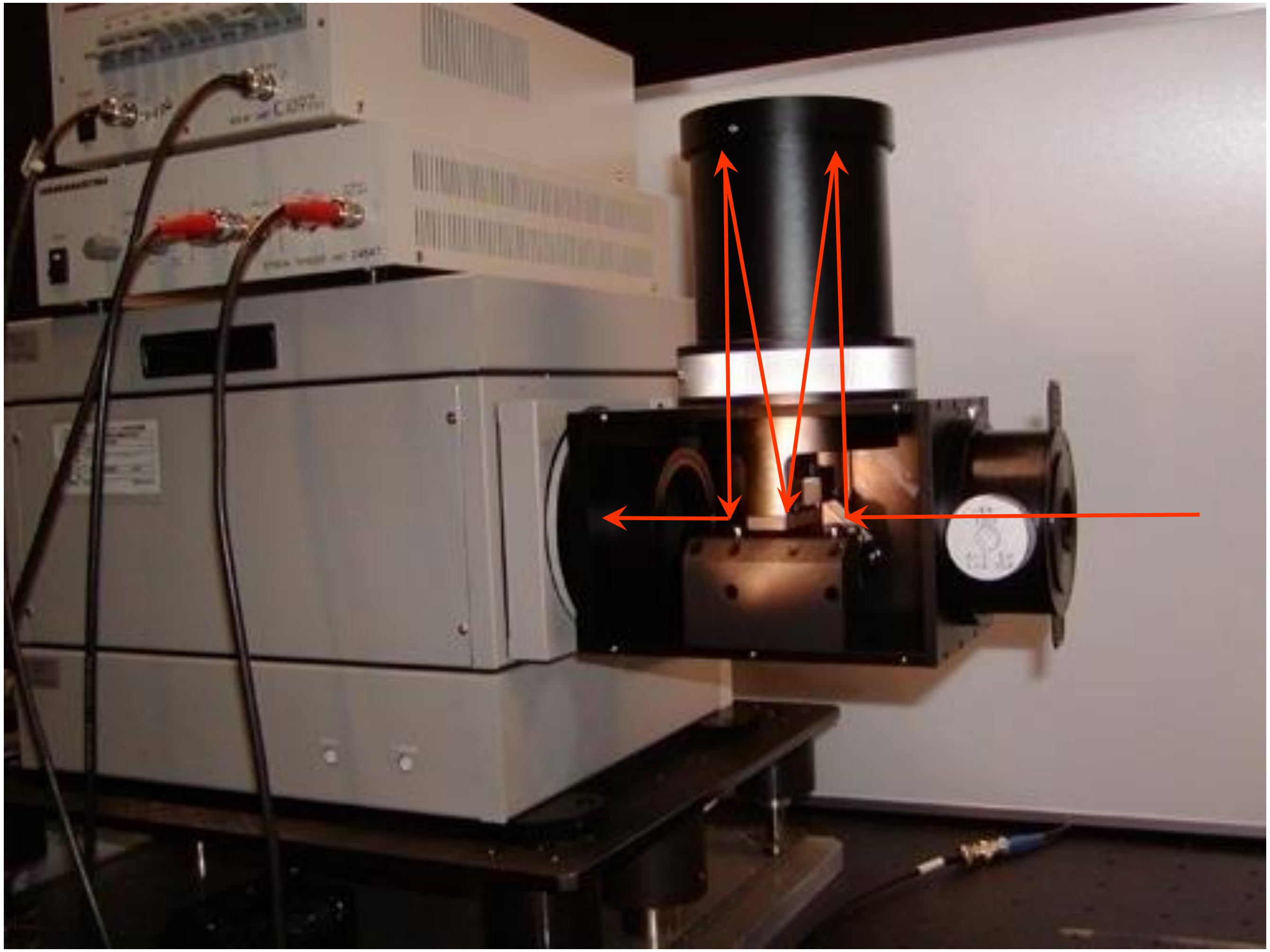


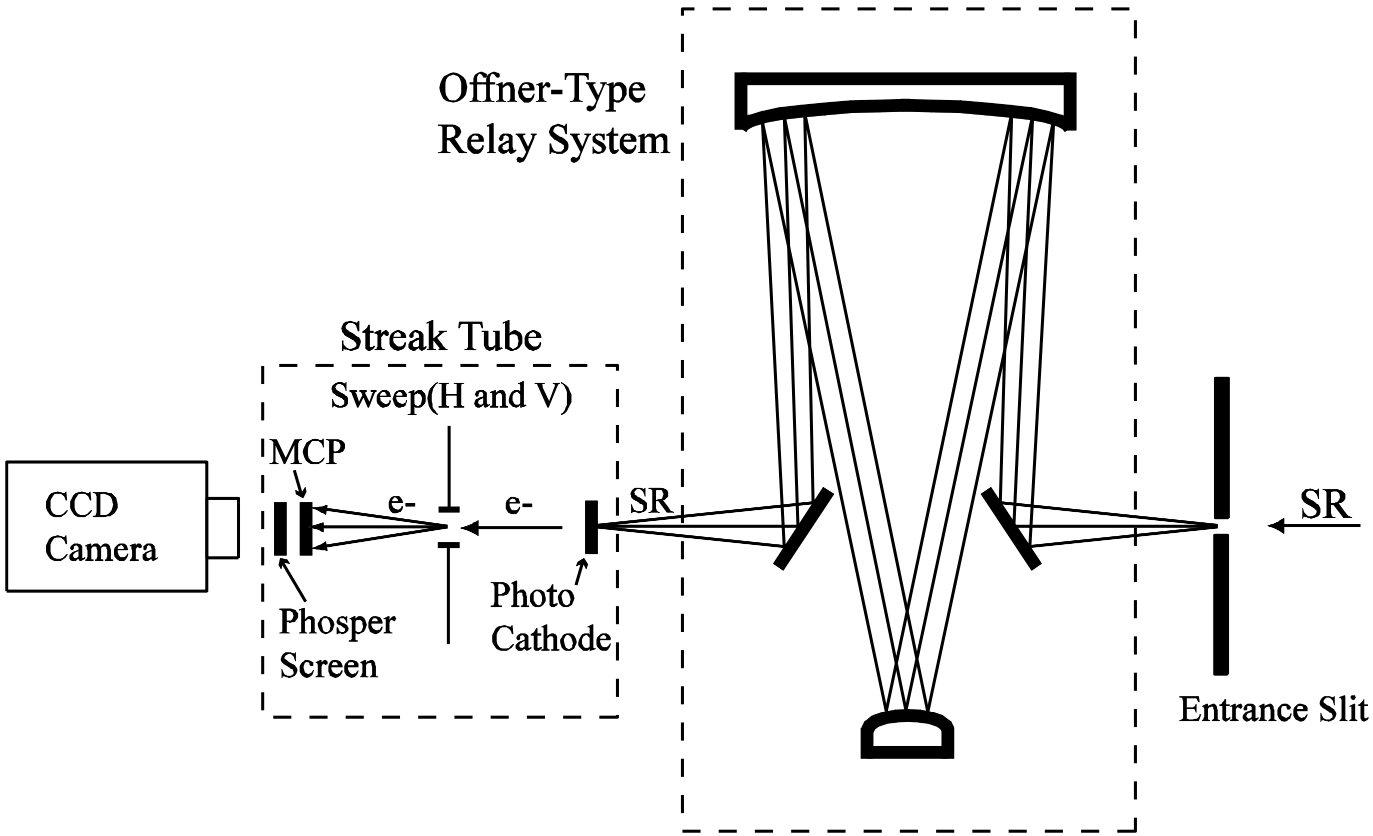
Streak camera to measure longitudinal profile



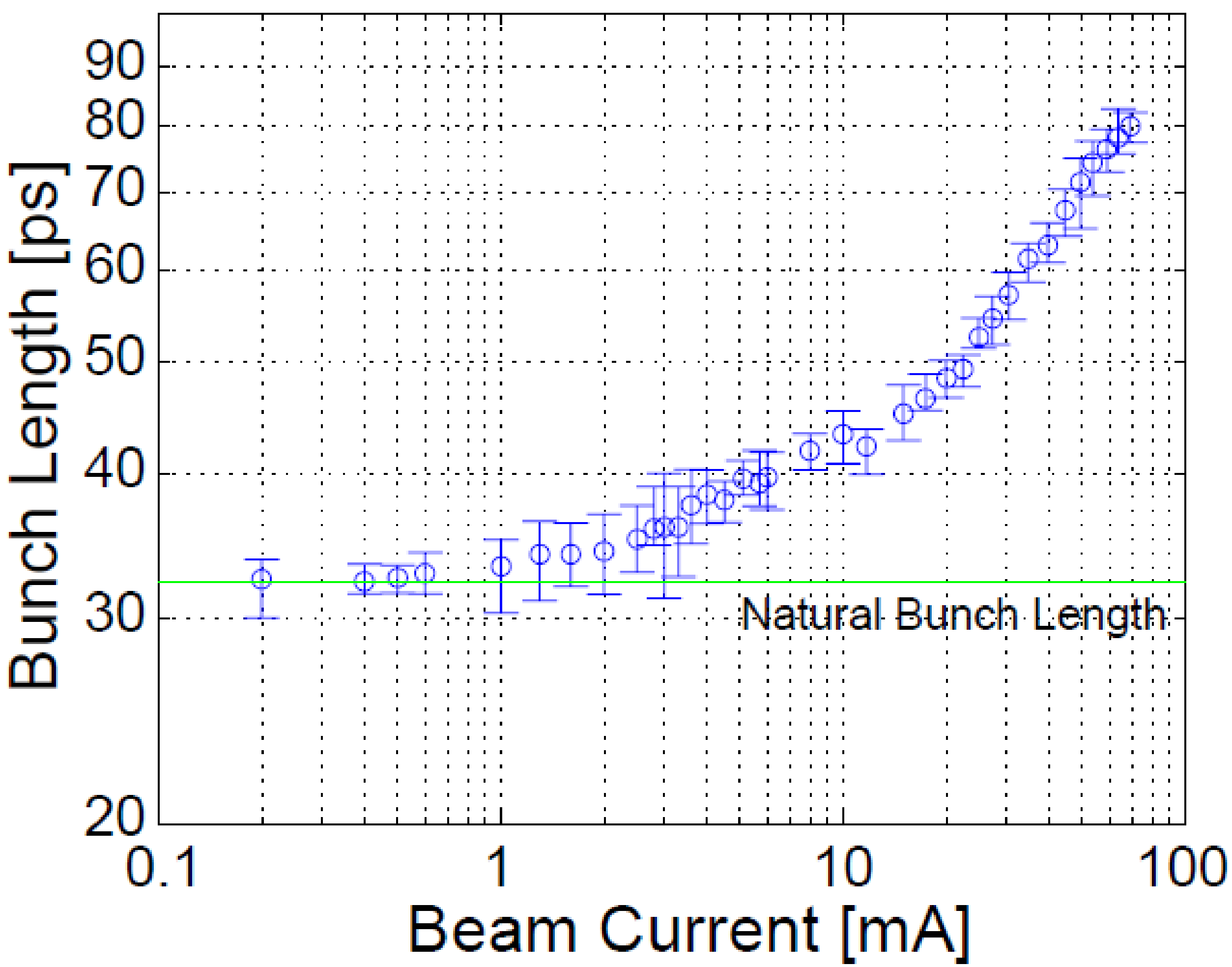


M1
M2
Optical Glass
Window(BK-7)
Beamline 27
SR
Be-mirror
Air
Vacuum
e-
Ring Tunnel
Optical Hutch
M3
M4
Newtonian
telescope
M5
Mask
M6
M7
Entrance Slit of Streak Camera

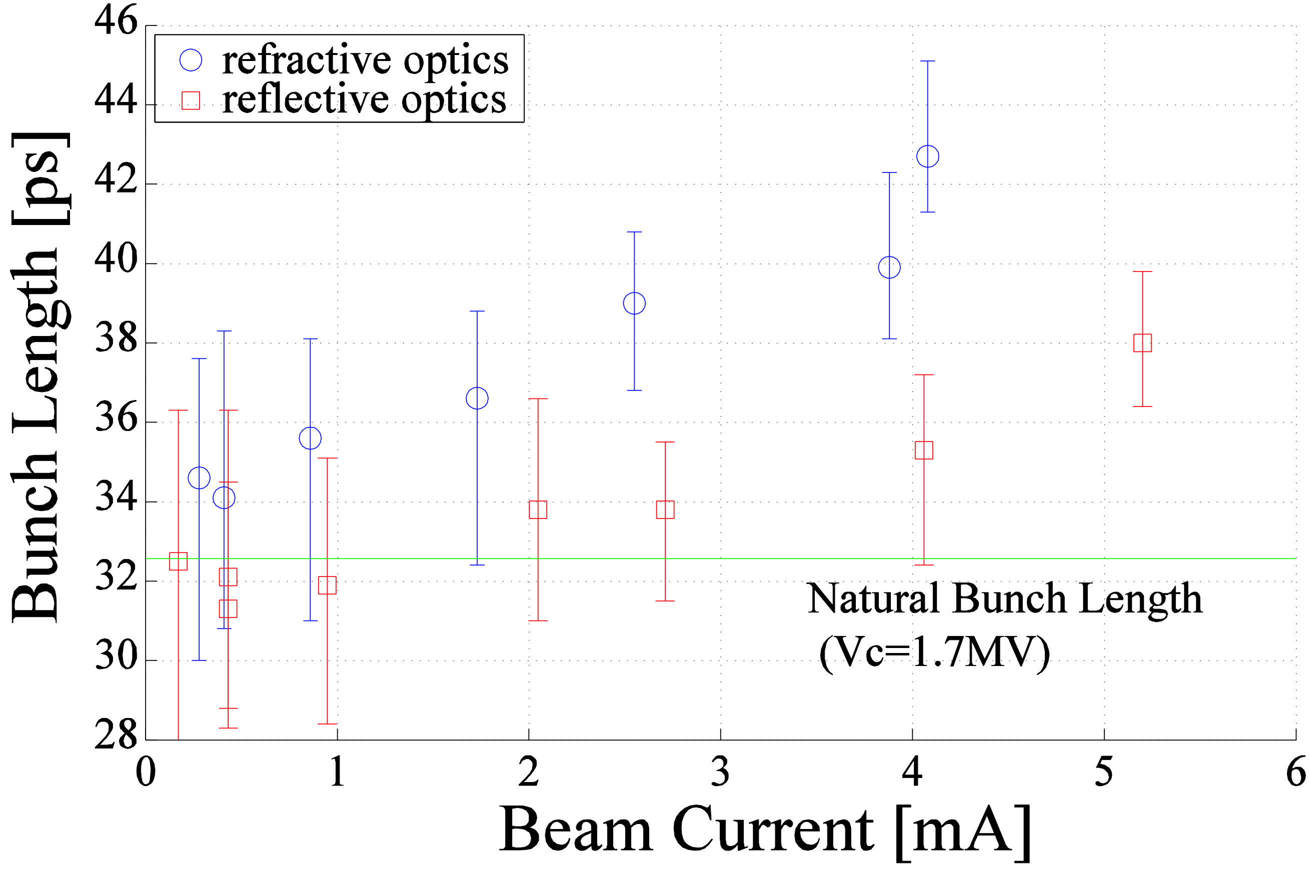



Offner-Type
Relay System
Streak Tube
Sweep(H and V)
MCP
CCD
Camera
e-
e-
SR
Photo
Cathode
Phosper
Screen
SR
Entrance Slit

Result of bunch length at rage between 0.2mA to 70mA

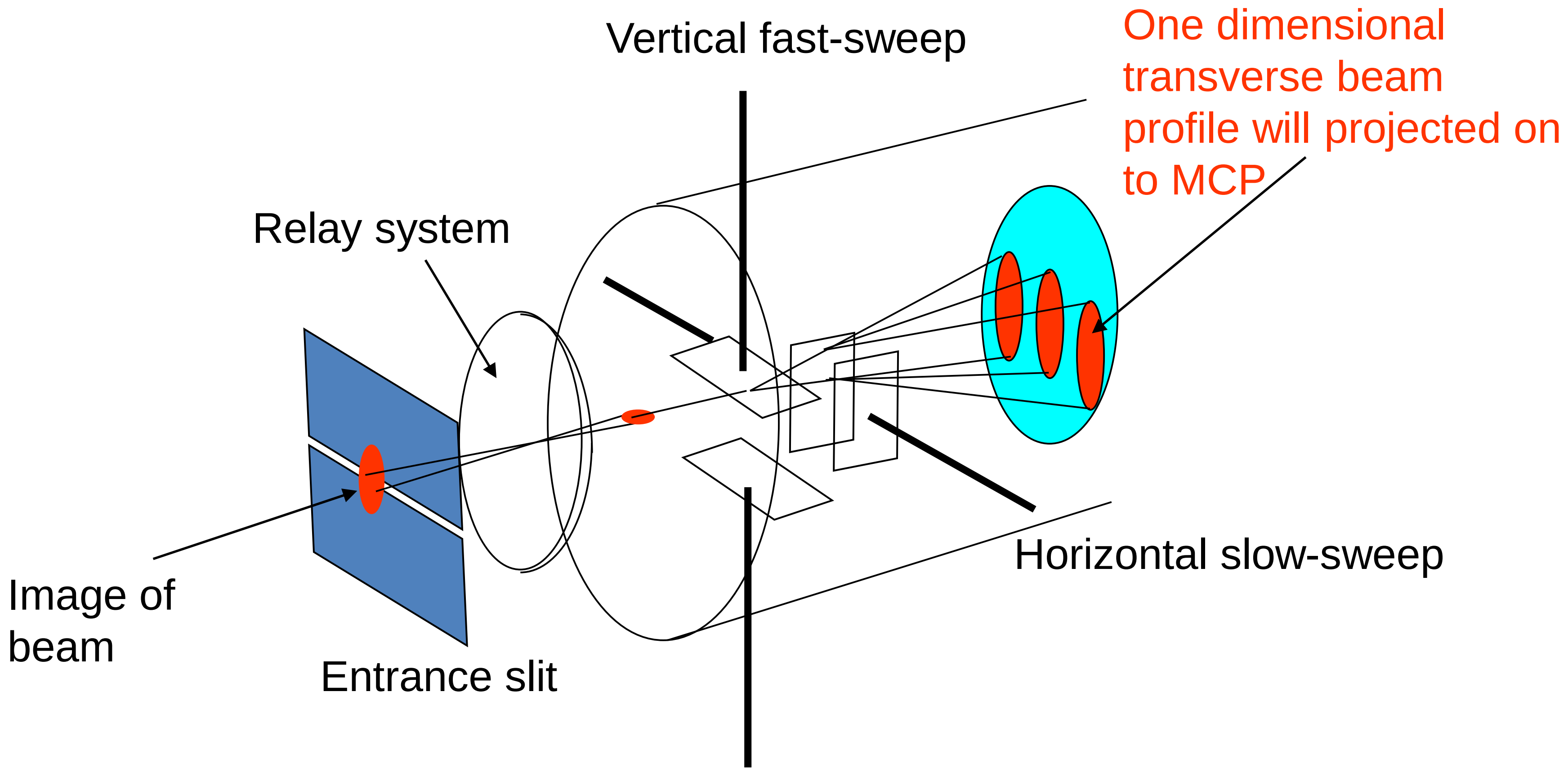


Error in bunch length measurement due to chromatic aberration

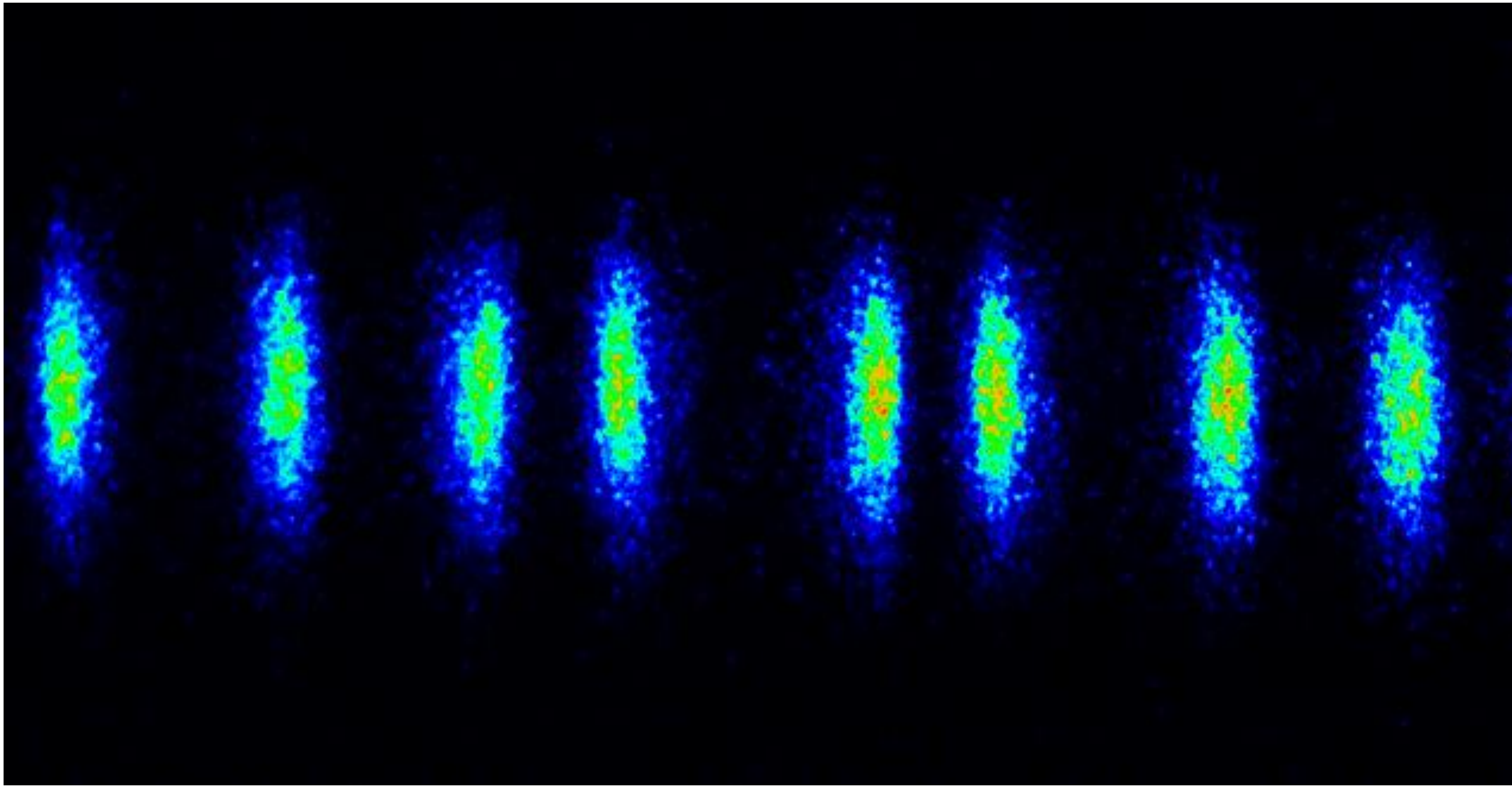


Result of bunch length measurement at the Photon Factory by white ray (non-monochromatic)

Dynamical observation for beam instability using the streak camera

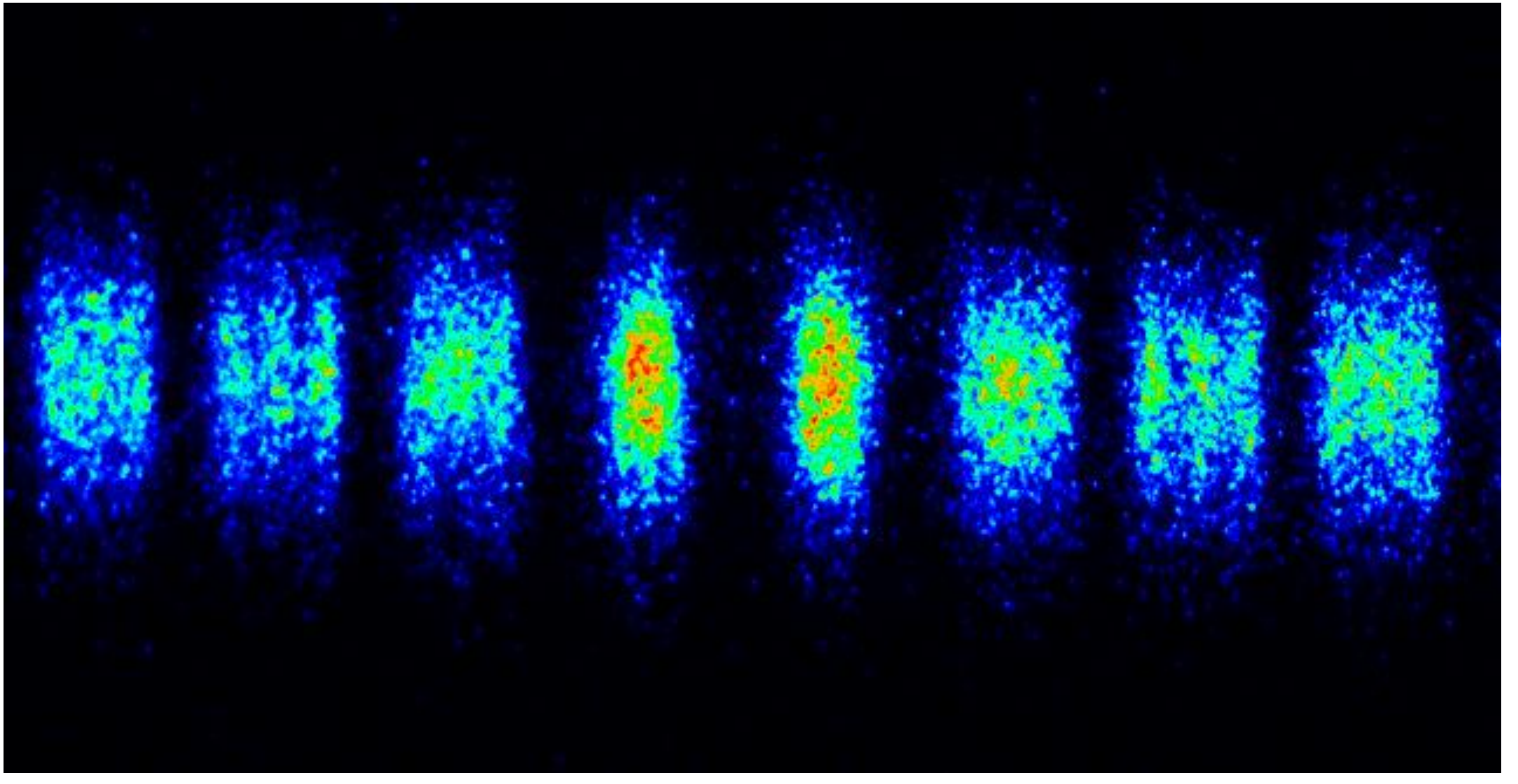


Fast temporal scan

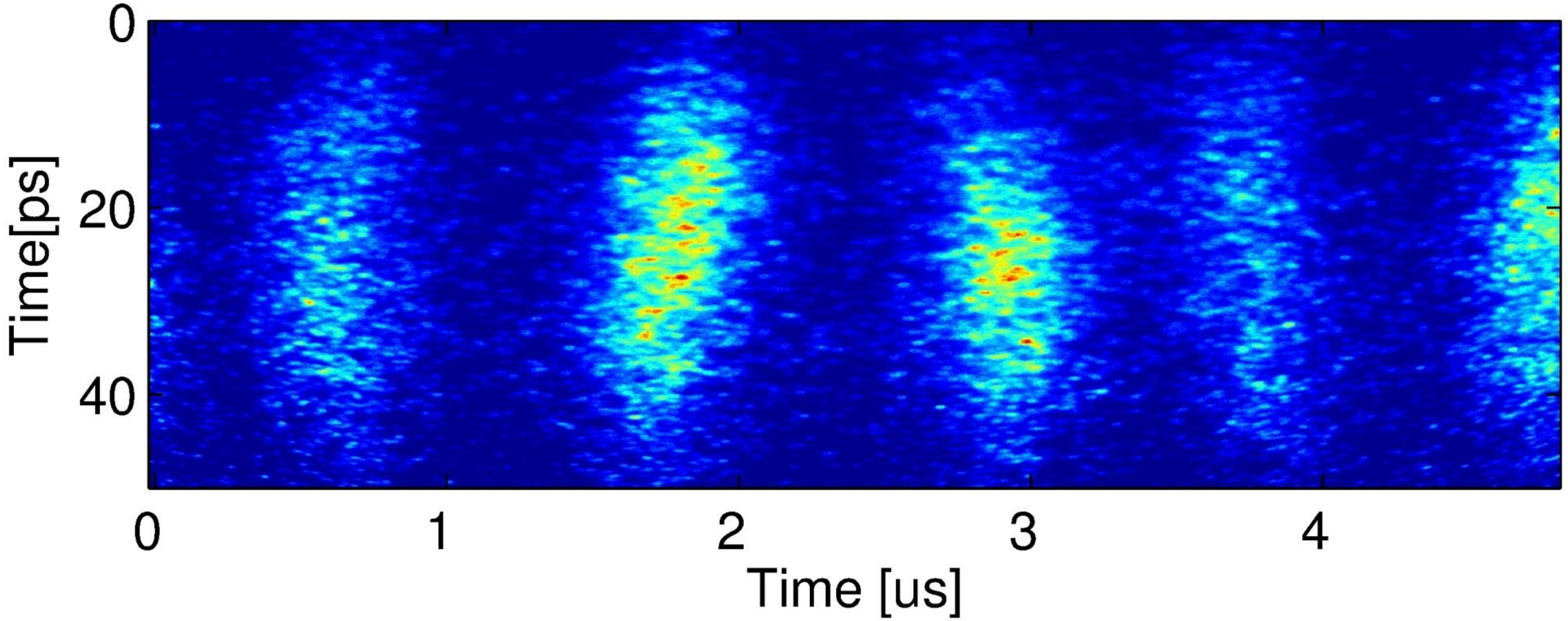


Turn by turn
Vertical beam
profile

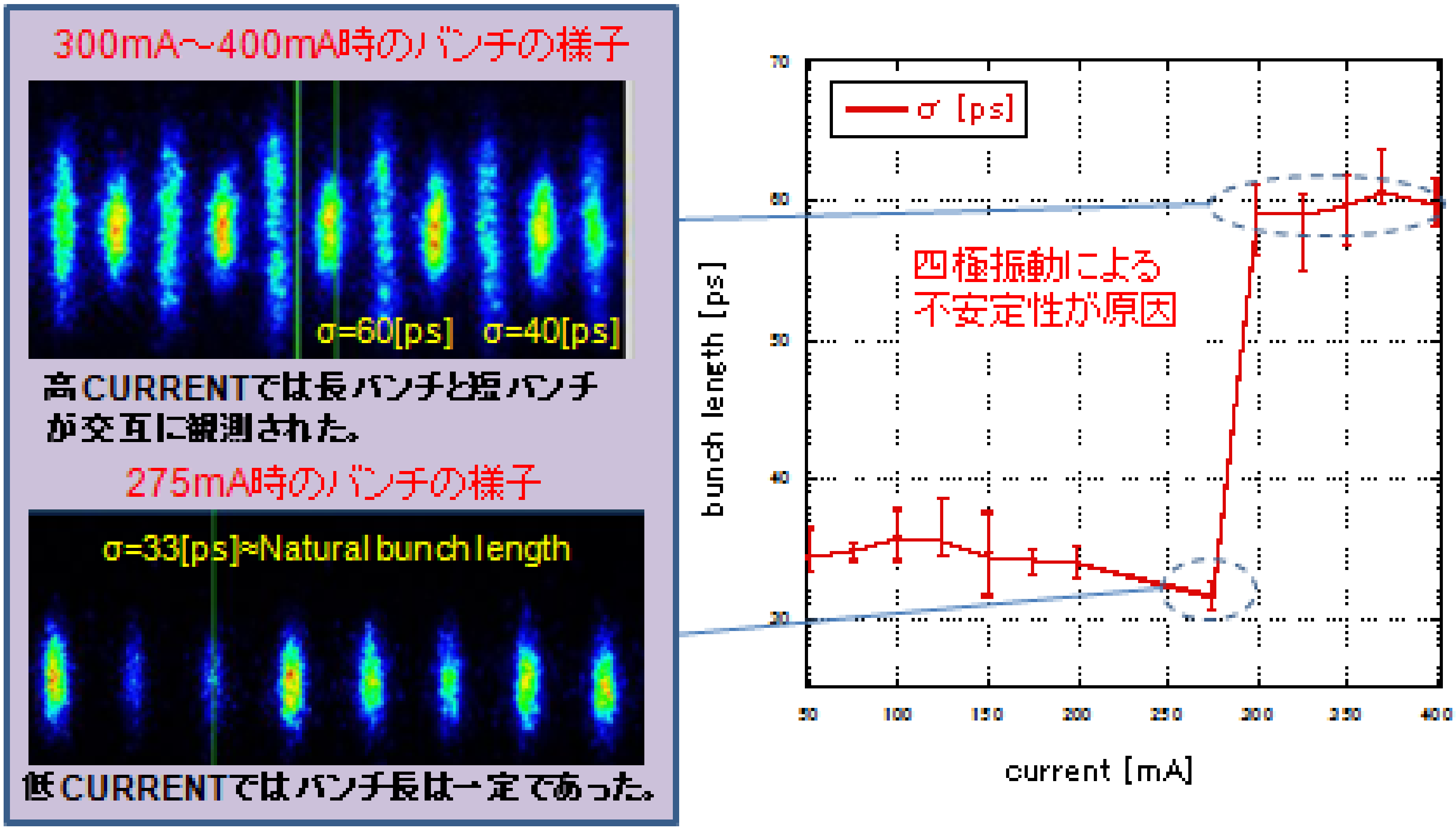
Observation of transverse quadruple motion in the vertical beam profile



Head-tail oscillation



Longitudinal profile oscillation



Focusing components are used everywhere in the SR monitor!

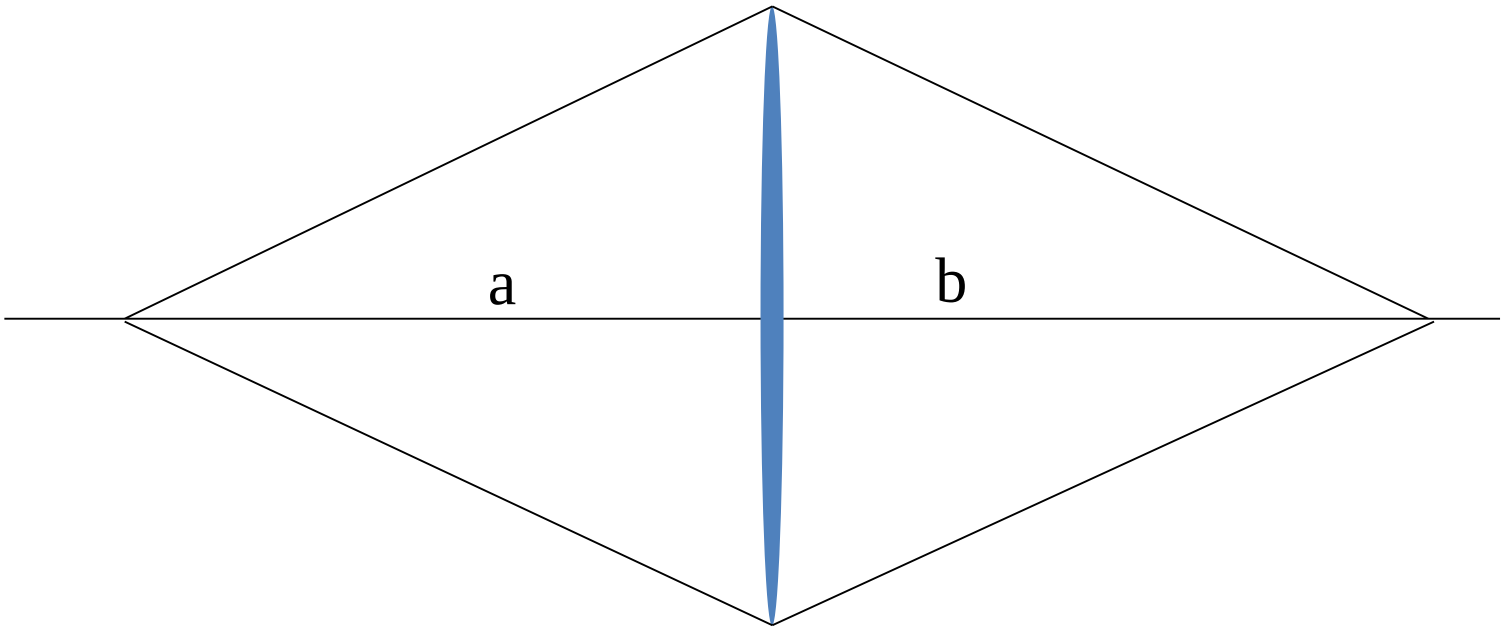
Optics to understand focusing system is most important issue in the SR monitor

Lecture 2

Geometrical optics of focusing system

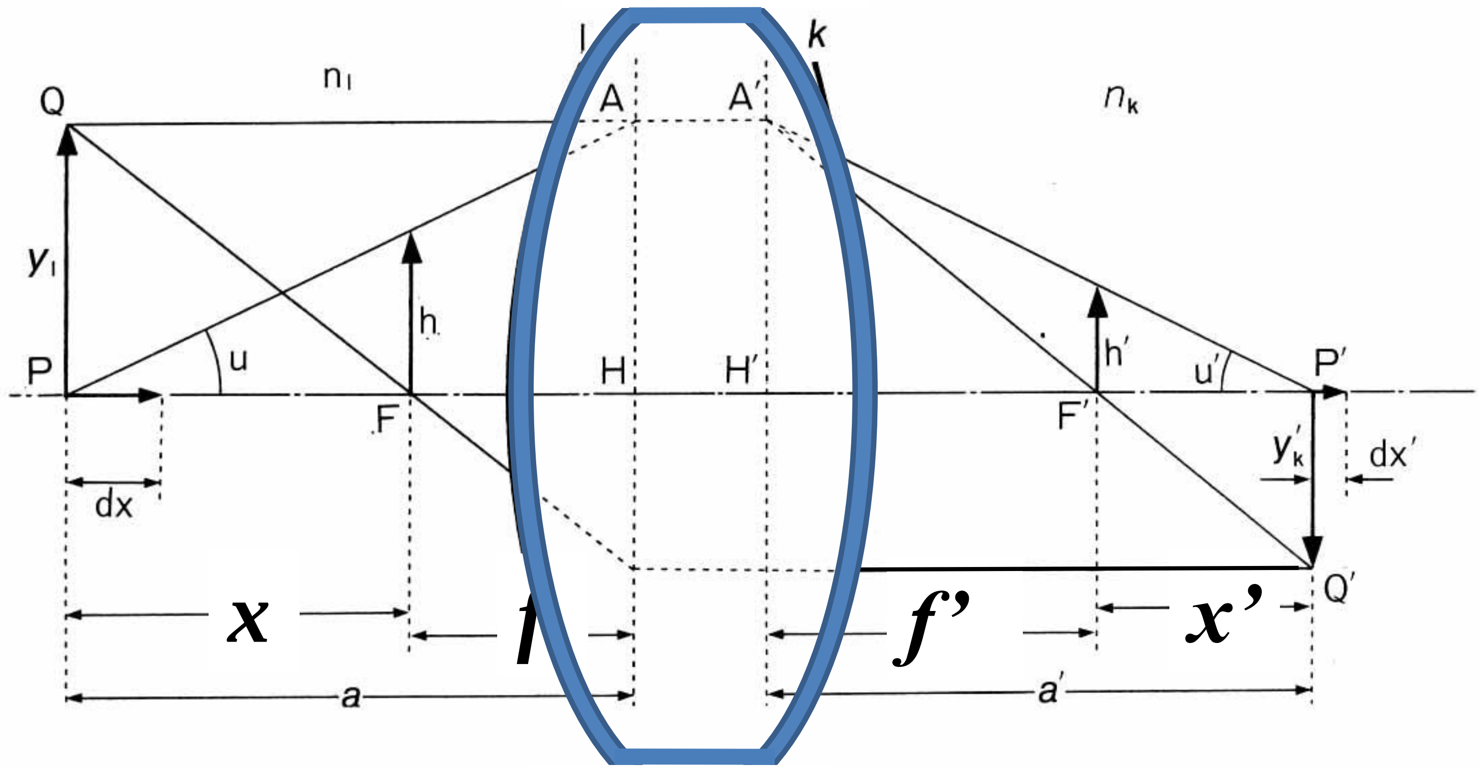
Geometrical optics of focusing system

Thin lens approximation



$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

Newton's equation for thick lens



$$\beta = \frac{y'_k}{y_1} = -\frac{x'}{f'} = \frac{f}{x}$$

Conjugation points

P: front focus point

P': back focus point

H: front principal point

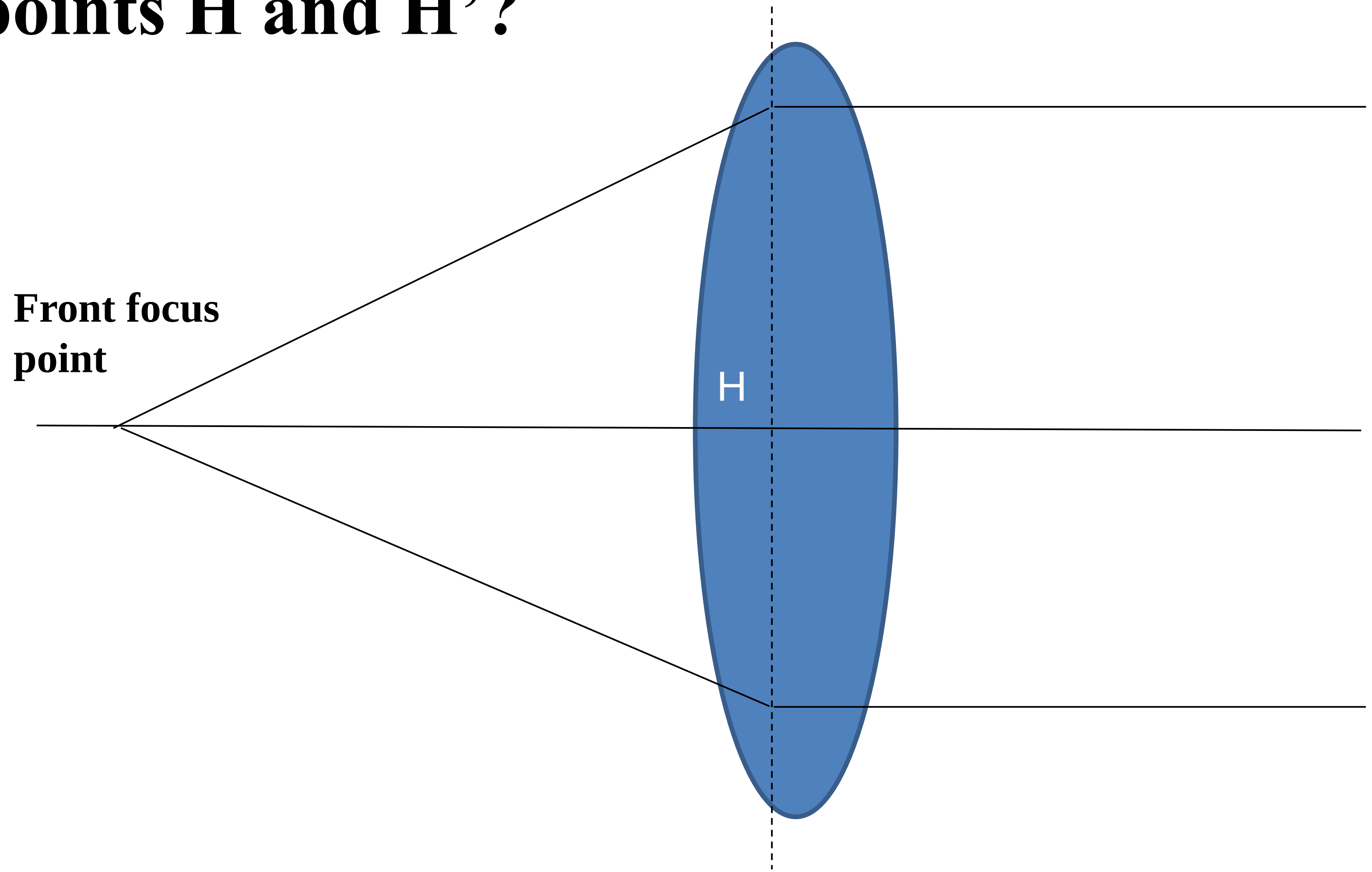
H': back principal point

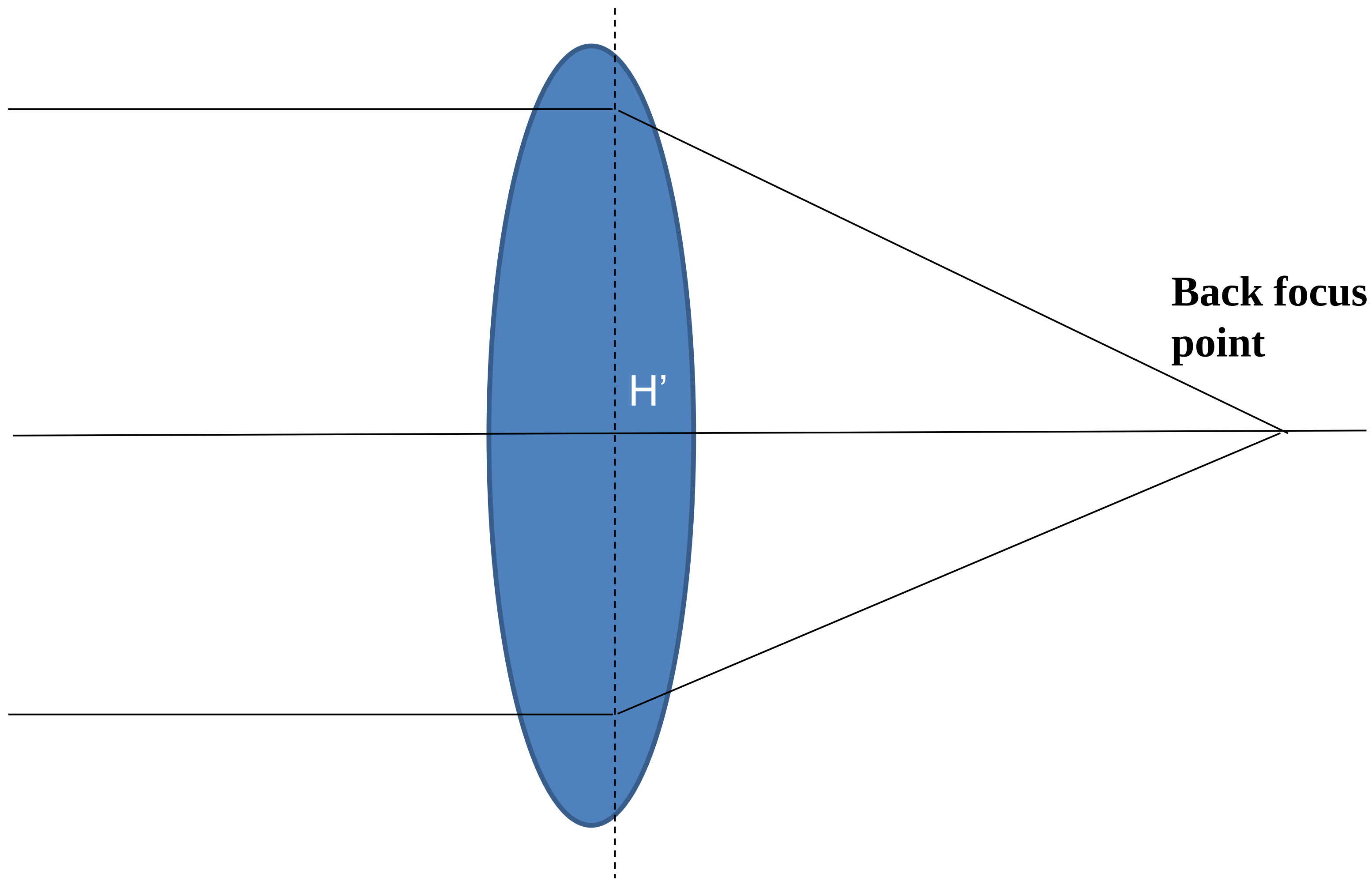
Longitudinal magnification

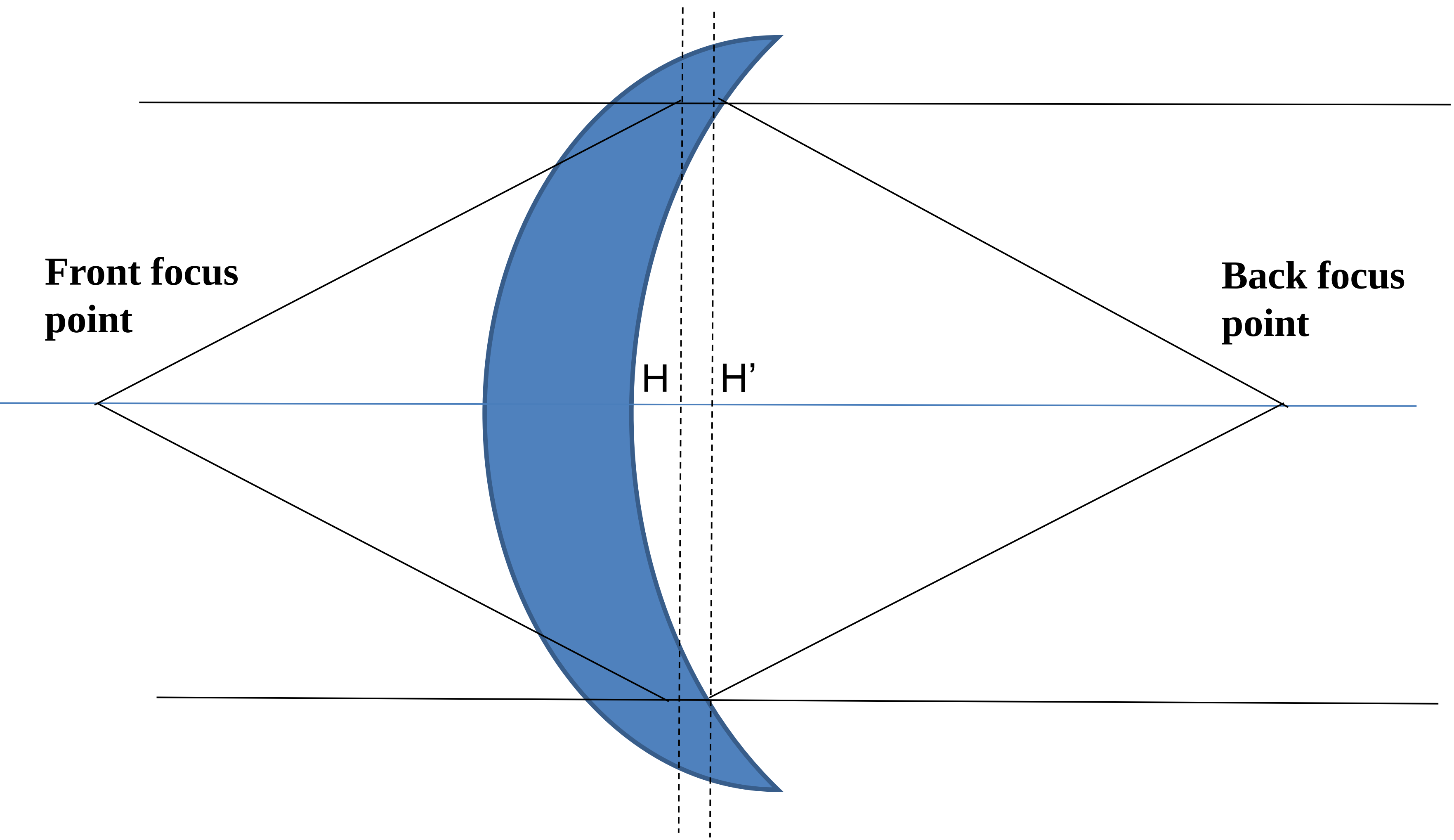
$$\frac{\Delta x_1}{\Delta x_2} = \frac{dx_1}{dx_2} = \frac{d}{dx_2}\left(-\frac{f^{\,2}}{x_2}\right) = \frac{f^{\,2}}{x_2^2} = \beta^2$$

Longitudinal magnification is given by square of transverse magnification

What is principle points H and H'?



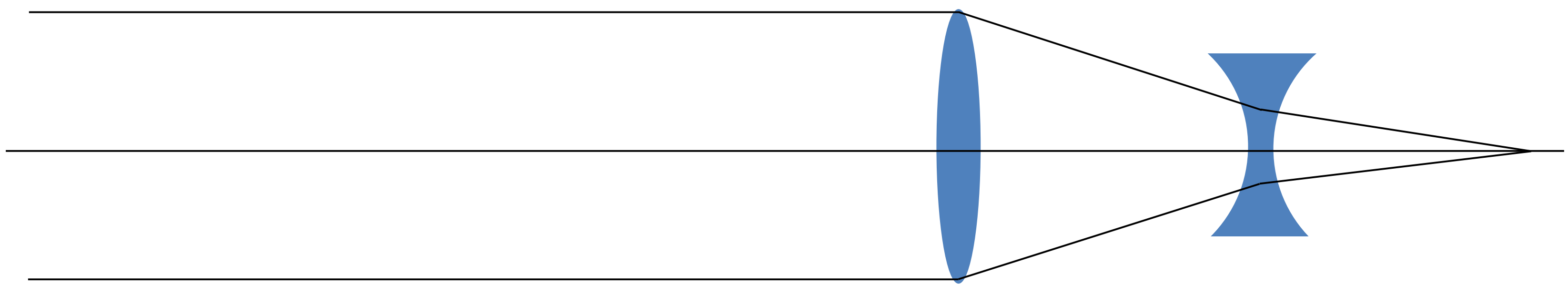

Back focus point
H'



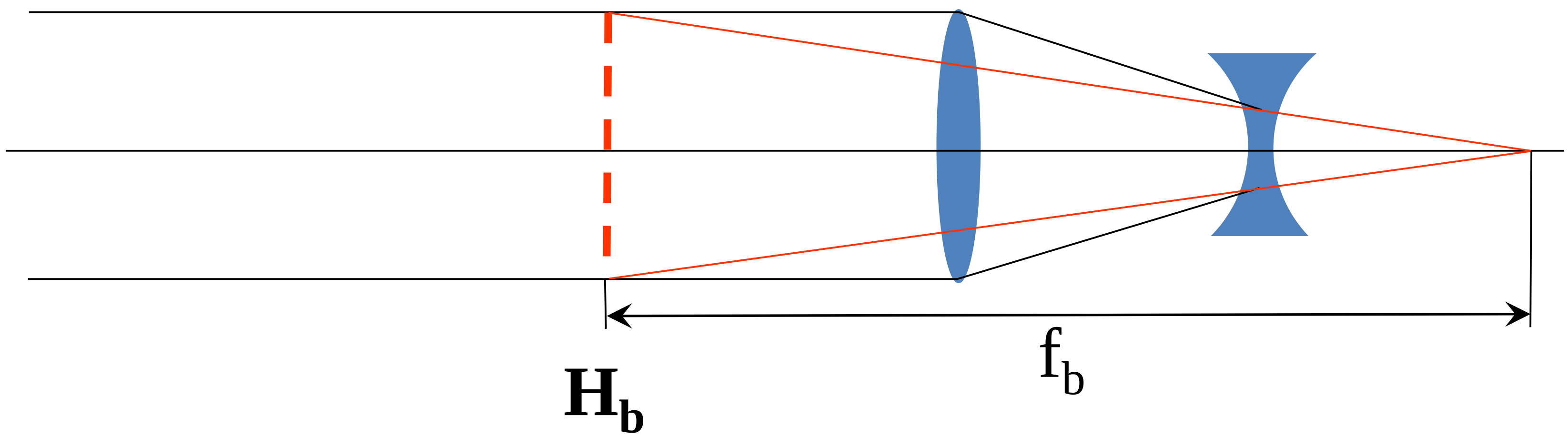
Front focus point
H
H'
Back focus point

How about this combination?

Where is back principal point?

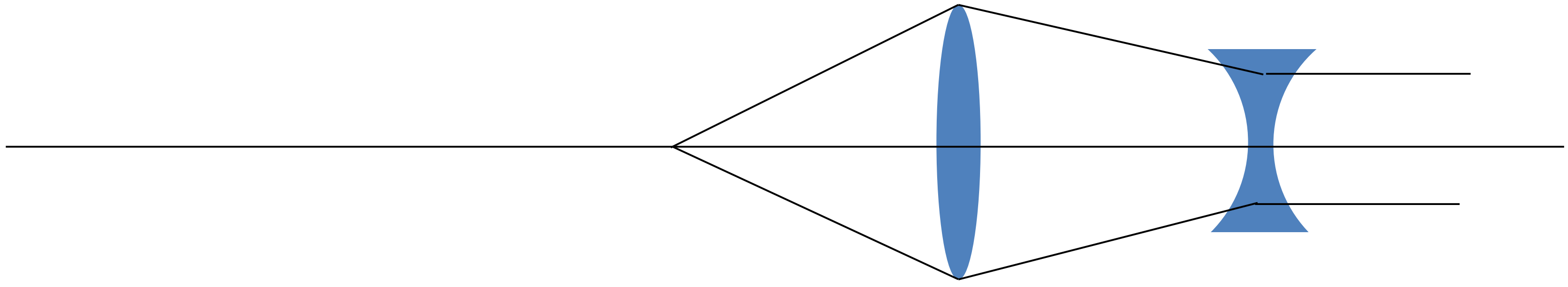


Where is back principal point?



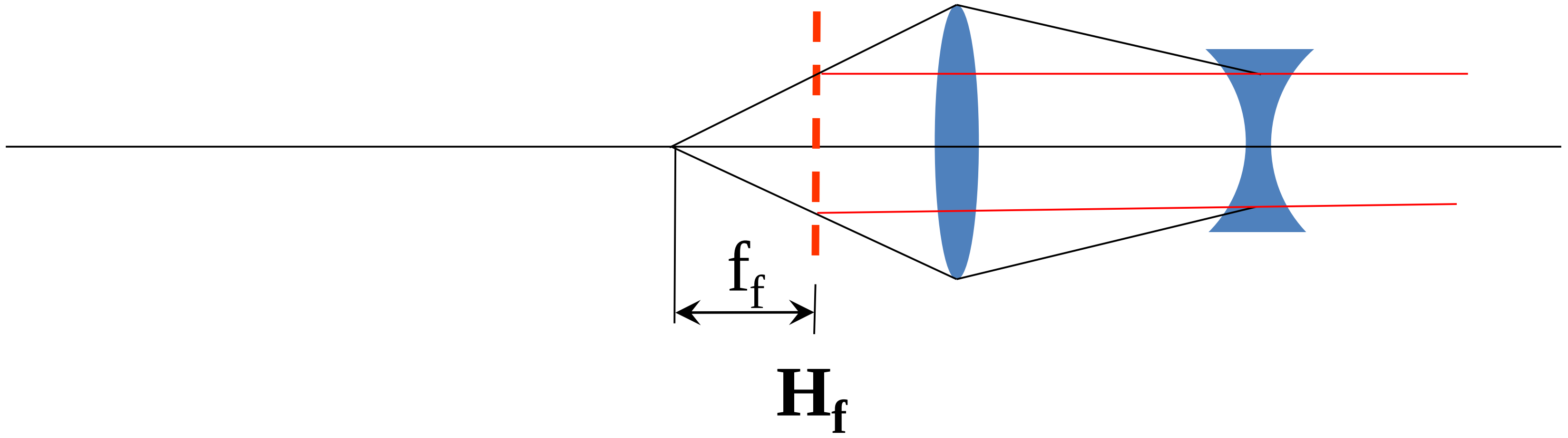
Telephoto lens

Where is front principal point?



Telephoto lens

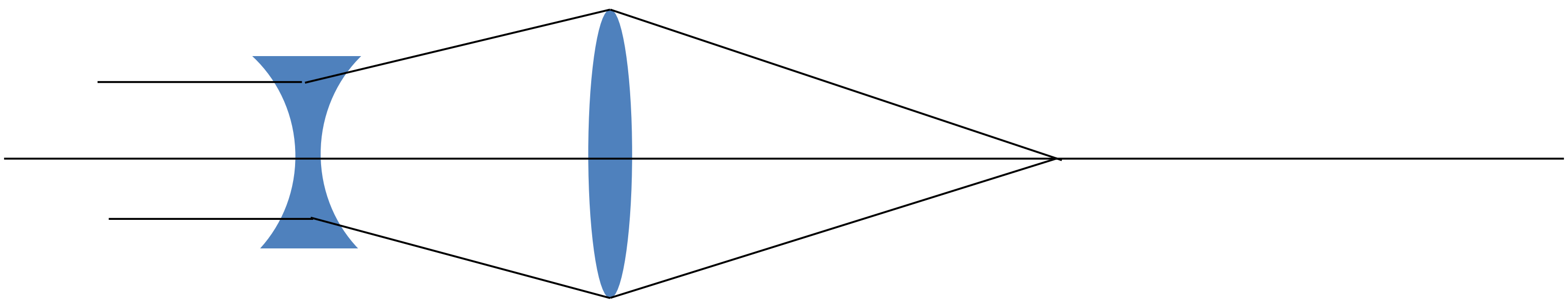
Where is front principal point?



Telephoto lens

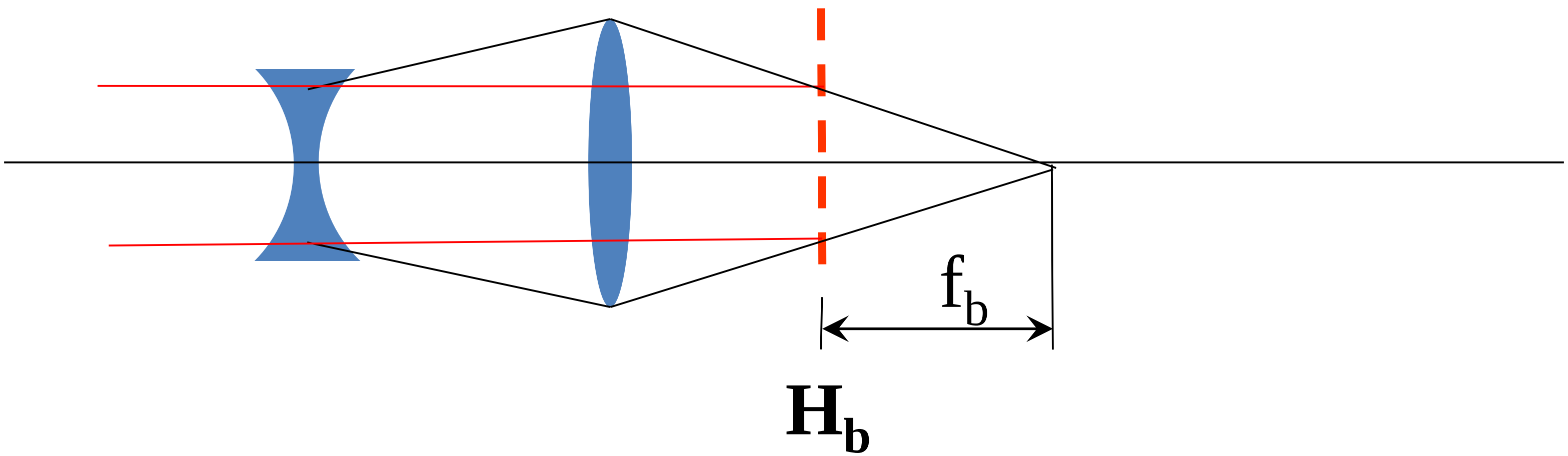
How about this combination?

How about this combination?



Letro focus lens

How about this combination?

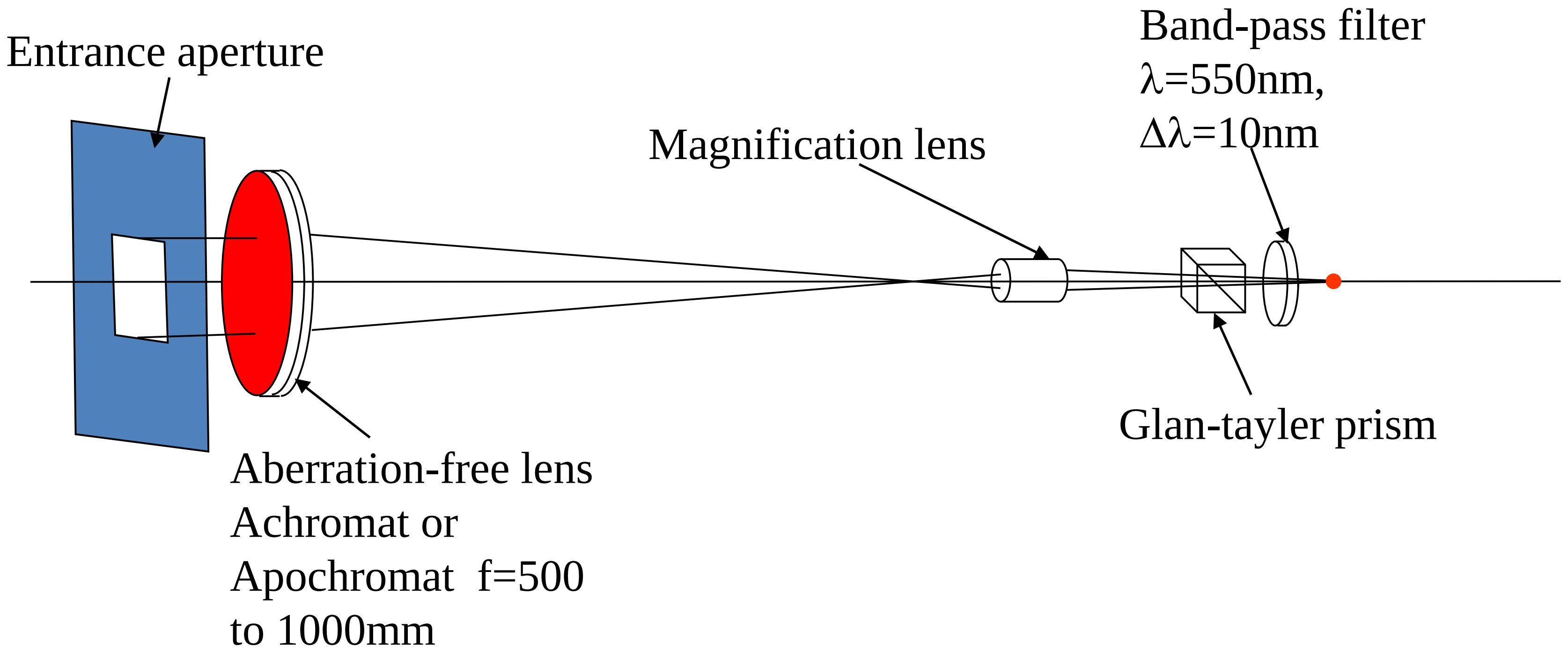


Letro focus lens

Aberration in Seidel’s region

1. Piston
2. Tilt
3. Spherical (appear in on axis)
4. Comma (appear in off axis)
5. Astigmatism (appear in on axis)
6. Distortion (uneven wavefront error)

In the focusing system which used in SR monitor, image will appear in narrow field around optical axis.

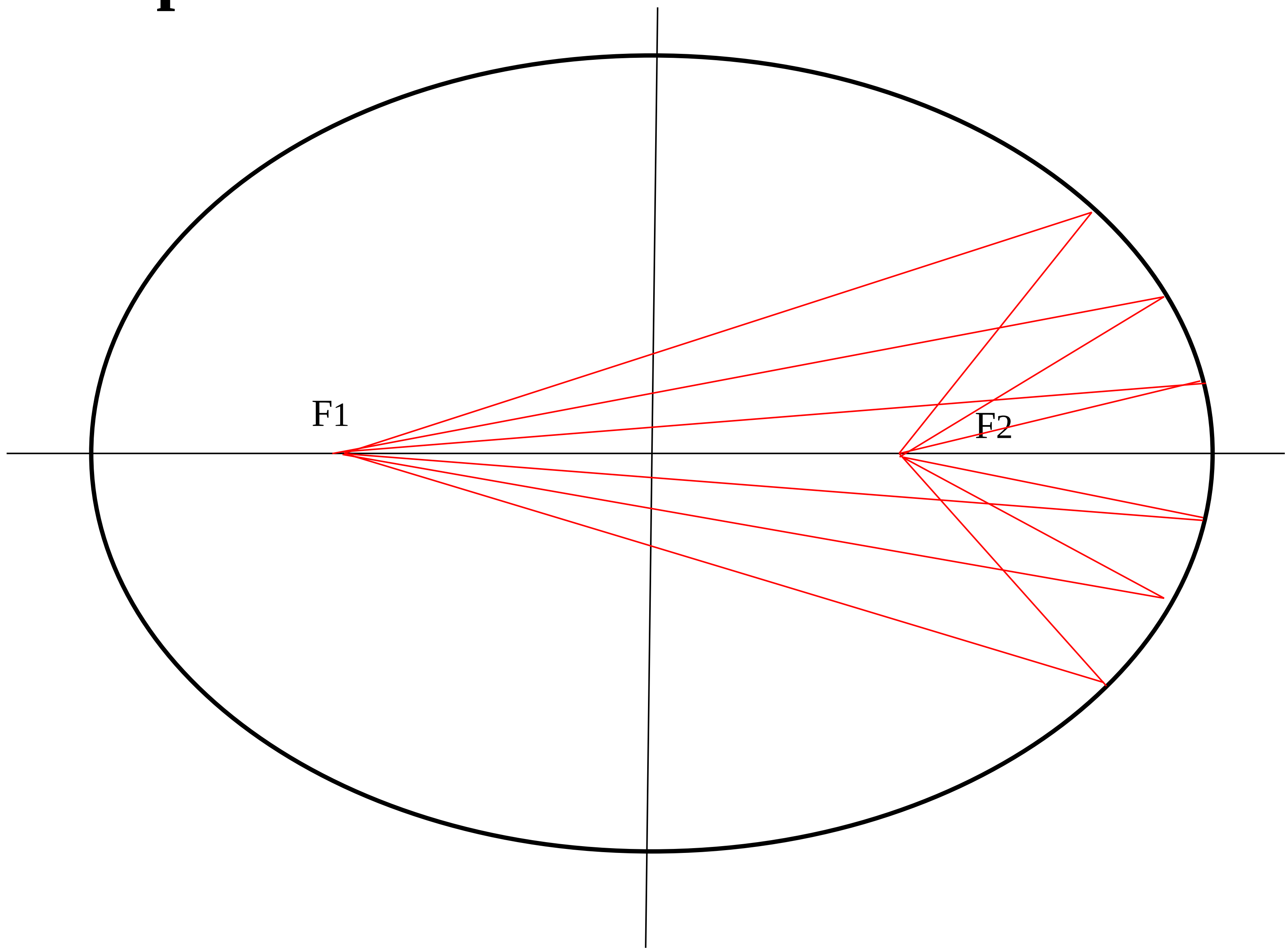


In this case, most important aberration is on-axis spherical aberration.

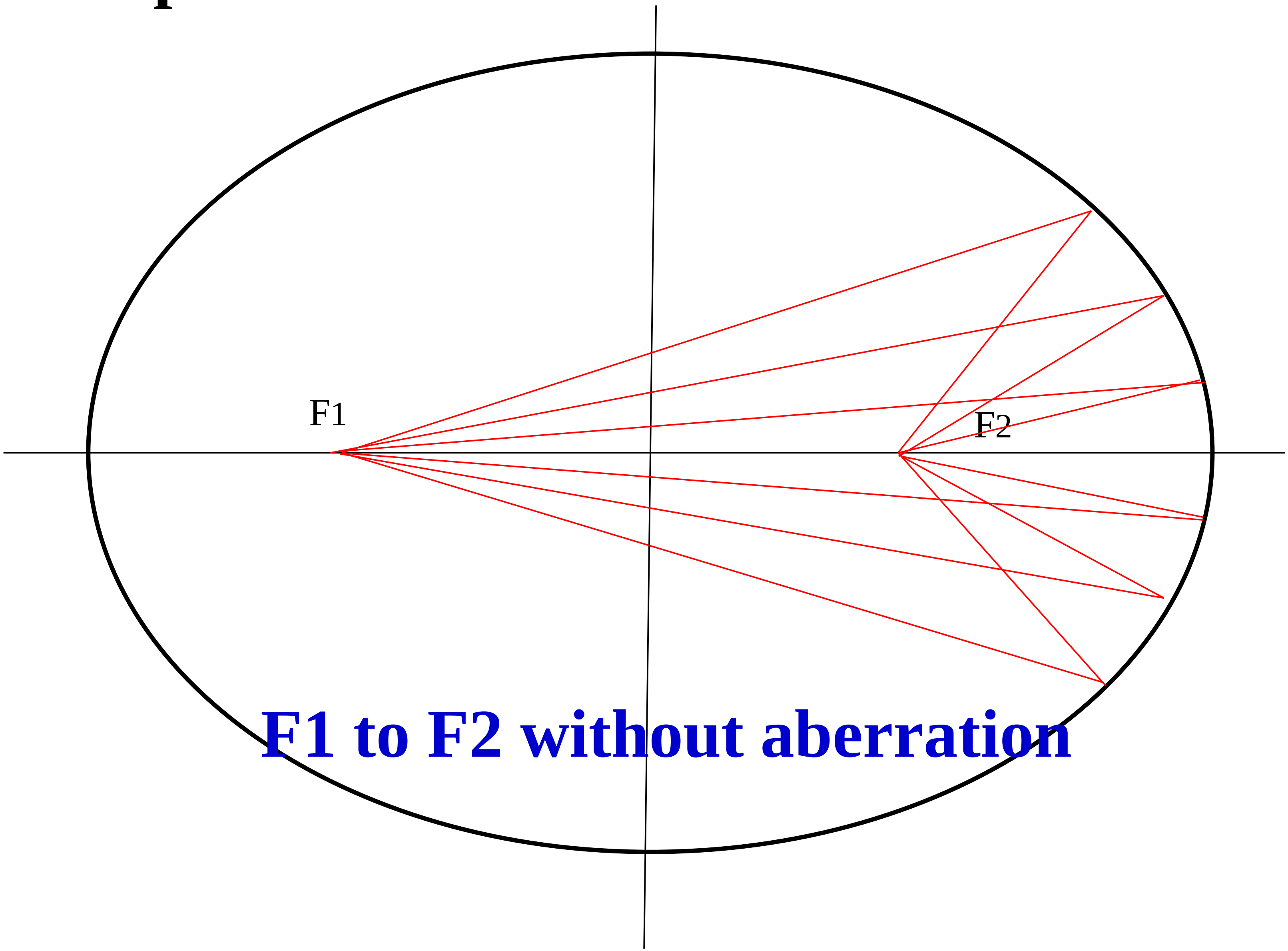
For understanding the spherical aberration, let us start with focusing mirror system by ellipsoidal surface.

Spherical aberration

1. Ellipsoidal surface

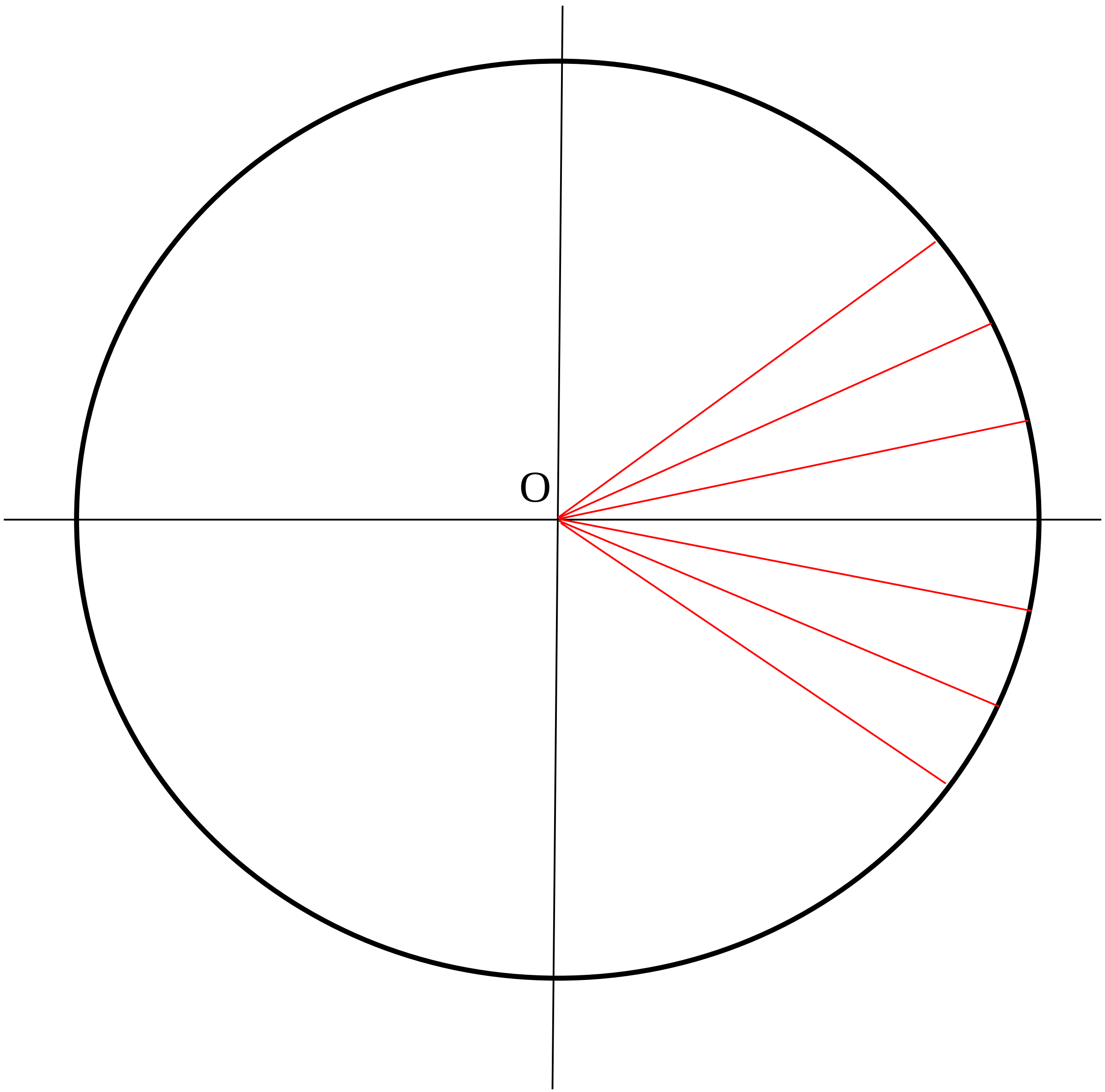


1. Ellipsoidal surface

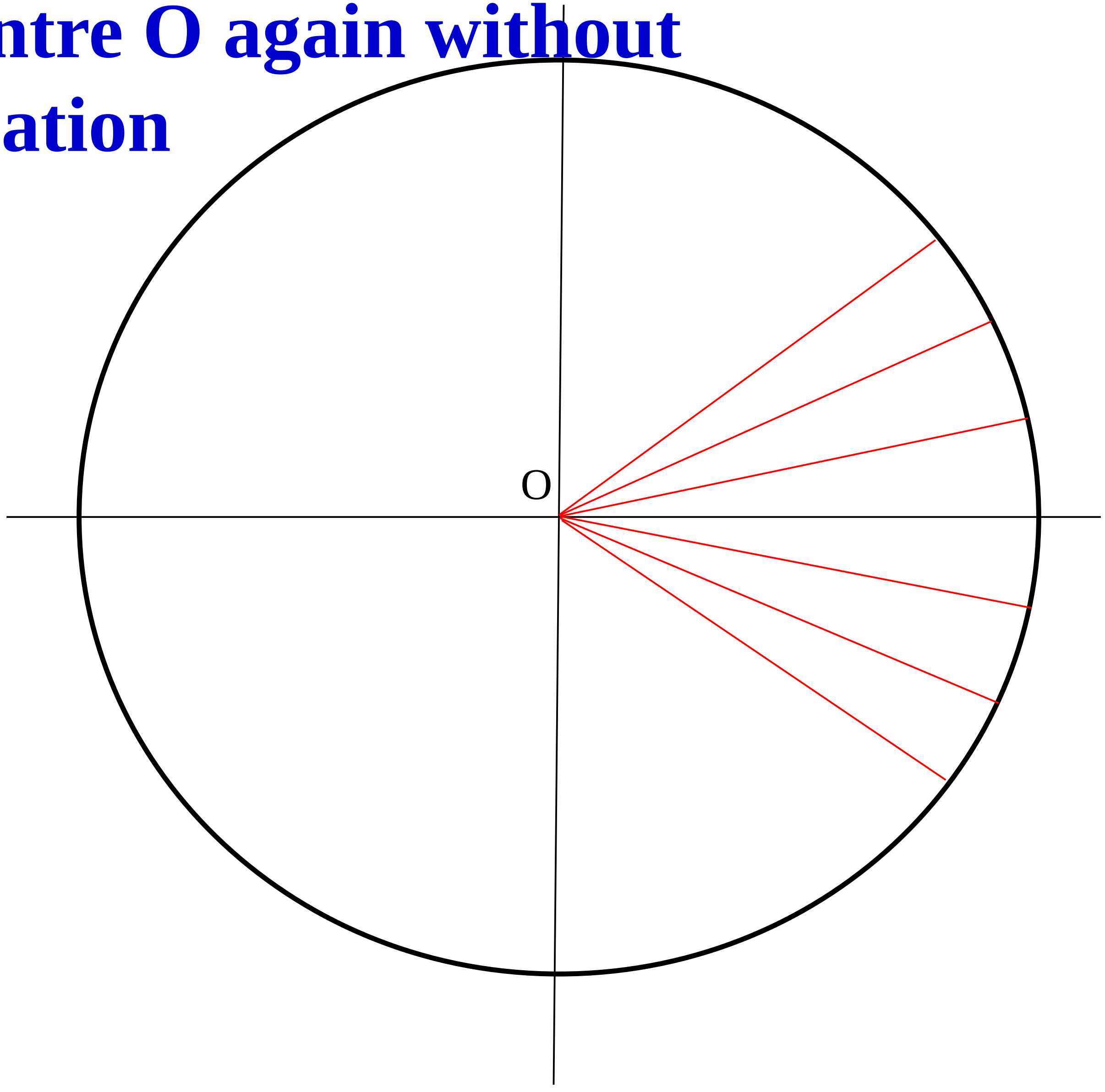


F1 to F2 without aberration

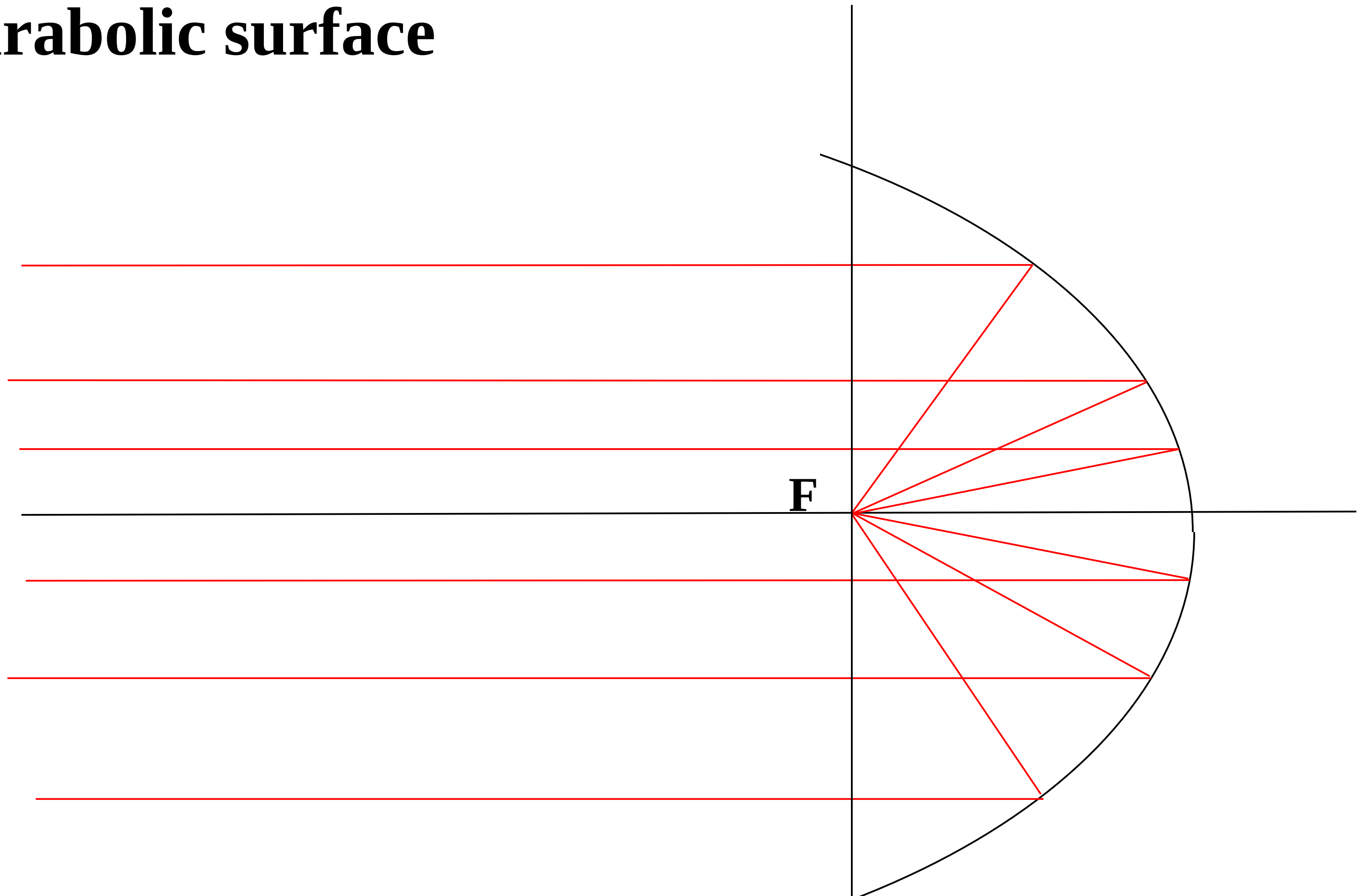
2.Spherical surface



Light from centre O will focus on centre O again without aberration

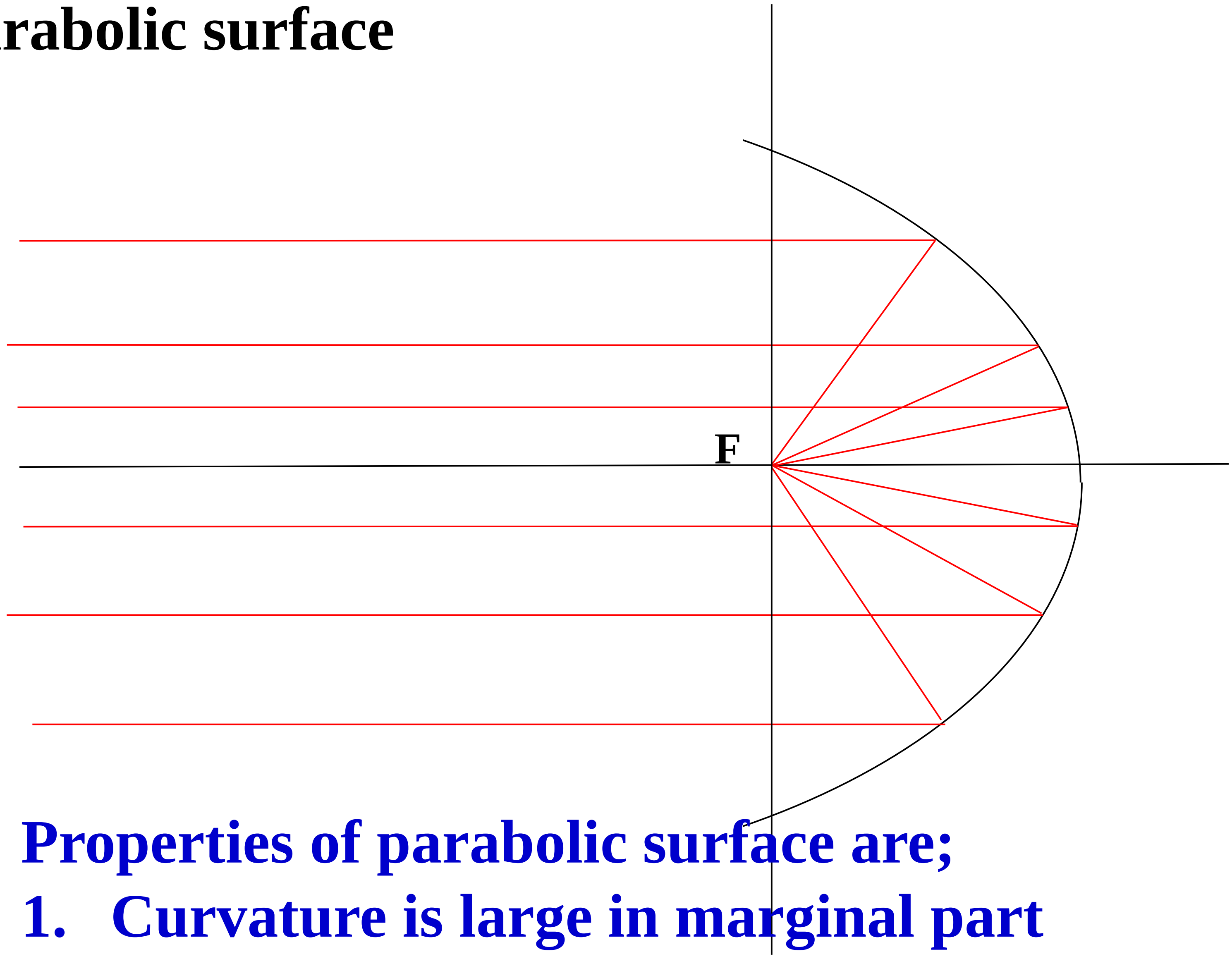


Parabolic surface



Parallel light will focus on F without aberration

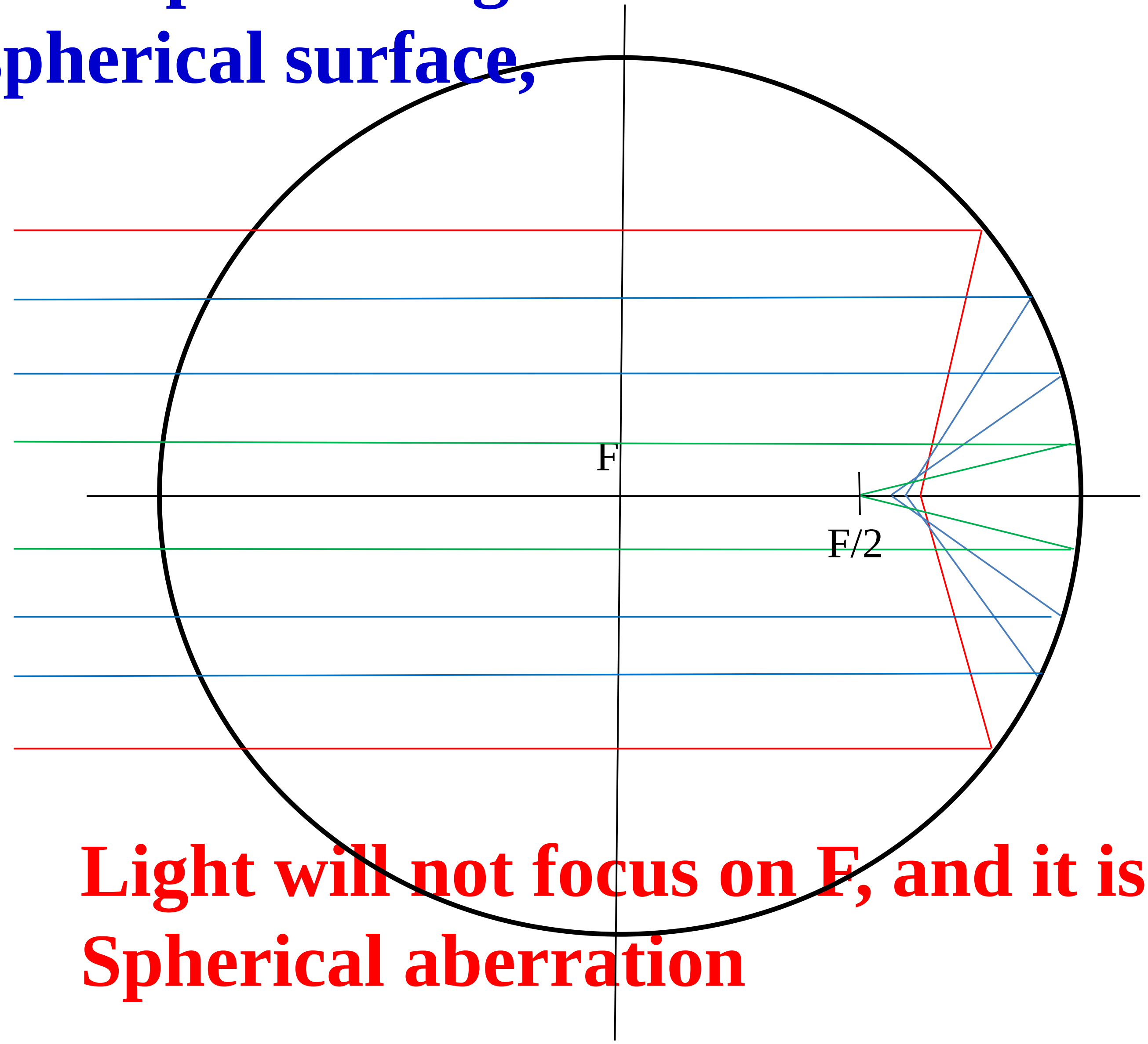
Parabolic surface



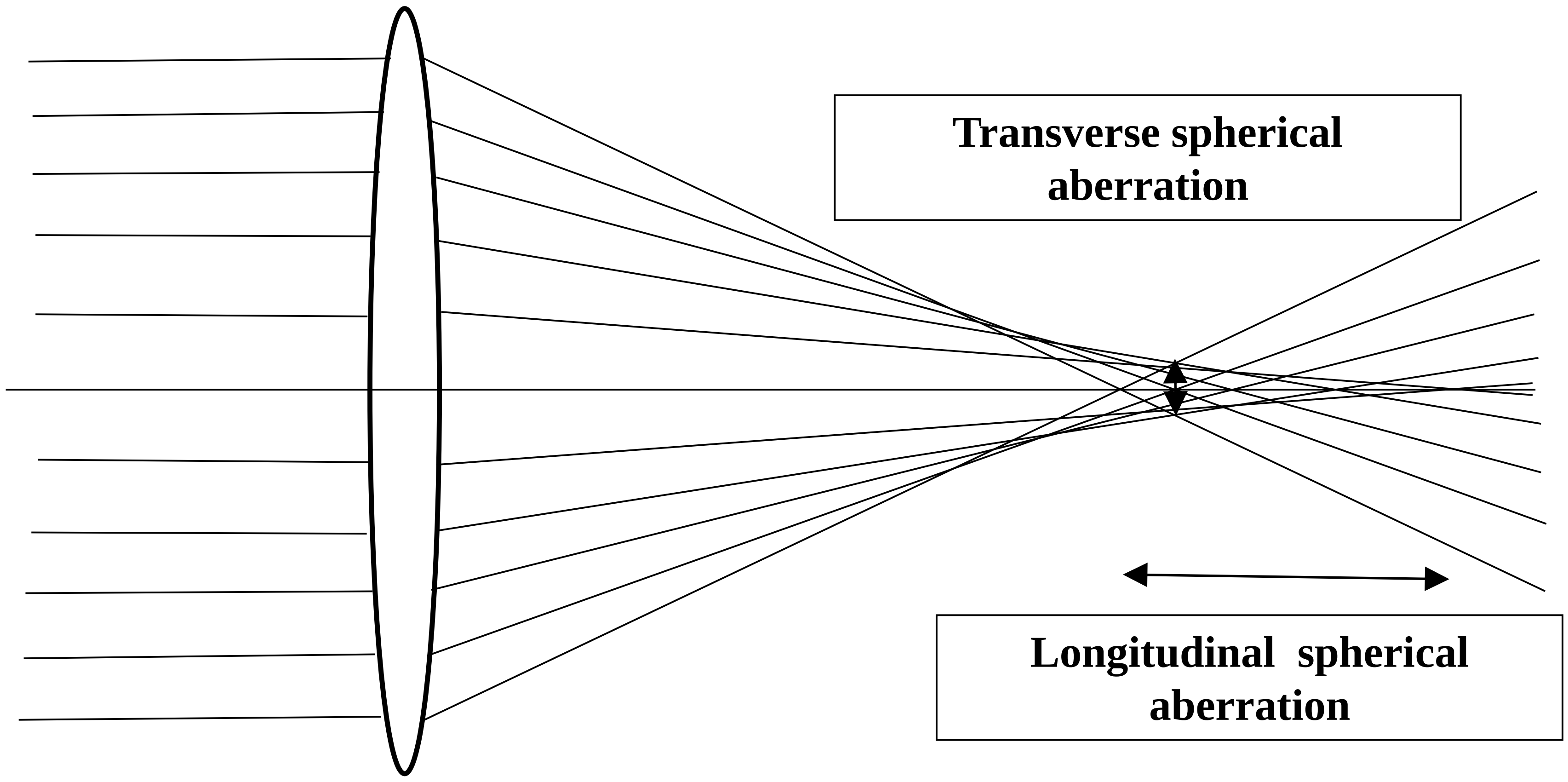
Properties of parabolic surface are;

1. **Curvature is large in marginal part**
2. **Curvature is small in paraxial part**

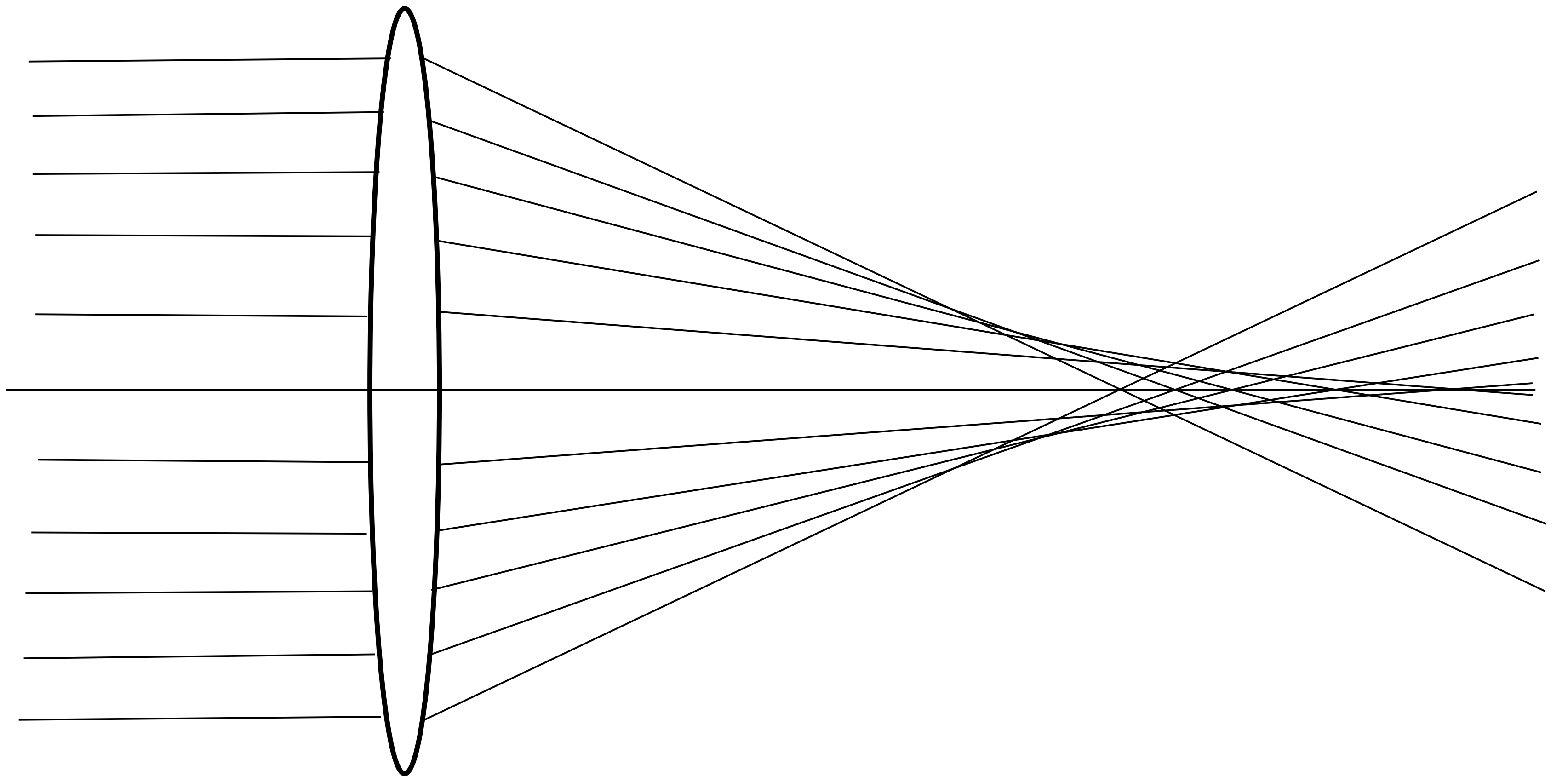
If the parallel light will come to spherical surface,



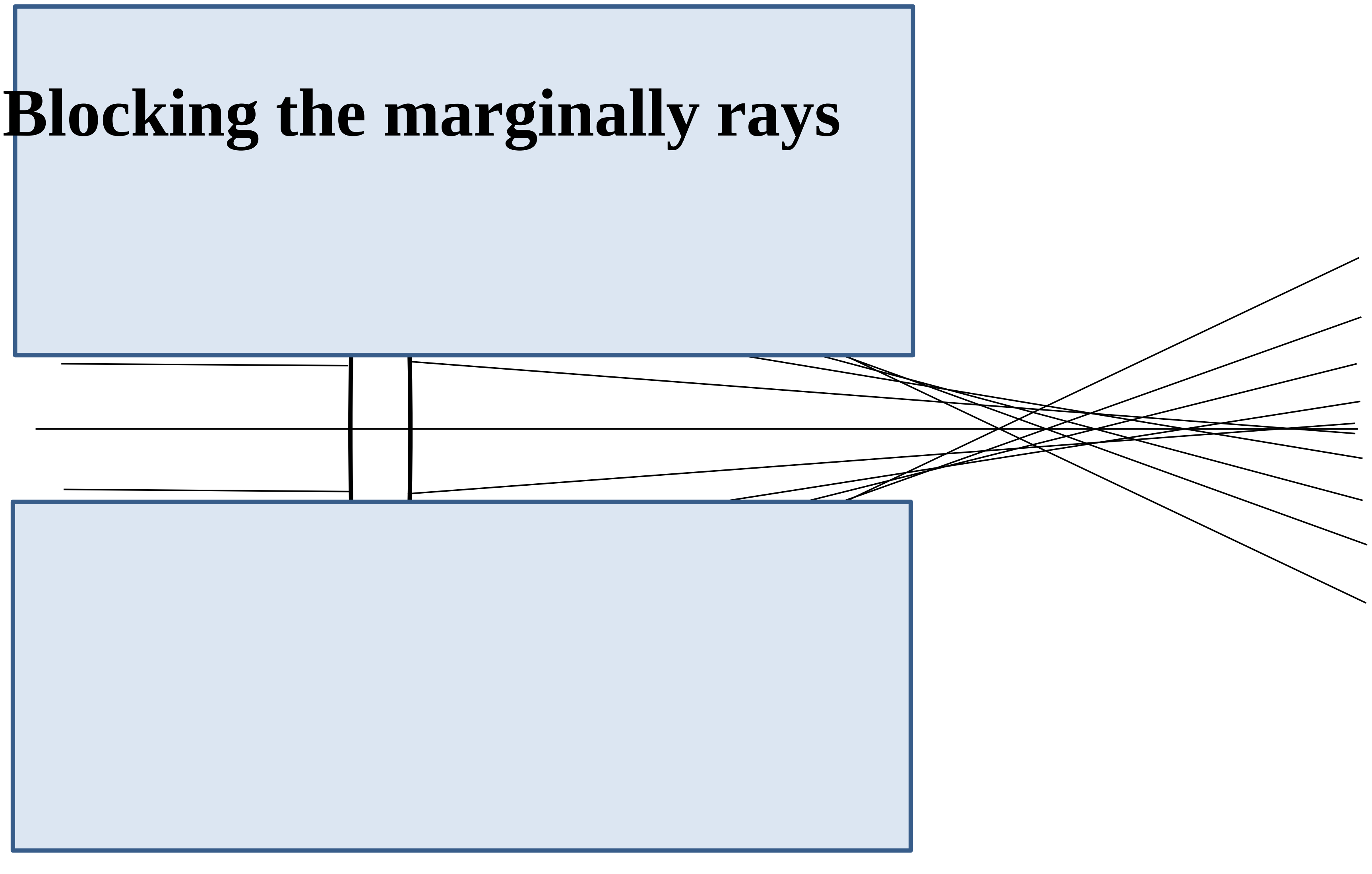
Light will not focus on F, and it is Spherical aberration


Transverse spherical aberration
Longitudinal spherical aberration

What is definition of focal length??

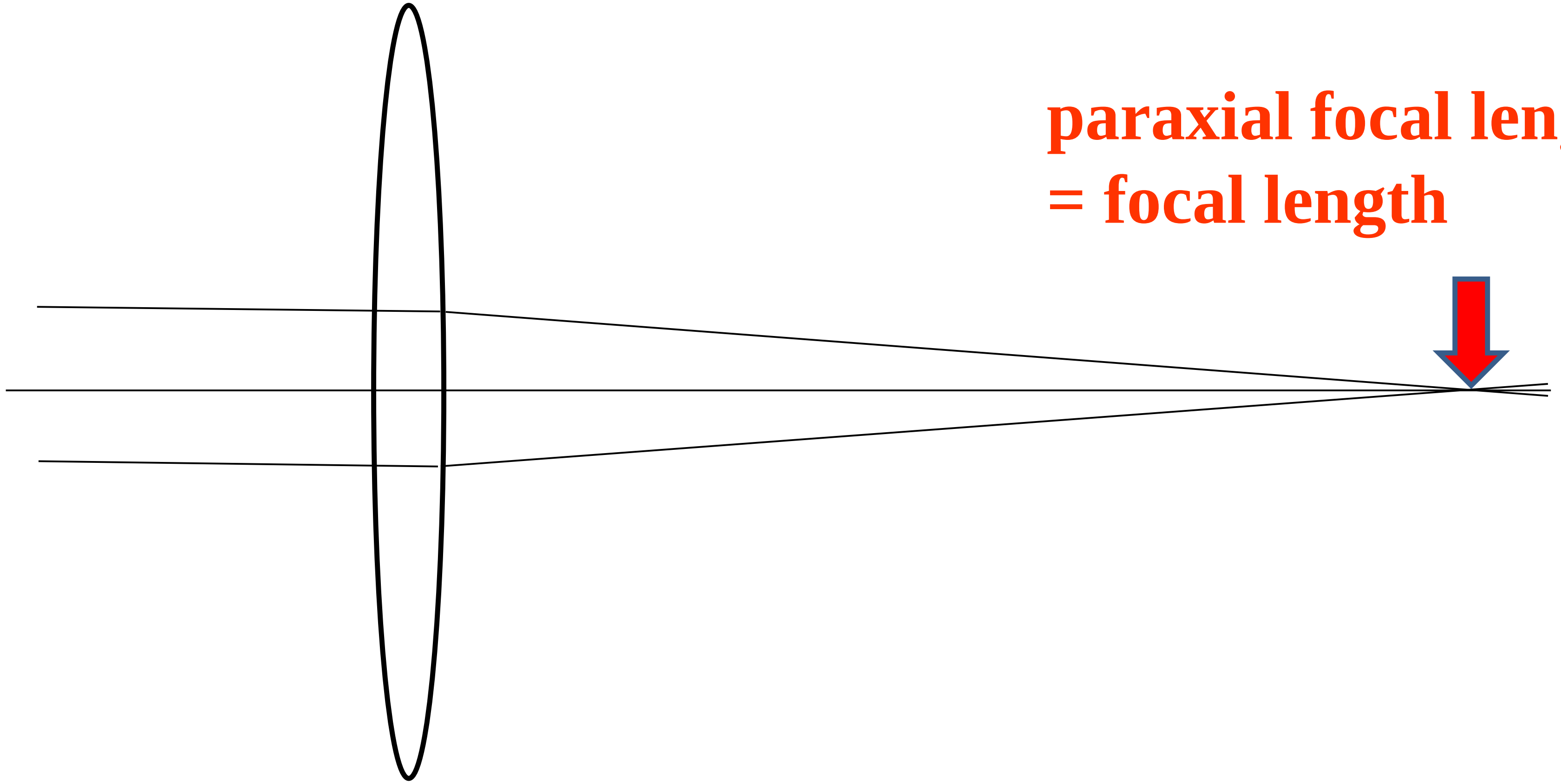


What is definition of focal length??



Only discuss with paraxial rays,
We can define focal length of the lens!

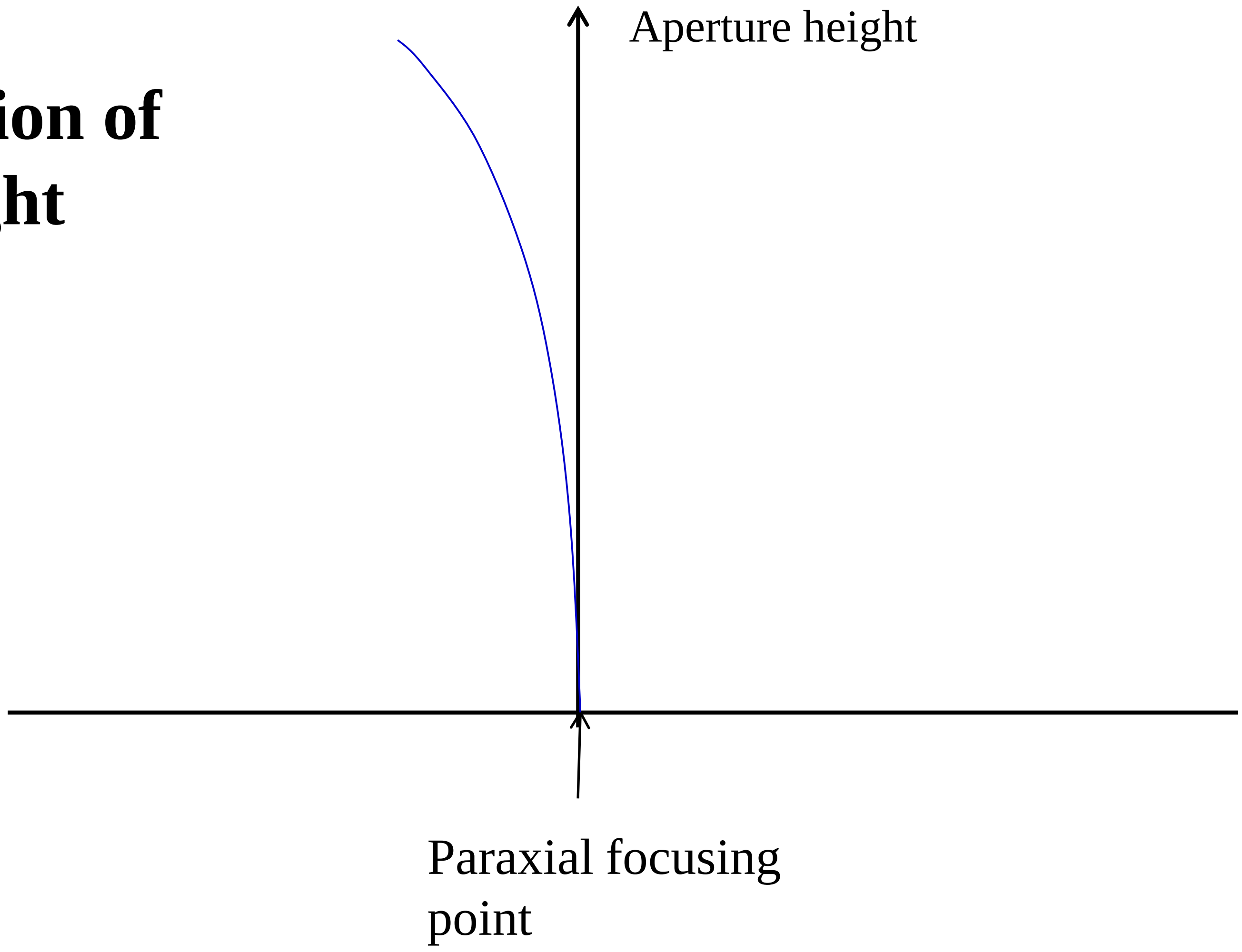
paraxial focal length
= focal length



At average focus point, spherical aberration Looks;

Representation of spherical aberration

Δf as a function of aperture height



How to correct spherical aberration?

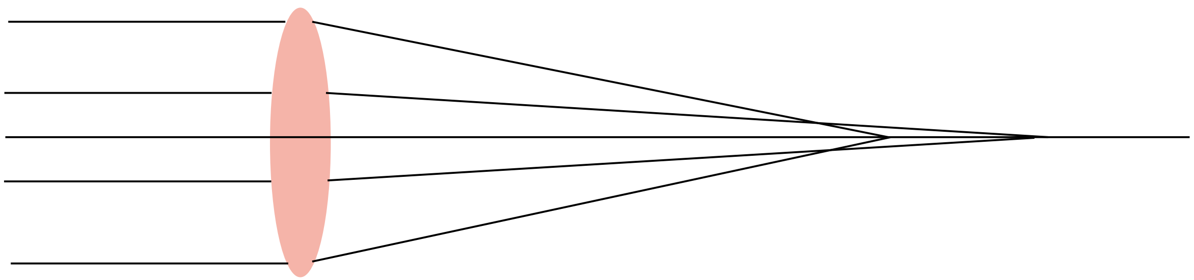
Off course, we shall use non-spherical surface for reflective system,

but

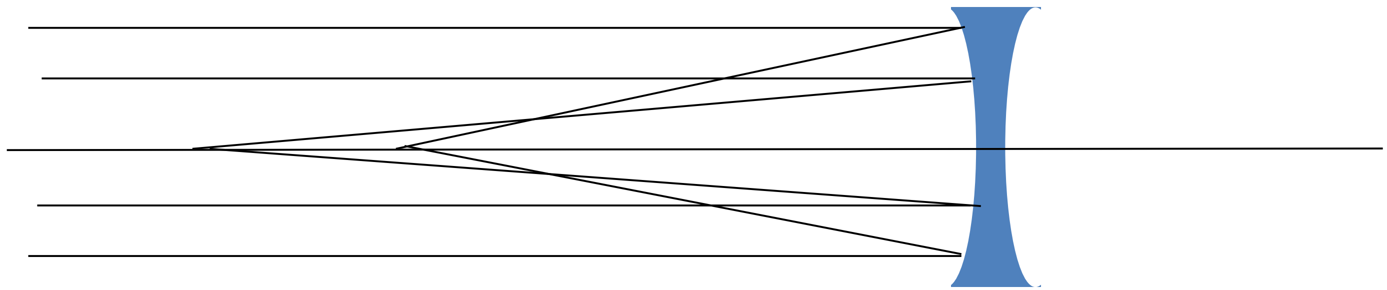
How we can do in the refractive (lens) system?

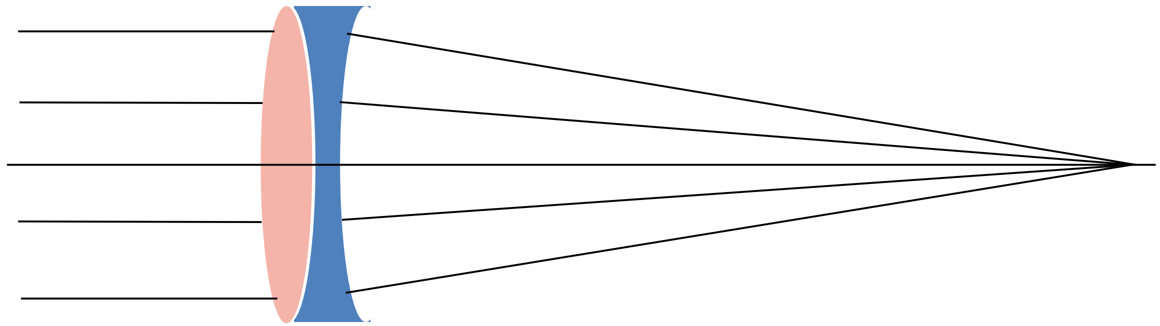
Spherical aberration

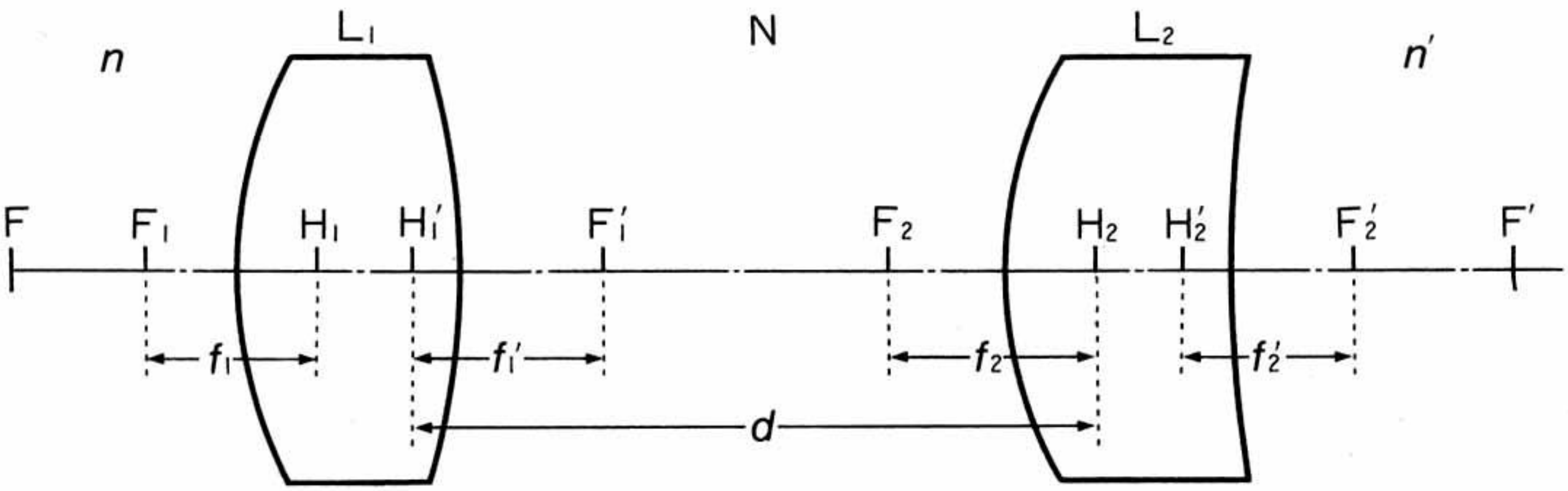
Focusing lens **+Spherical aberration**



Defocusing lens **-Spherical aberration**





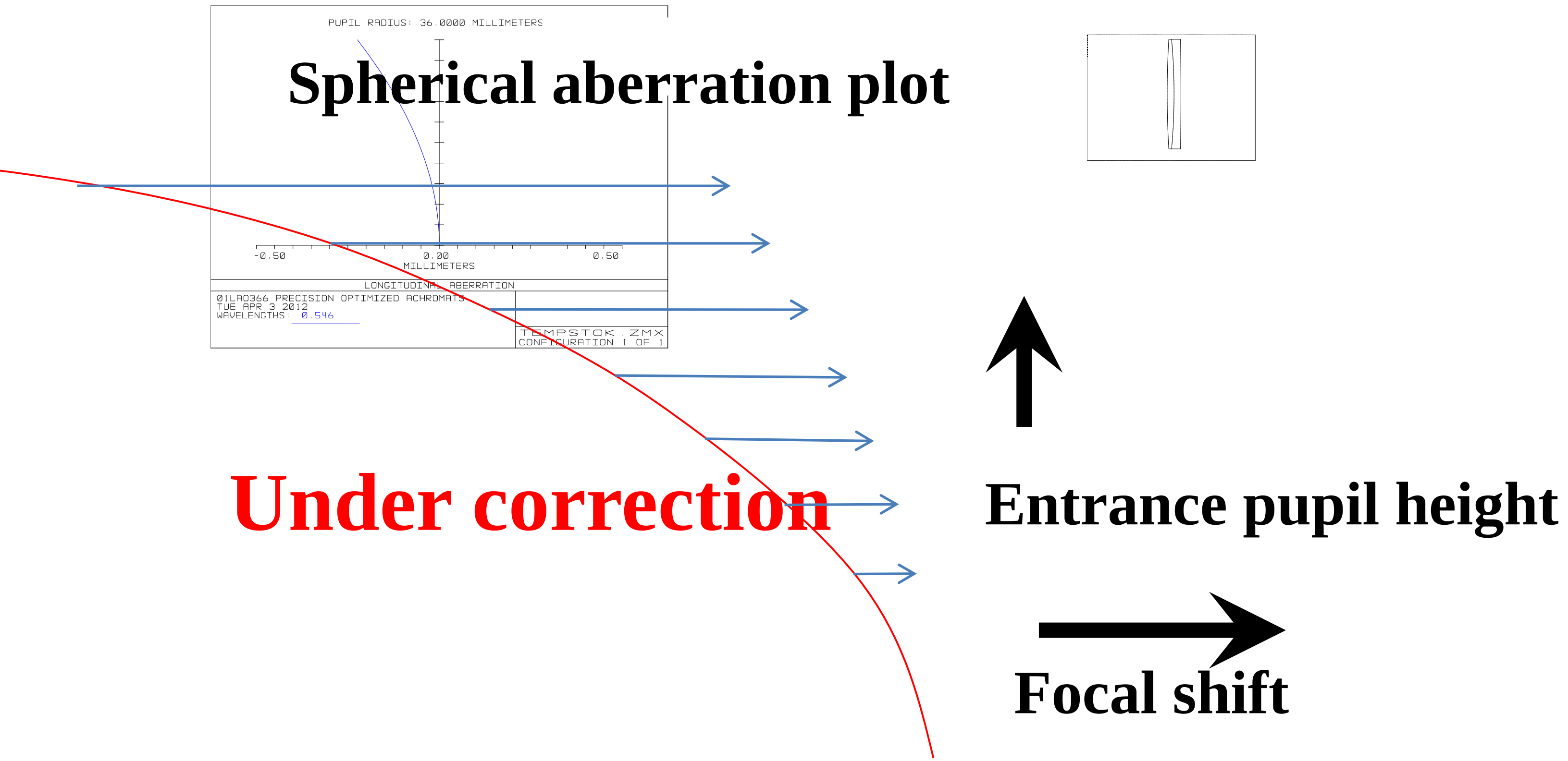


Front focus $$f = \frac{f_1 \cdot f_2}{f'_1 + f_2 - d}$$

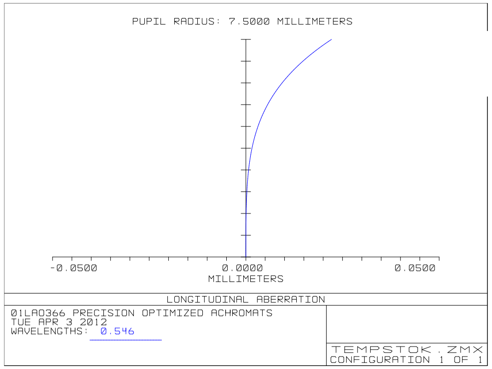
d≠0

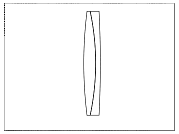
***f*, *f'*≠∞**

Back focus $$f' = \frac{f'_1 \cdot f'_2}{f'_1 + f'_2 - d}$$



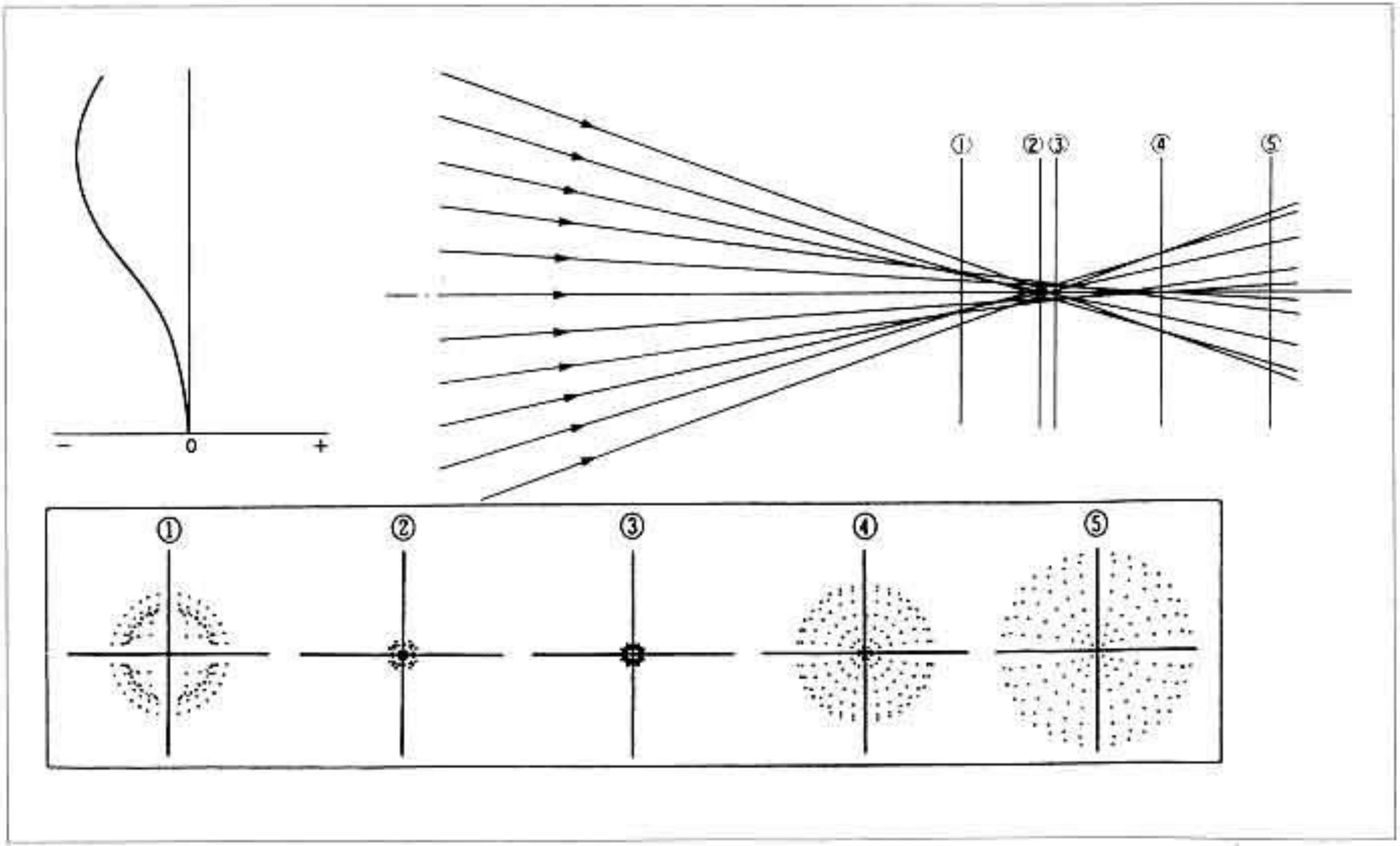
Spherical aberration plot
PUPIL RADIUS: 36.0000 MILLIMETERS
-0.50
0.00
0.50
MILLIMETERS
LONGITUDINAL ABERRATION
01LAO366 PRECISION OPTIMIZED ACHROMATS
TUE APR 3 2012
WAVELENGTHS: 0.546
TEMPSTOK.ZMX
CONFIGURATION 1 OF 1
Under correction
Entrance pupil height
Focal shift



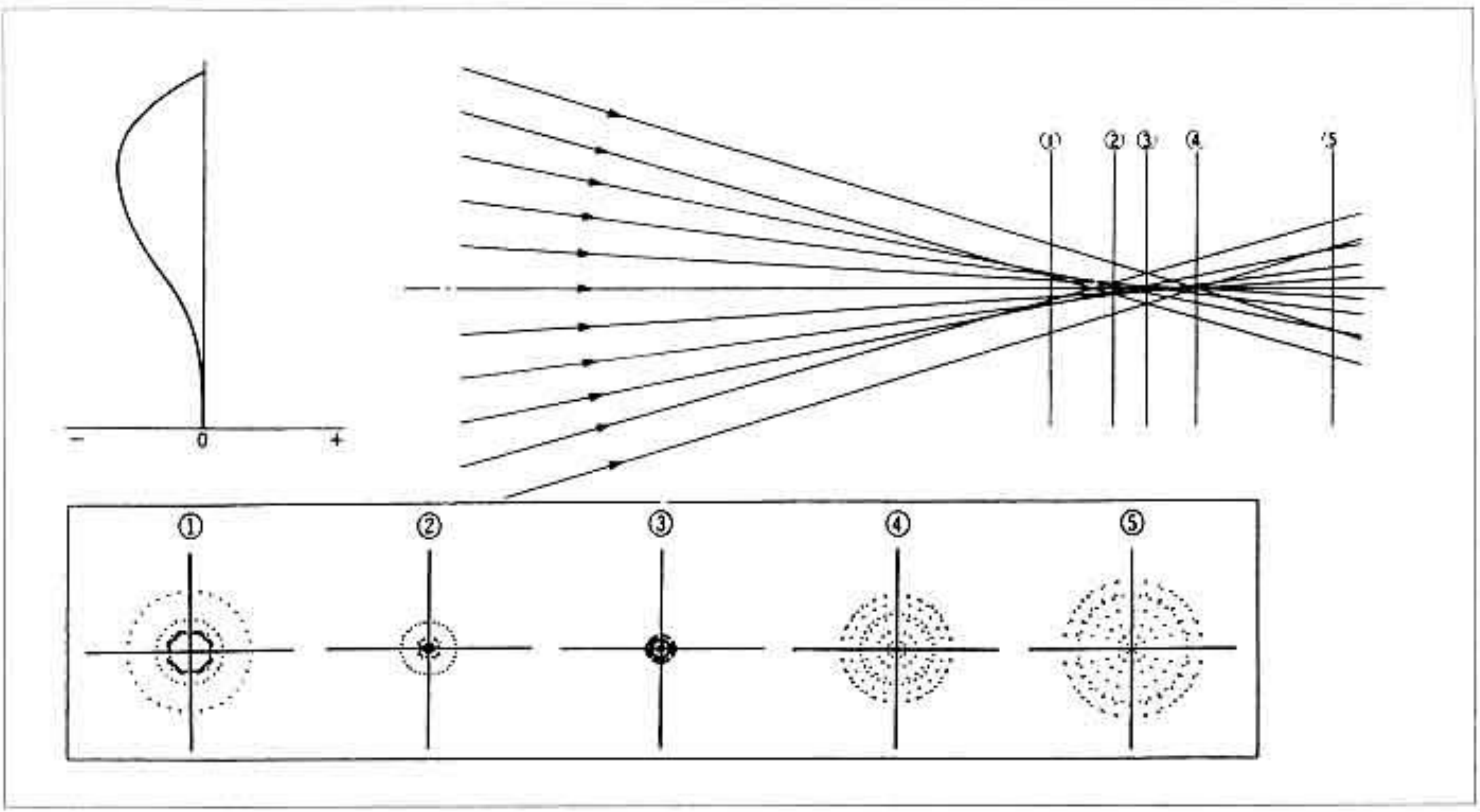


Over correction

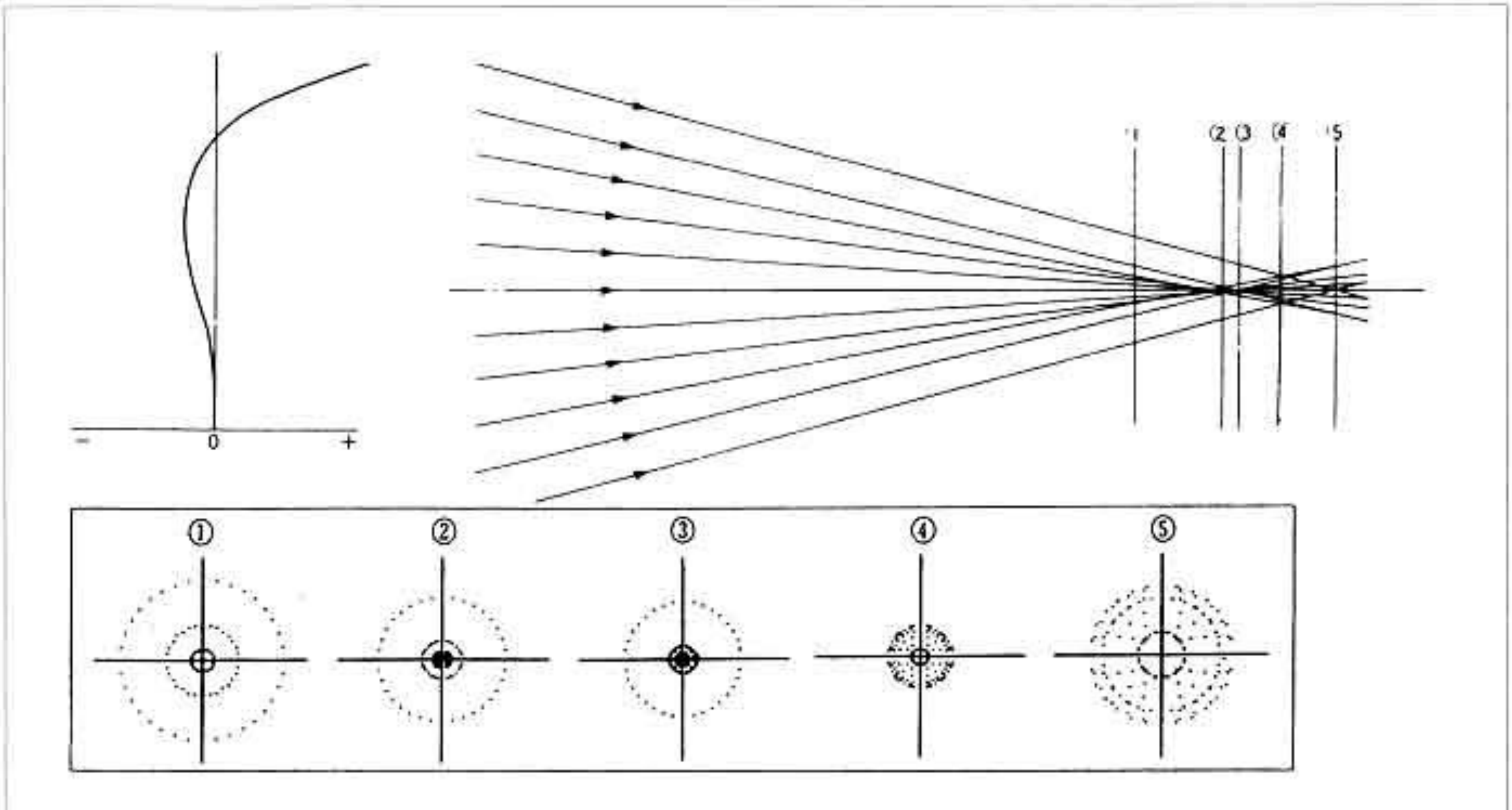
Under correction



Full correction

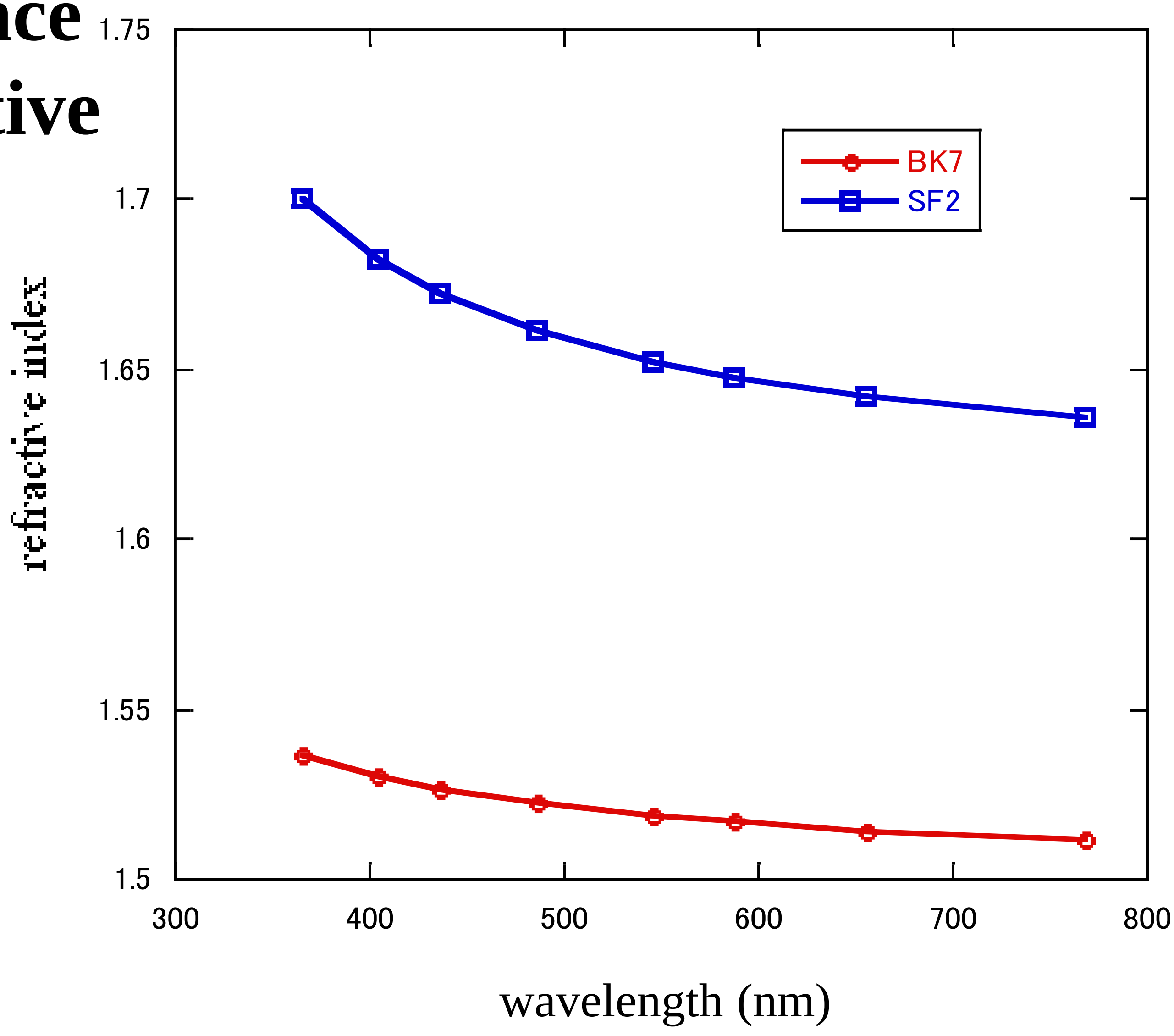


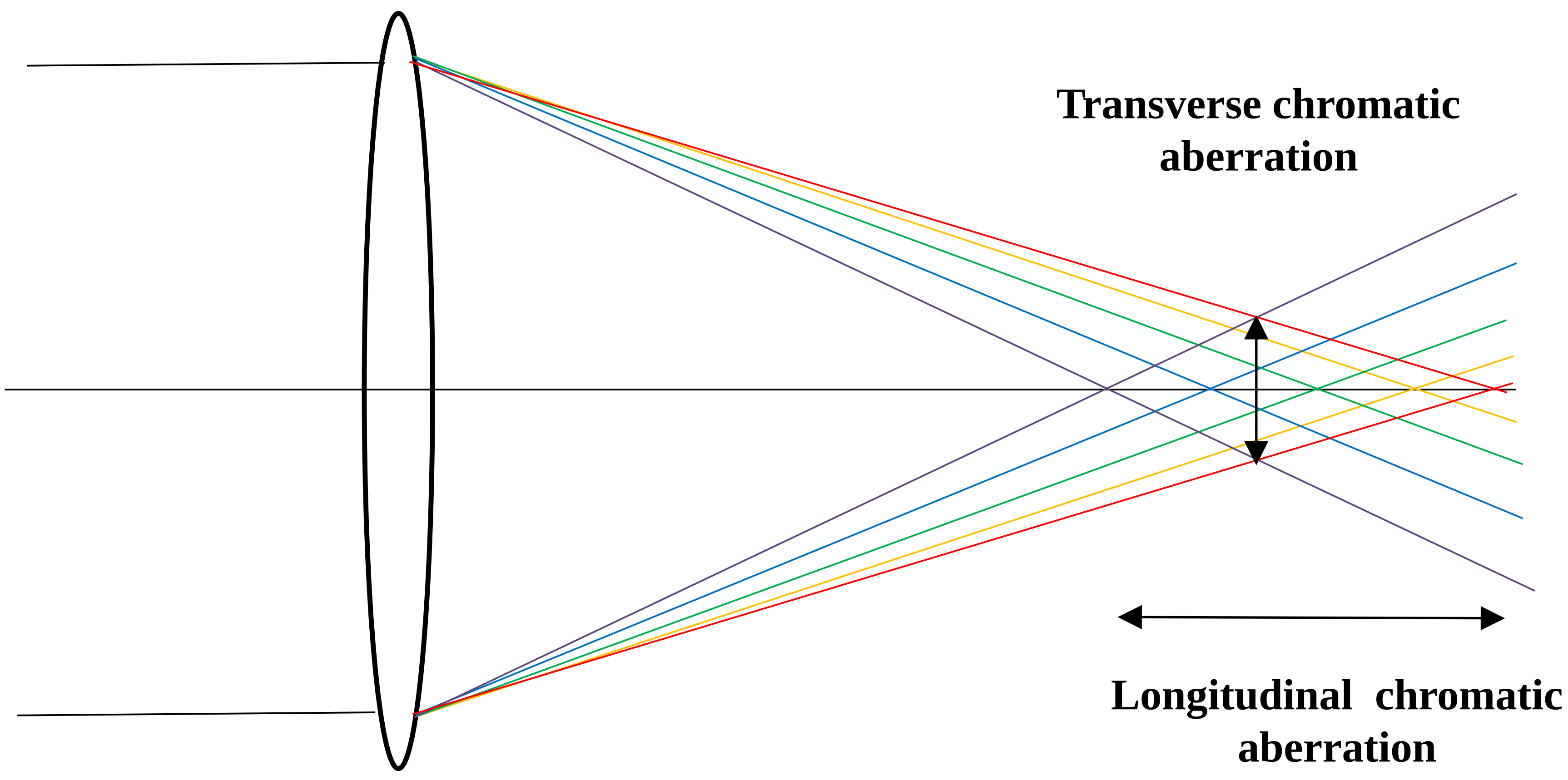
Over correction



Chromatic aberration

Wave length dependence of refractive index

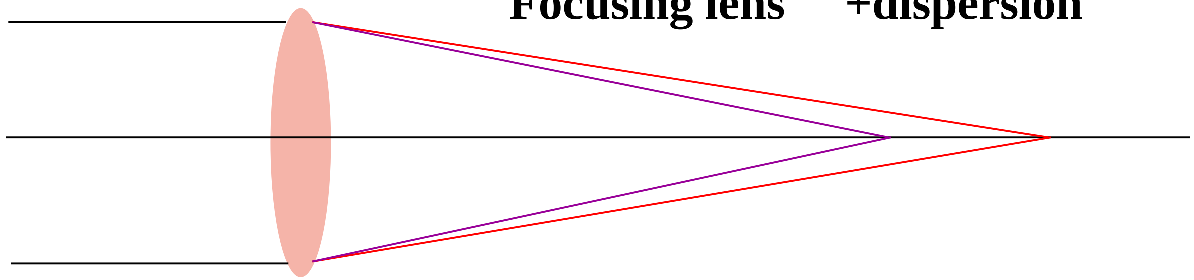




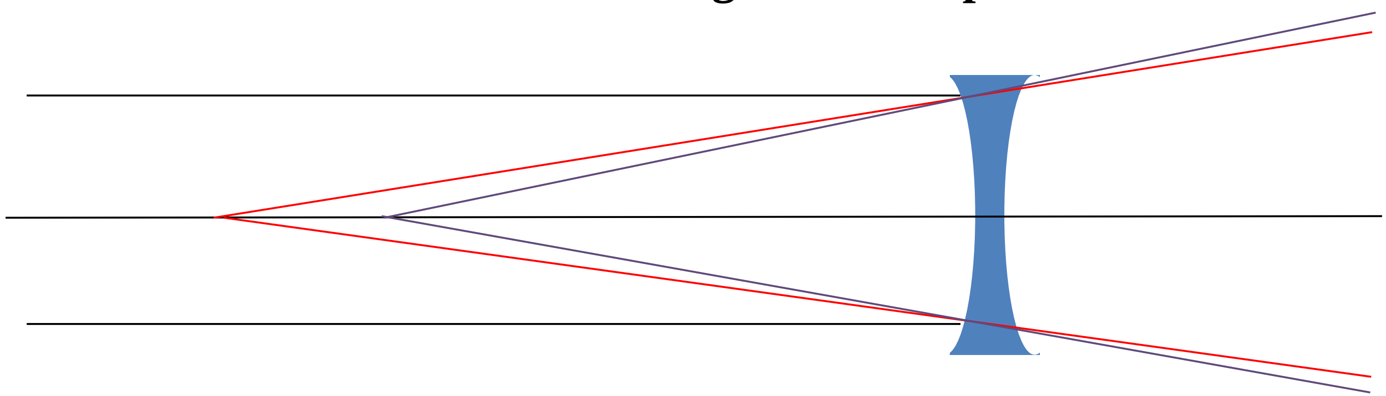
Transverse chromatic aberration
Longitudinal chromatic aberration

Concept of achromatic lens

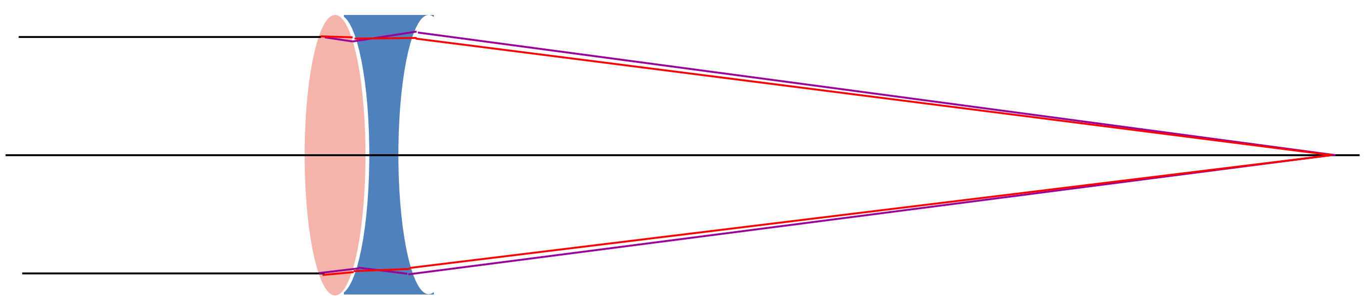
Focusing lens +dispersion



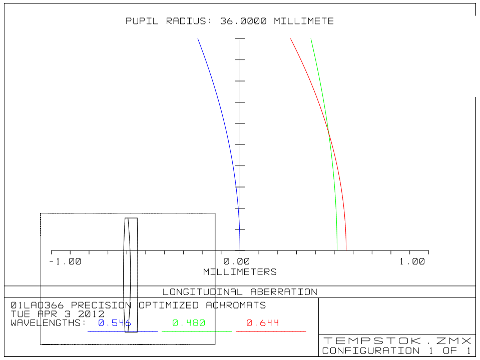
Defocusing lens -dispersion

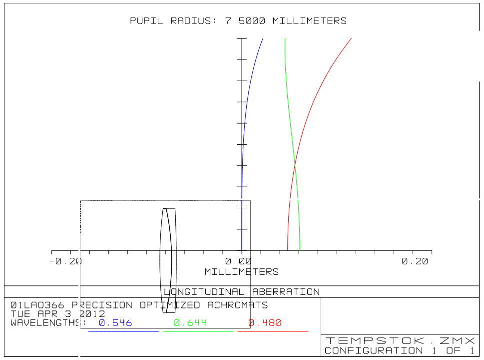


Certain combination of focusing and defocusing lens can cancel focal shift

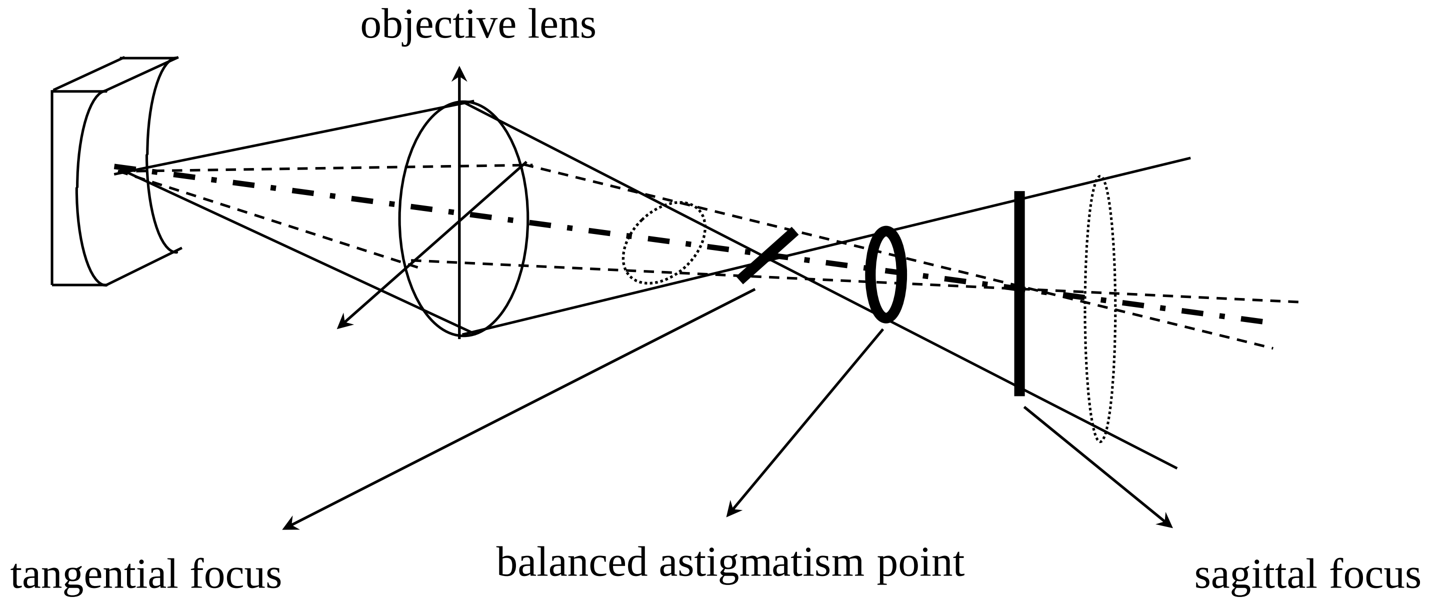


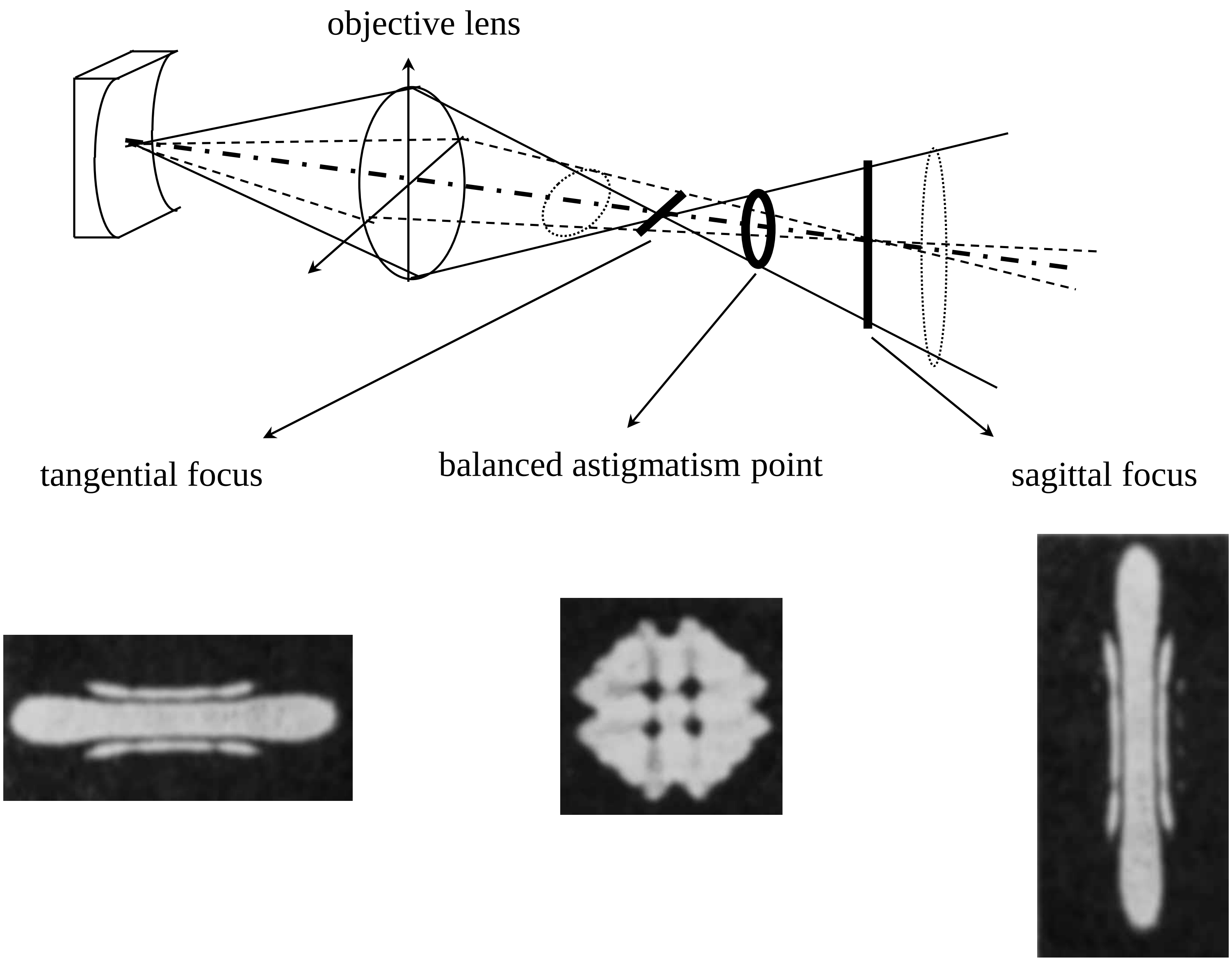
Chromatic aberration plot

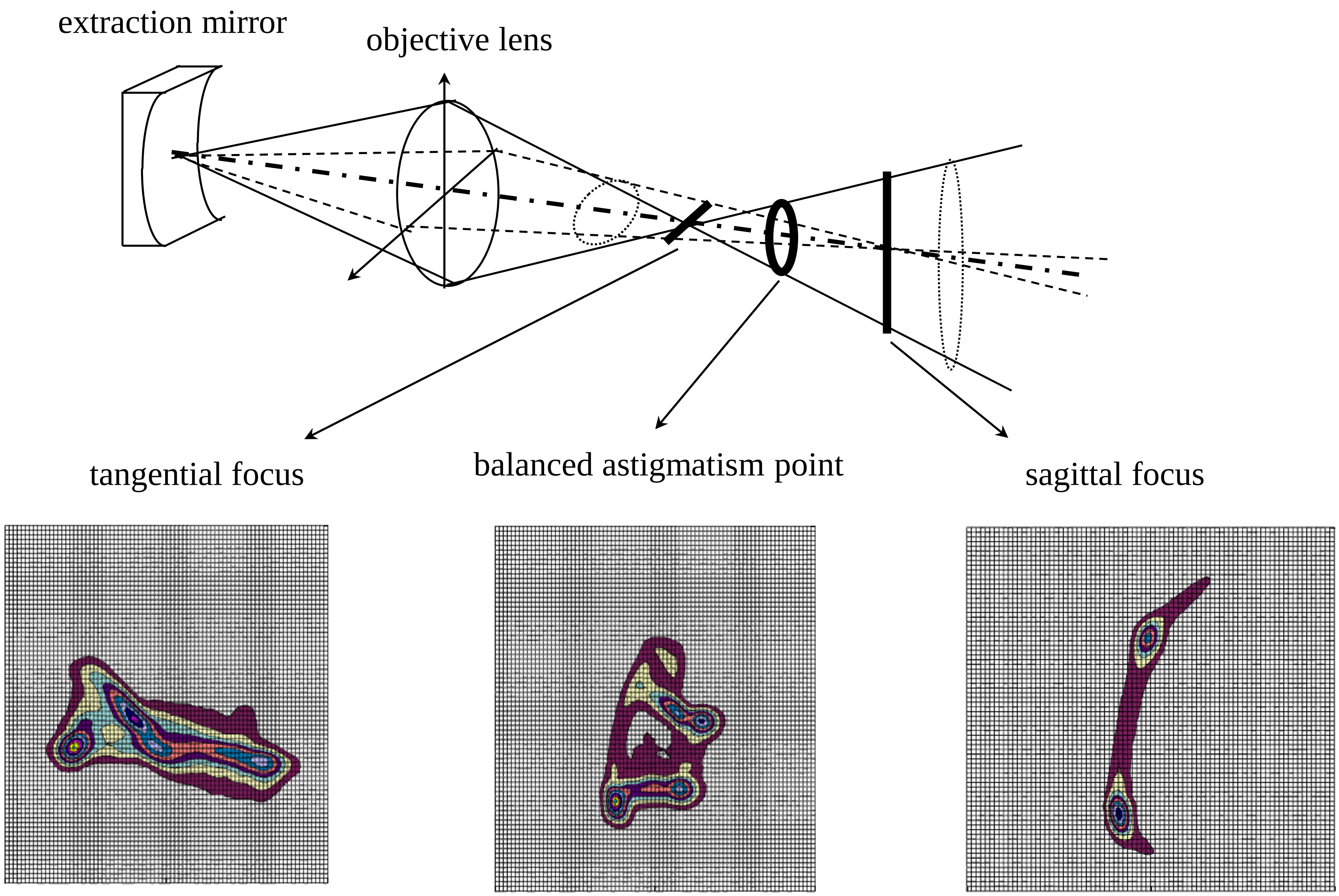



PUPIL RADIUS: 7.5000 MILLIMETERS
-0.20
0.00
0.20
MILLIMETERS
LONGITUDINAL ABERRATION
01LAO366 PRECISION OPTIMIZED ACHROMATS
TUE APR 3 2012
WAVELENGTHS: 0.546 0.644 0.480
TEMPSTOK.ZMX
CONFIGURATION 1 OF 1

Astigmatism
due to
troidal focusing power


objective lens
tangential focus
balanced astigmatism point
sagittal focus

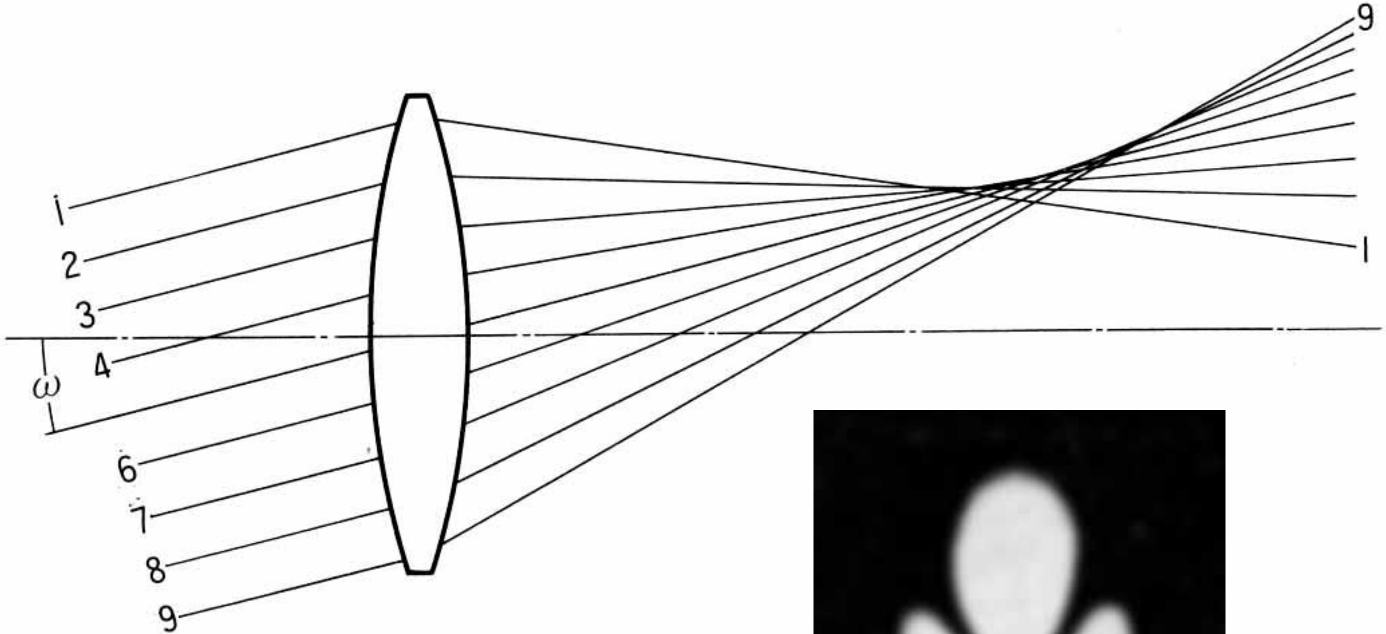

objective lens
tangential focus
balanced astigmatism point
sagittal focus

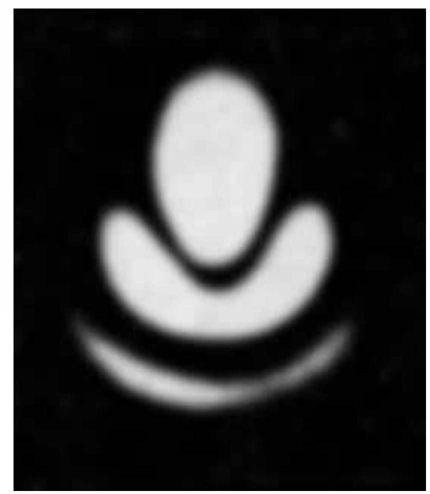


extraction mirror
objective lens
tangential focus
balanced astigmatism point
sagittal focus

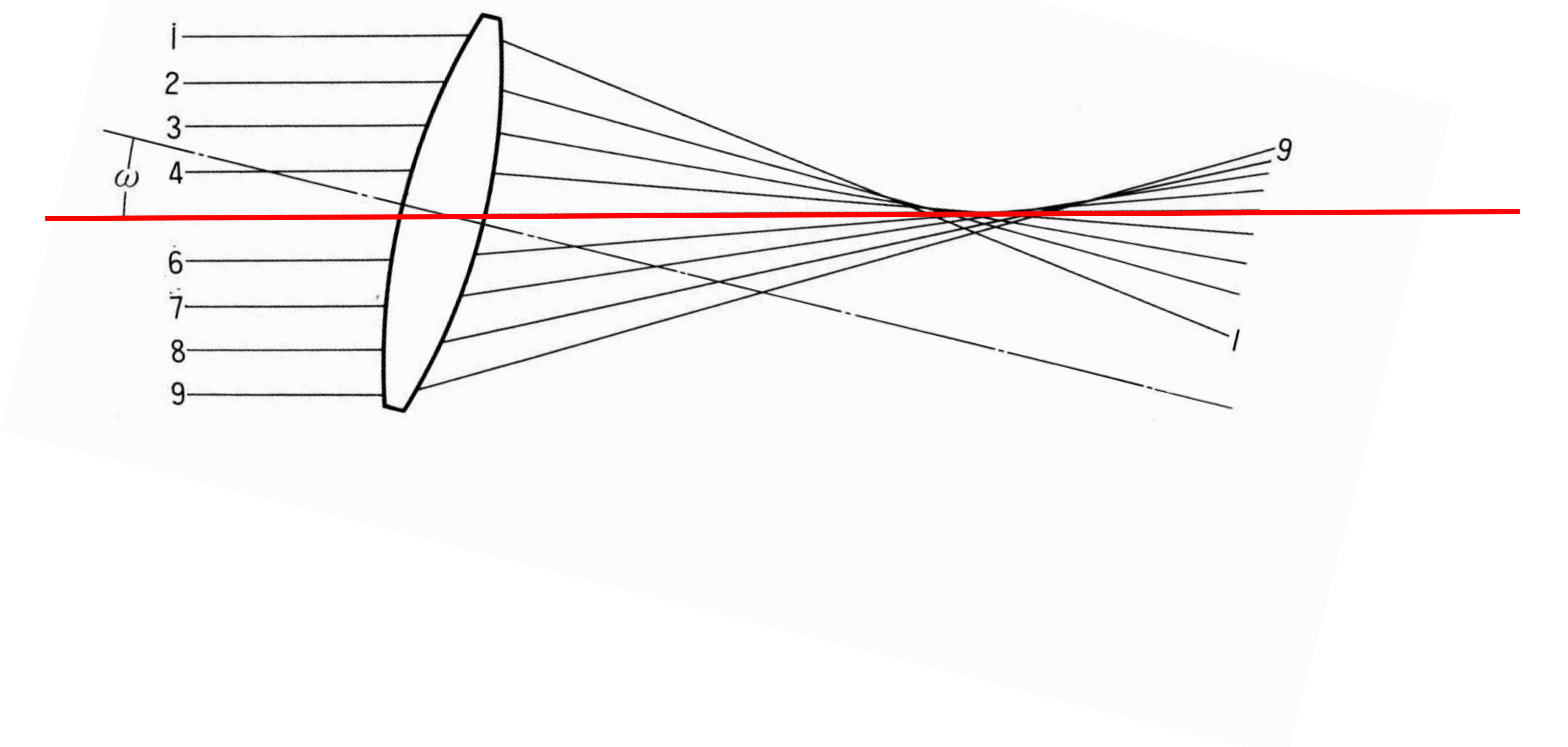
Comma
due to
tilted incidence

Input ray with incident angle of ω

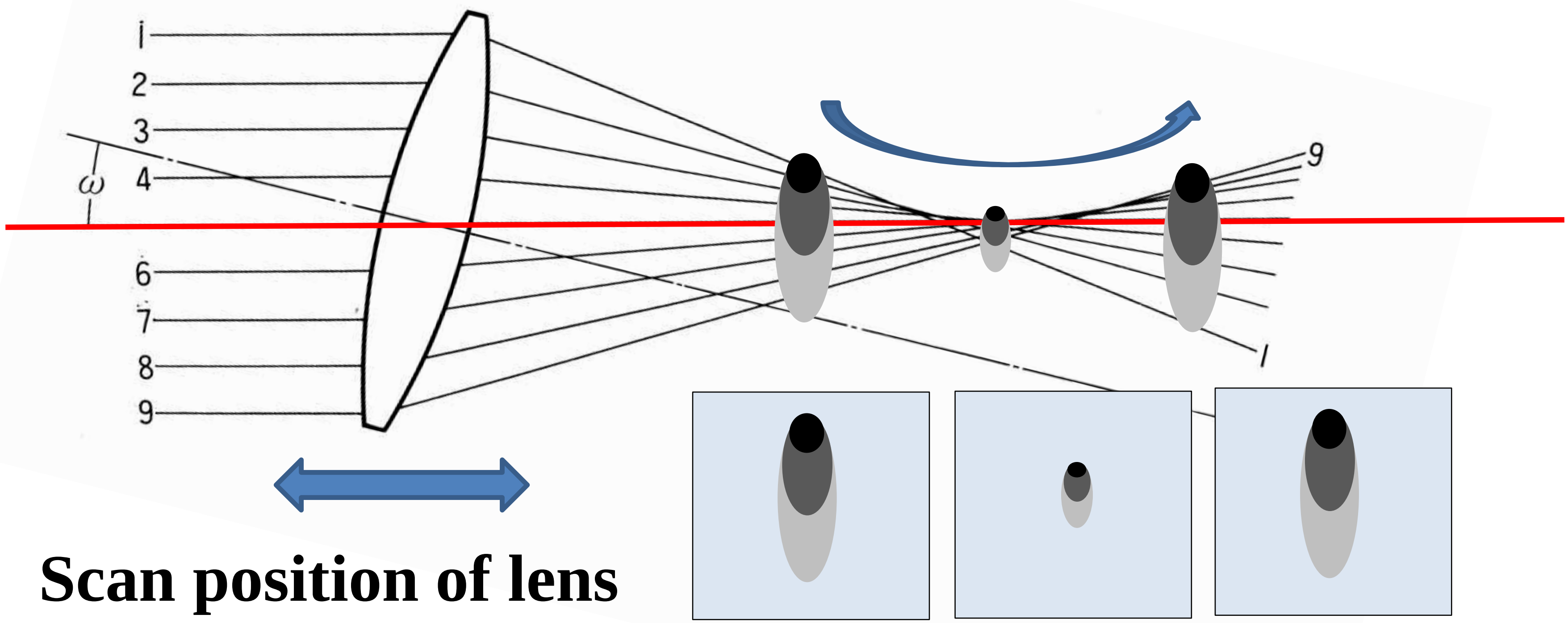




Alignment error (tilt) of lens also produce comma

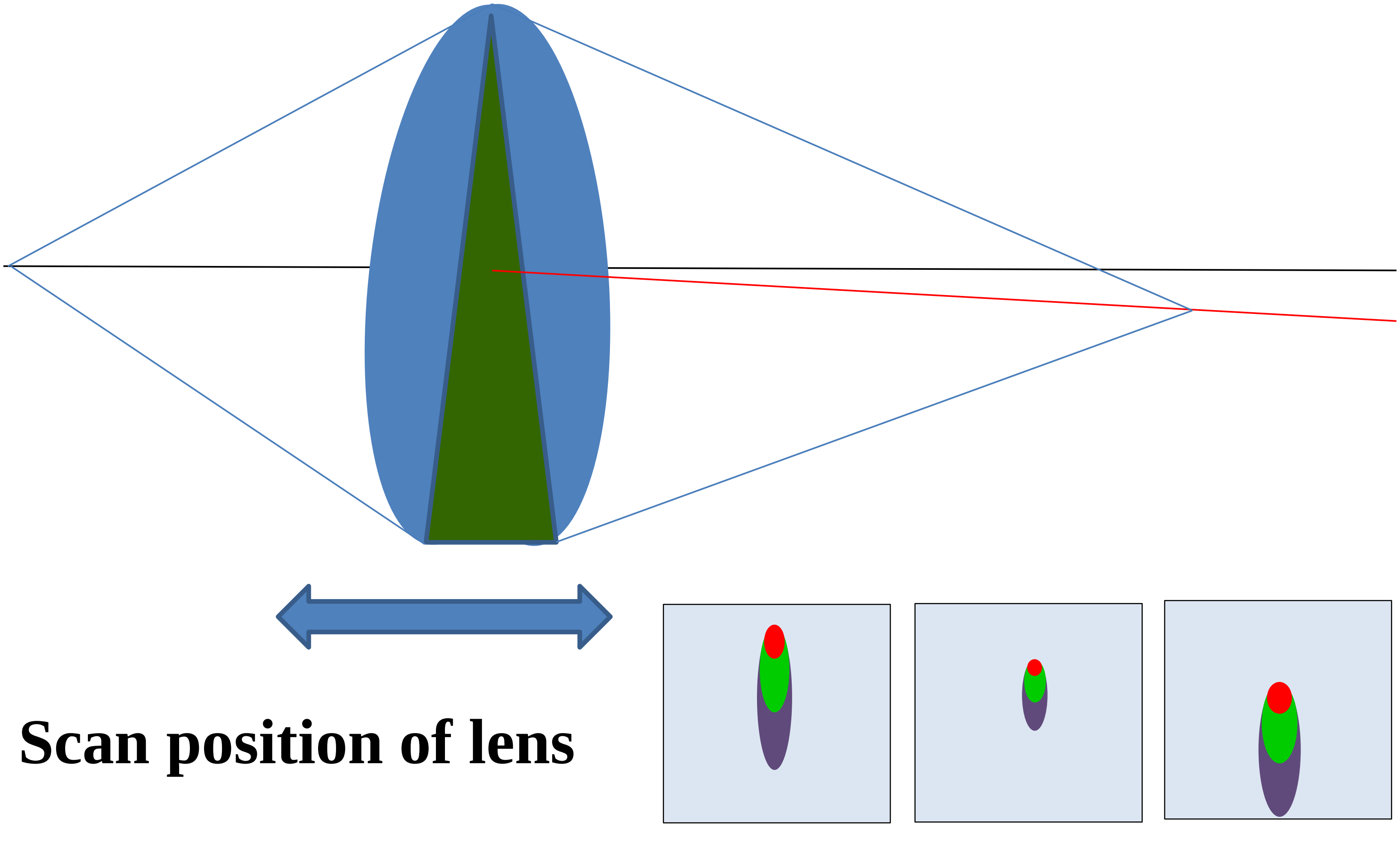


Alignment error (tilt) of lens also produce comma



Scan position of lens

Lens has often has a wedge component!



Scan position of lens

More general theory for aberrations

Zernike’s circule polynomial and aberration coefficients.

Zernike's circule polynomial for expansion of wavefront sag.

$$W(x,y) = W(\rho \sin\theta, \rho\cos\theta) = W(\rho,\theta)$$

$$= \sum_{n=0}^{k}\sum_{m=0}^{n} A_{nm} \cdot R_n^{n-2m}(\rho) \cdot \begin{cases} \cos|n-2m|\theta & : n-2m \geq 0 \\ \sin|n-2m|\theta & : n-2m < 0 \end{cases}$$

$$R_n^{n-2m}(\rho) = \sum_{s=0}^{m} (-1)^s \frac{(n-s)!\,\rho^{n-2s}}{s!(m-s)!(n-m-s)!}$$

Aberration coefficients

A_0^0: **Piston** A_4^0 **:spherical**

B_1^1 **: Tilt in y**

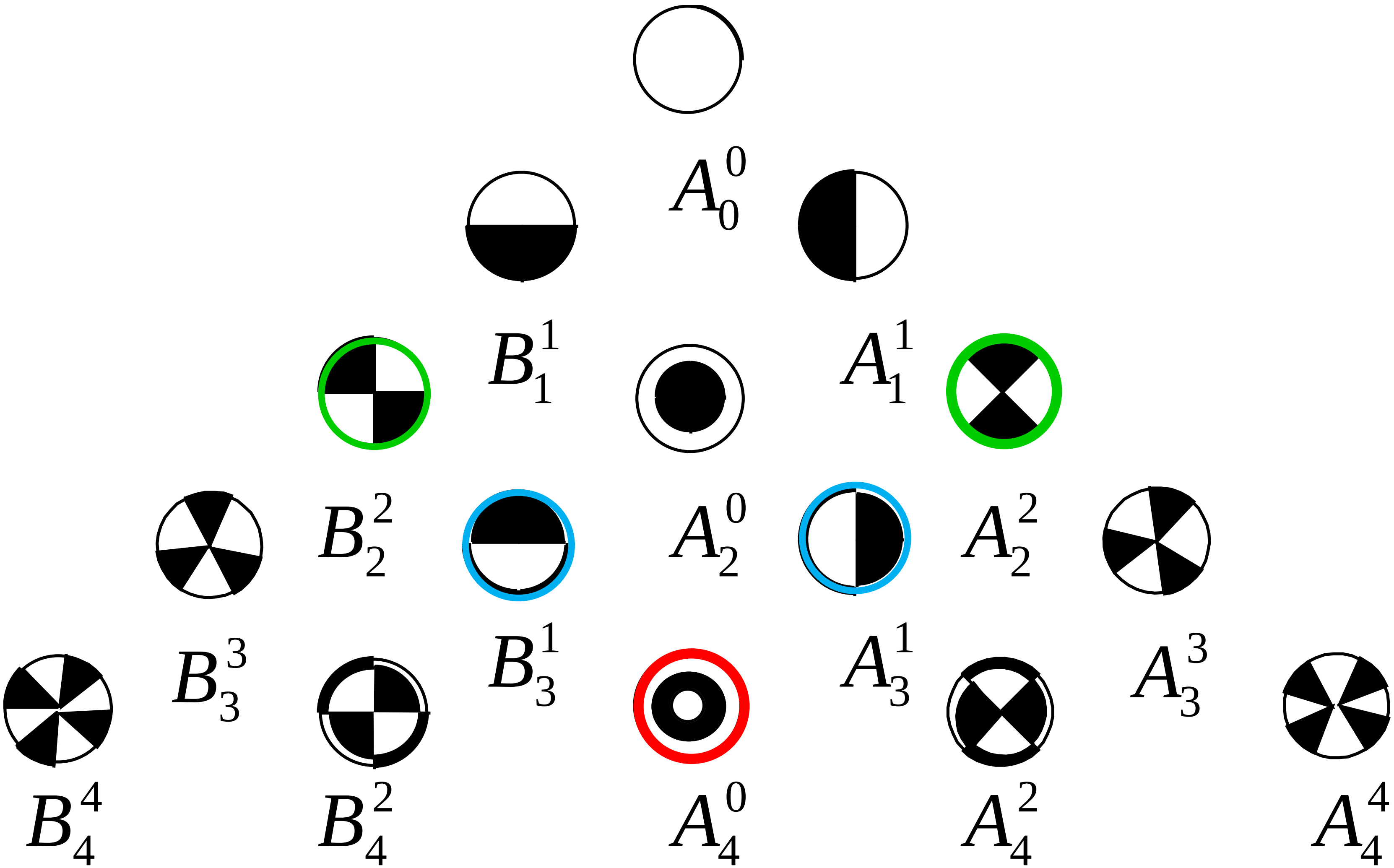
A_1^1**: Tilt in x**

A_2^0**: Focus shift**

B_2^2**: Astigmatism in diagonal**

A_2^2**: Astigmatism**

Graphical drawing of Zernick's aberration function



Conclusions from Geometrical Optics

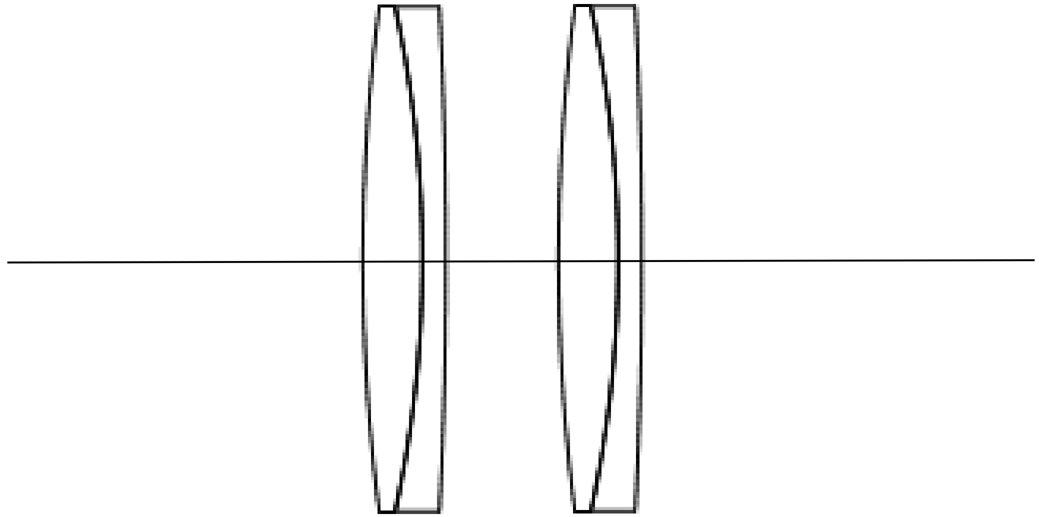
Focusing system has many aberrations, even on optical axis!

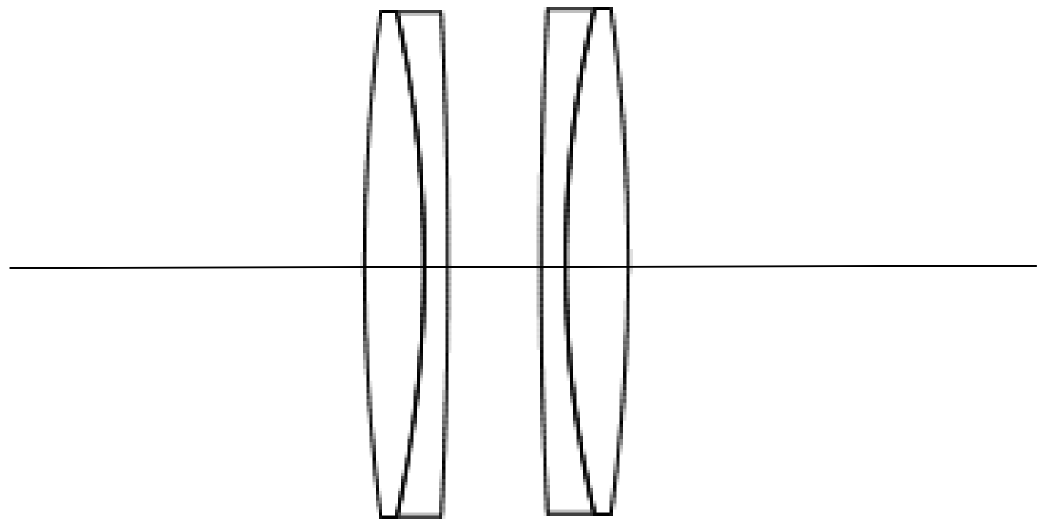
This error is very often not smaller than diffraction width!

Careful analysis of aberrations is important!

Excise

Which is correct combination of two doublets?





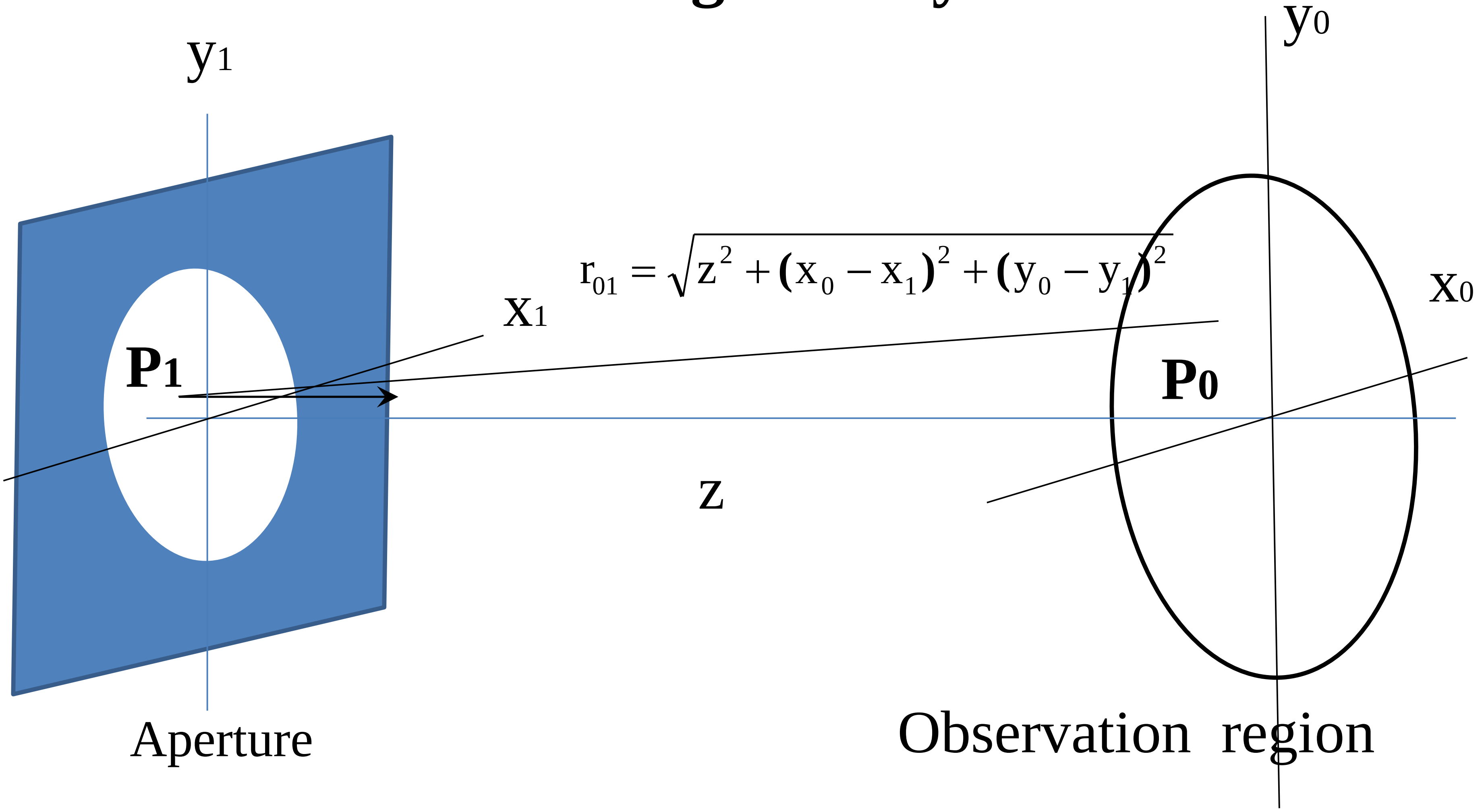
Lecture 3

Wave optics of focusing system

Wave optics for focusing system

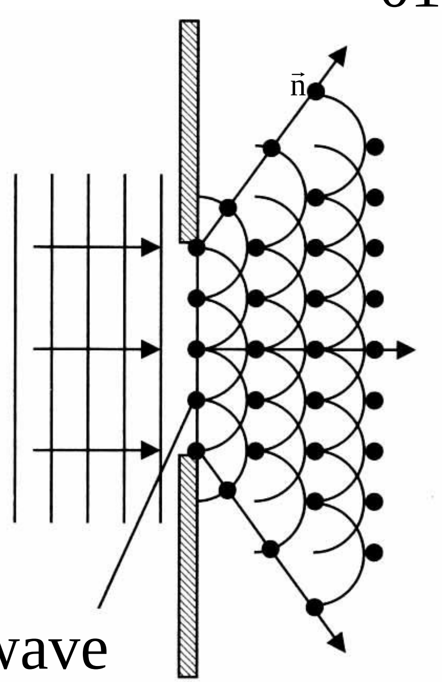
Diffraction

Diffraction geometry

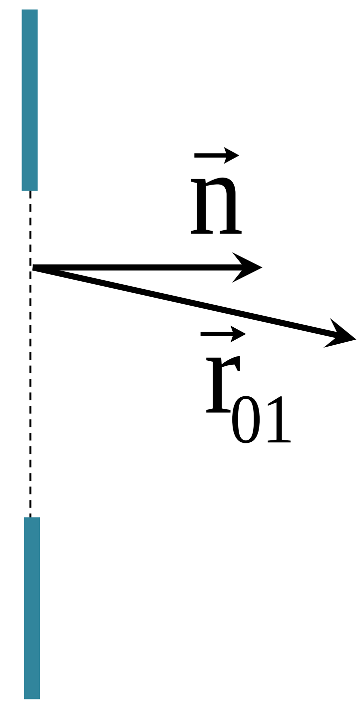


$$\mathbf{U}(p_0) = \iint_{\Sigma} \mathbf{h}(P_0, P_1)\, \mathbf{U}(P_1)\, dx_1 dy_1$$

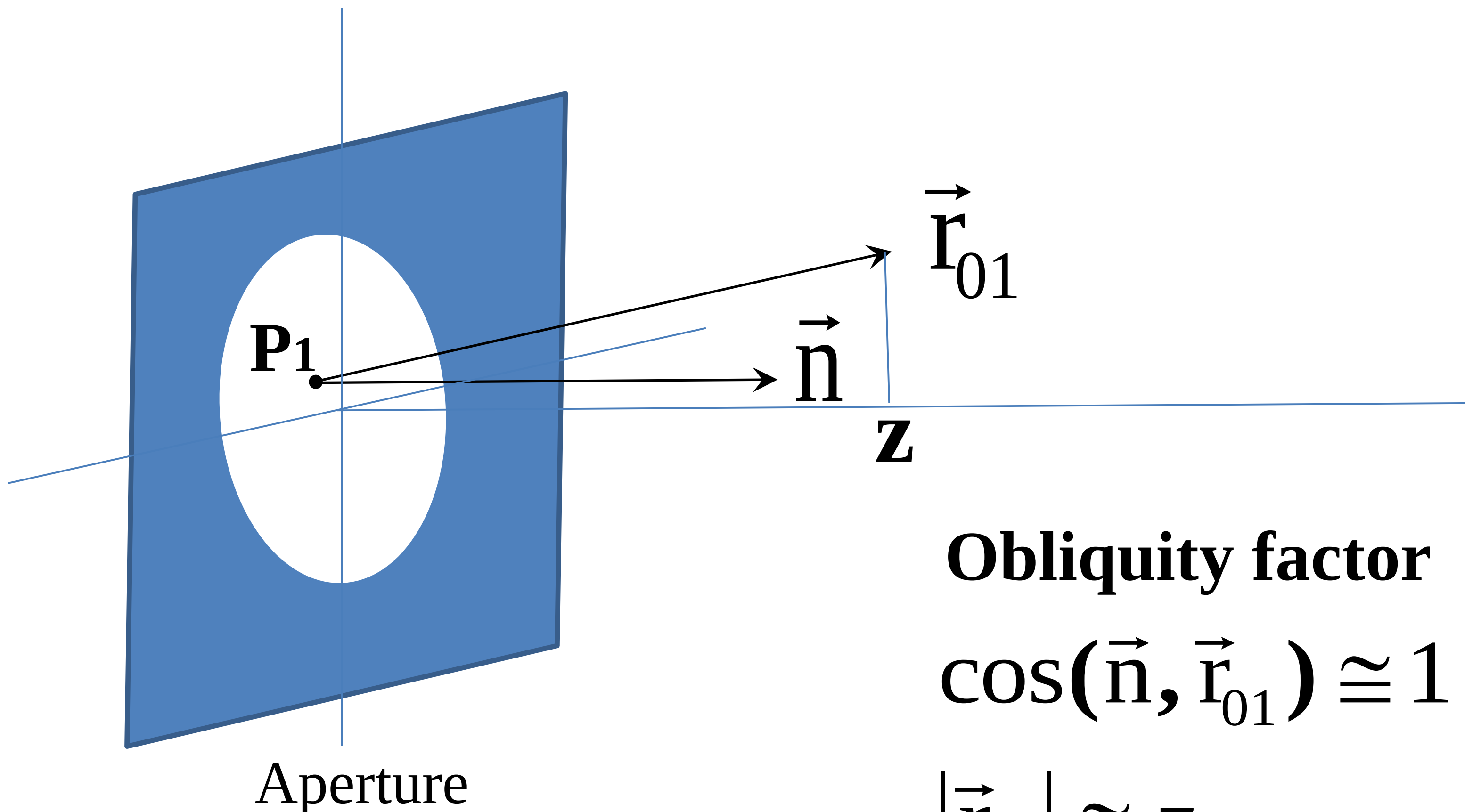
$$\mathbf{h}(P_0, P_1) = \frac{1}{i \cdot \lambda} \frac{\exp(i \cdot k \cdot r_{01})}{r_{01}} \cos(\vec{n}, \vec{r}_{01})$$



Secondary spherical wave



Paraxial approximation



Obliquity factor

$$\cos(\vec{n}, \vec{r}_{01}) \cong 1$$

$$|\vec{r}_{01}| \cong z$$

$$\mathbf{U}(\mathrm{p}_0) = \iint_{\Sigma} \mathbf{h}(\mathrm{P}_0, \mathrm{P}_1)\, \mathbf{U}(\mathrm{P}_1)\, \mathrm{d}x_1 \mathrm{d}y_1$$

$$\mathbf{h}(\mathrm{P}_0, \mathrm{P}_1) = \frac{1}{\mathrm{i} \cdot \lambda} \frac{\exp(\mathrm{i} \cdot \mathrm{k} \cdot \mathrm{r}_{01})}{\mathrm{r}_{01}} \boxed{\cos(\vec{\mathrm{n}}, \vec{\mathrm{r}}_{01})}$$

$$\approx 1$$

$$\mathrm{h}(\boldsymbol{P}_0, \boldsymbol{P}_1) = \frac{1}{\boldsymbol{i} \cdot \lambda} \frac{exp(\boldsymbol{i} \cdot \boldsymbol{k} \cdot \boldsymbol{r}_{01})}{\boldsymbol{r}_{01}}$$

The Fresnel approximation for r_{01} in phase factor

$$r_{01} = \sqrt{z^2 + (x_0 - x_1)^2 + (y_0 - y_1)^2}$$

$$= z\sqrt{1 + \left(\frac{x_0 - x_1}{z}\right)^2 + \left(\frac{y_0 - y_1}{z}\right)^2}$$

$$\cong z\left[1 + \frac{1}{2}\left(\frac{x_0 - x_1}{z}\right)^2 + \frac{1}{2}\left(\frac{y_0 - y_1}{z}\right)^2\right]$$

Spherical $\Rightarrow$ Quadratic phase factor

$$\mathbf{h}(x_0, y_0 : x_1, y_1) = \frac{\exp(ikz)}{i\lambda z} \boxed{\exp\left[\frac{ik}{2z}\left[(x_0 - x_1)^2 + (y_0 - y_1)^2\right]\right]}$$

Quadratic wave

$$\boxed{\begin{aligned} &= \frac{\exp(ikz)}{i\lambda z} \mathbf{exp}\left[\frac{ik}{2z}\left[x_0{}^2 + y_0{}^2\right]\right] \mathbf{exp}\left[\frac{ik}{2z}\left[x_1{}^2 + y_1{}^2\right]\right] \\ &\quad \times \mathbf{exp}\left[\frac{ik}{2z}\left[x_0 x_1 + y_0 y_1\right]\right] \end{aligned}}$$

Fresnel diffraction

The Fraunhofer approximation

Very long z

$$\frac{ik}{2z}\left[x_1^{\ 2}+y_1^{\ 2}\right] << 1$$

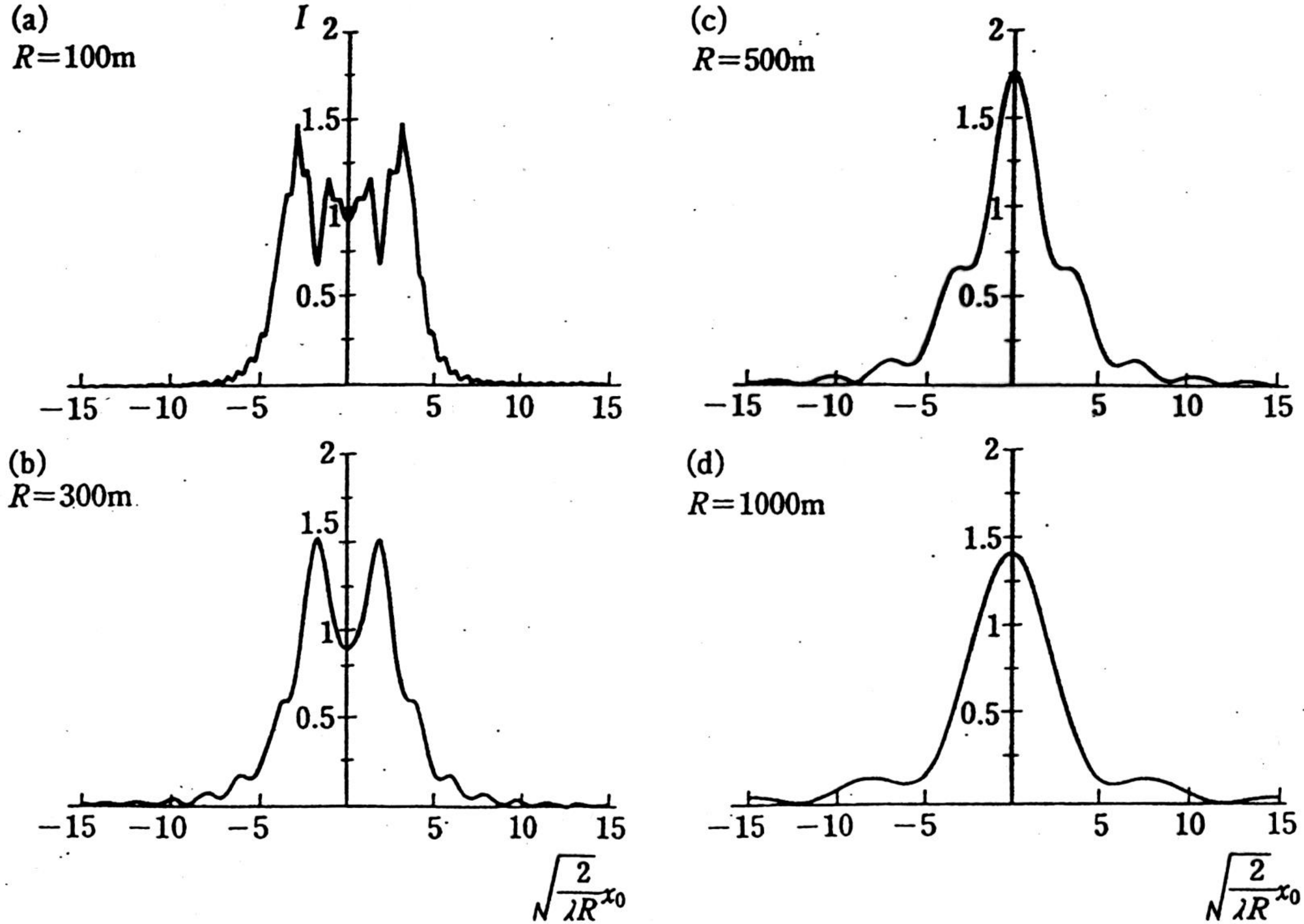
$$\exp\left[\frac{ik}{2z}\left[x_1^{\ 2}+y_1^{\ 2}\right]\right] \cong 1$$

$$\mathbf{h}(x_0, y_0 : x_1, y_1) = \exp\frac{\exp(ikz)}{i\lambda z}\exp\left[\frac{ik}{2z}\left[x_0^{\ 2}+y_0^{\ 2}\right]\right]$$

$$\times \boxed{\exp\left[\frac{ik}{2z}\left[x_0 x_1 + y_0 y_1\right]\right]}$$

Fraunhofer diffraction

Diffraction by slit   height 10mm



Propagation of light in free space by few 10m

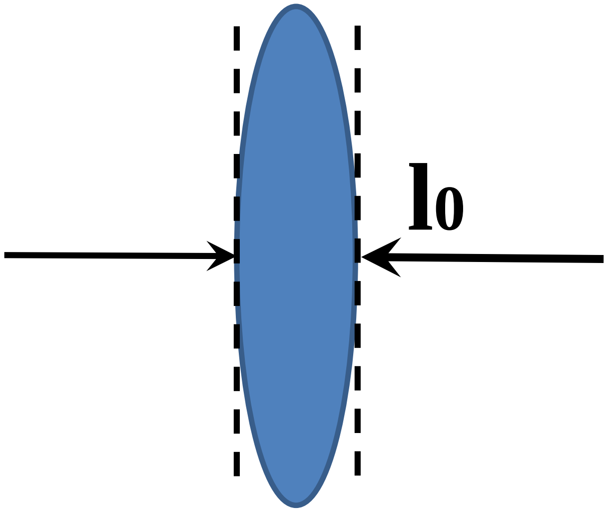
Fresnel diffraction!

Fraunhofer diffraction: few km

Pupil with Lens

Pupil with Lens

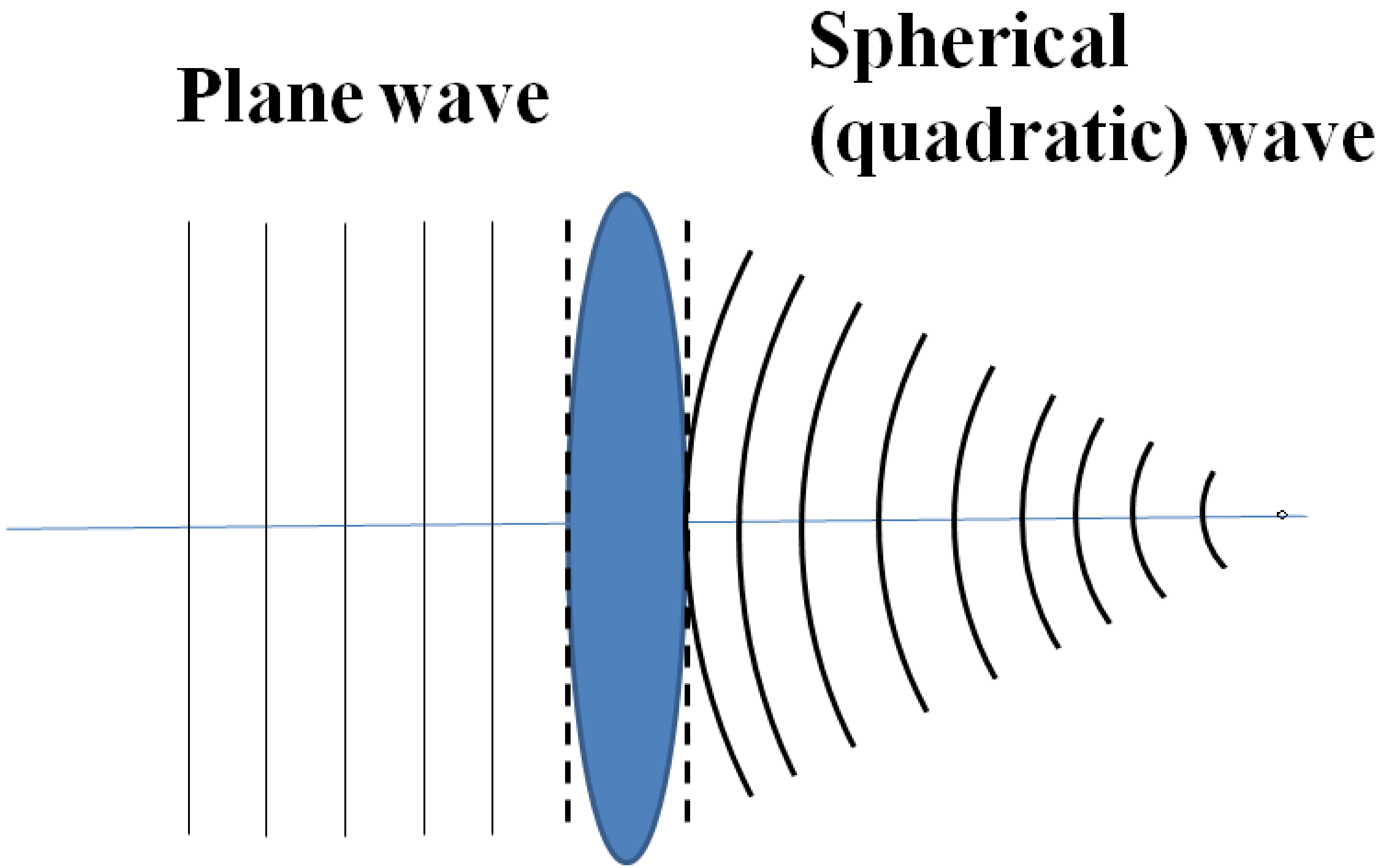
Paraxial Lens transfer function t_l



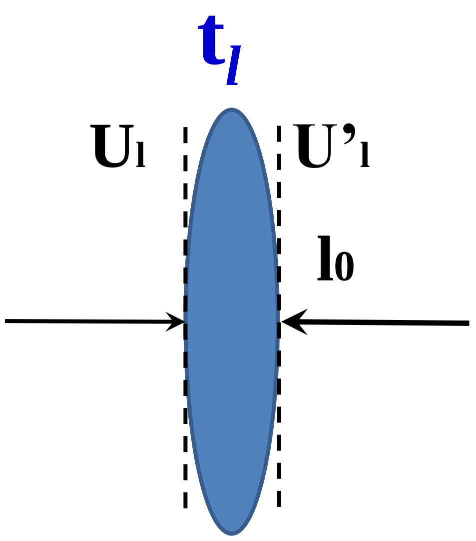
$$t_l(\boldsymbol{x},\boldsymbol{y}) = exp\left[\boldsymbol{ik}\left(\boldsymbol{l}_0 + (\boldsymbol{n}-1)\boldsymbol{l}_0 - \frac{1}{2\boldsymbol{f}}\left(\boldsymbol{x}^2+\boldsymbol{y}^2\right)\right)\right]$$

$$= exp(\boldsymbol{ikl}_0)\,exp[\boldsymbol{ik}(\boldsymbol{n}-1)\boldsymbol{l}_0]\,exp\left(-\boldsymbol{i}\frac{\boldsymbol{k}}{2\boldsymbol{f}}\left(\boldsymbol{x}^2+\boldsymbol{y}^2\right)\right)$$

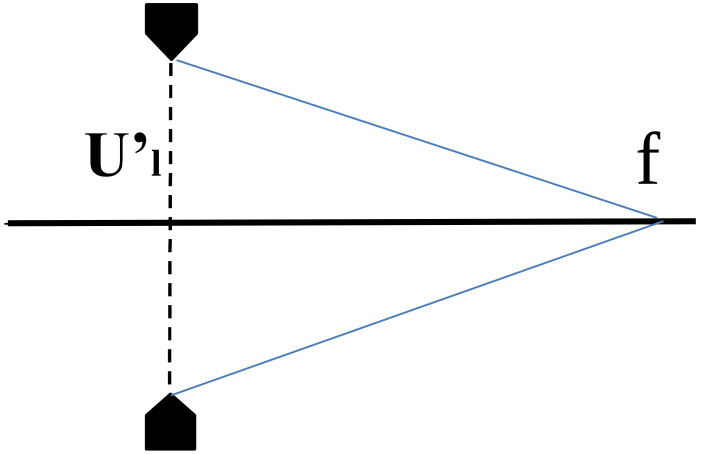
Physical meaning of paraxial lens transfer function



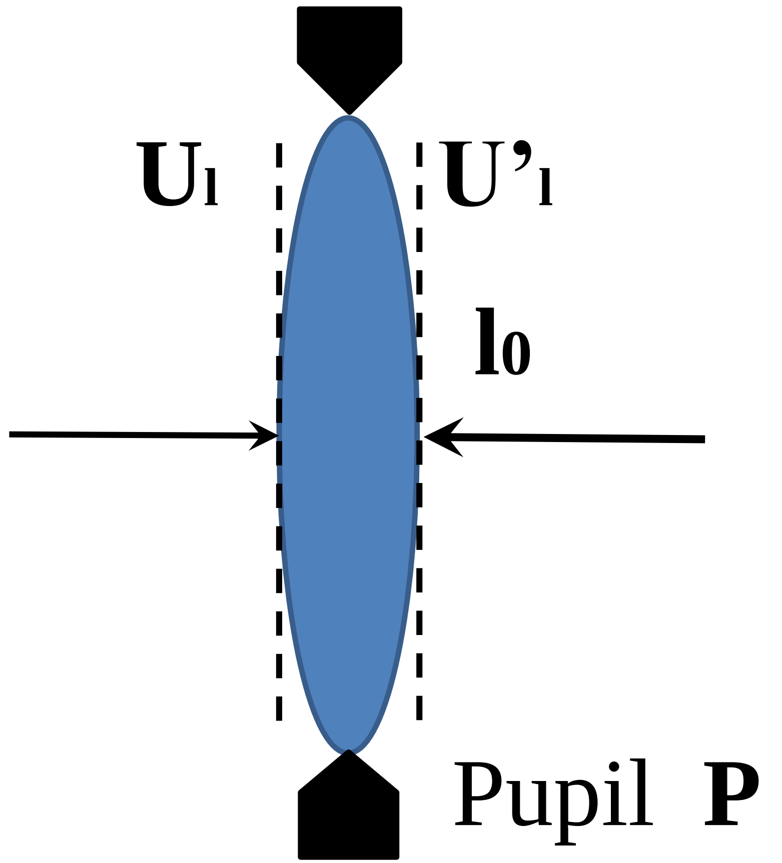
Diffraction with lens



Step 1
Paraxial lens transform of input U_l



Step 1
Fresnel Transform of U'_l



$$\mathbf{U}'_{l}(x,y) = \mathbf{P}(x,y)\,t_{l}(x,y)\,\mathbf{U}_{l}(x,y)$$

$$= \mathbf{P}(x,y)\,\mathbf{U}_{l}(x,y)\exp\left[-i\frac{k}{2f}\left(x^{2}+y^{2}\right)\right]$$

$$U_f(x_f, y_f) = \frac{\exp\left[i\frac{k}{2f}\left(x_f^2 + y_f^2\right)\right]}{i\lambda f}$$

$$\times \iint_\Sigma U_l(x,y)\exp\left(-i\frac{k}{2f}\left(x^2+y^2\right)\right)\boxed{\exp\left(i\frac{k}{2f}\left(x^2+y^2\right)\right)}\exp\left[\frac{ik}{2f}\left[xx_f + yy_f\right]\right]dxdy$$

Lens transfer function

Quadratic wave

As the result;

$$U_f(x_f, y_f) = \frac{\exp\left[i\frac{k}{2f}\left(x_f^2 + y_f^2\right)\right]}{i\lambda f}$$

Plane wave

$$\times \iint U_l(x,y)P(x,y)\boxed{\exp\left[\frac{ik}{2f}\left[xx_f + yy_f\right]\right]}dxdy$$

$$U_f(x_f, y_f) = \frac{\exp\left[i \frac{k}{2f}\left(x^2{}_f + y^2{}_f\right)\right]}{i\lambda f}$$

$$\times \iint_{\Sigma} U_l(x,y) \underbrace{\exp\left(-i \frac{k}{2f}\left(x^2 + y^2\right)\right)}_{\text{Lens transfer function}} \underbrace{\boxed{\exp\left(i \frac{k}{2f}\left(x^2 + y^2\right)\right)}}_{\text{Quadratic wave}} \exp\left[\frac{ik}{2f}\left[xx_f + yy_f\right]\right] dxdy$$

As the result;

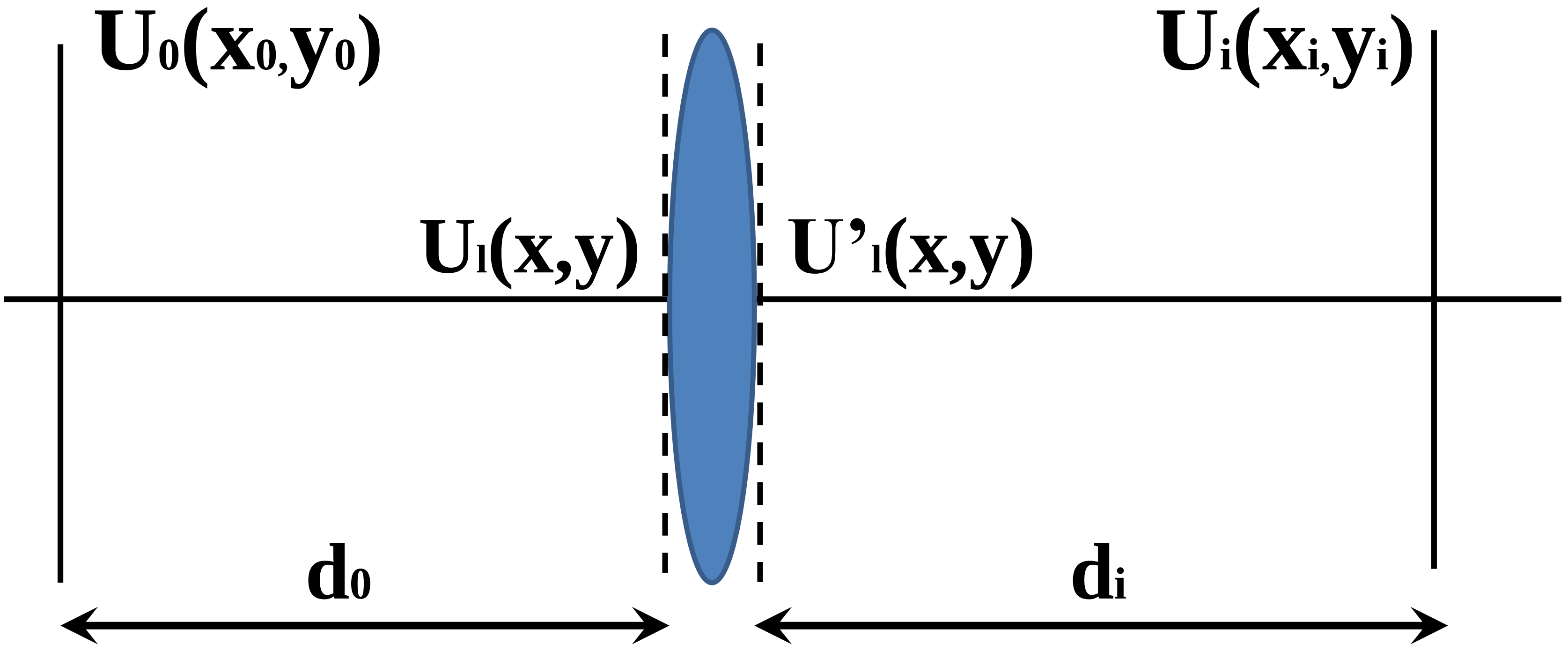
Fourier transform of pupil function

$$\mathbf{U}_f\,(x_f, y_f) = \frac{\exp\left[i \frac{k}{2f}\left(x^2{}_f \right.\right.}{i\lambda f}$$

Plane wave

$$\times \iint \mathbf{U}_l(x,y)\mathbf{P}(x,y) \boxed{\exp\left[\frac{ik}{2f}\left[xx_f + yy_f\right]\right]} dxdy$$

Diffraction on image plane



1. $\mathbf{U_0(x_0,y_0) \longrightarrow U_l(x,y)}$

Fresnel transform

$$\mathbf{U}_l(x,y)=\iint \mathbf{U}_0(x_0,y_0)\boxed{\exp\left(i\frac{k}{2d_0}\left(x^2+y^2\right)\right)\exp\left[\frac{ik}{2d_0}\left[x_0x+y_0y\right]\right]dx_0dy_0}$$

2. $\mathbf{U_1(x,y) \longrightarrow U'_l(x,y)}$

Lens transform

$$\mathbf{U'}_l(x,y)=t(x,y)\mathrm{U}_l(x,y)$$

$$=\iint \mathbf{U'}_l(x,y)\mathbf{P}(x,y)\boxed{\exp\left(-i\frac{k}{2f}\left(x^2+y^2\right)\right)}\exp\left(i\frac{k}{2d_0}\left(x^2+y^2\right)\right)$$

$$\times\exp\left[\frac{ik}{2d_0}\left[x_0x+y_0y\right]\right]dx_0dy_0$$

3. $U'_1(x,y) \longrightarrow U_i(x_i,y_i)$

Fresnel **tran**sform

$$U_i(x_i, y_i) = \iint U'_1(x, y) \boxed{\exp\left(i\frac{k}{2d_i}\left(x^2 + y^2\right)\right)}$$

$$\boxed{\times \exp\left[\frac{ik}{2d_i}\left[x x_i + y y_i\right]\right]} dxdy$$

Then, h is given by;

$$\mathbf{h}(x_0, y_0 : x_i, y_i) = \iint \mathbf{U}'_1(x,y)\,\mathbf{P}(x,y) \exp\left(i\frac{k}{2di}(x^2+y^2)\right)\exp\left(i\frac{k}{2d_0}(x^2+y^2)\right)$$

$$\times \exp\left(-i\frac{k}{2f}(x^2+y^2)\right)\exp\left[\frac{ik}{2d_0}[x_0x + y_0y]\right]\exp\left[\frac{ik}{2di}[xx_i + yy_i]\right]dxdy$$

Tidy up the equation;

$$= (\text{Quadratic phase factor}) \times \iint \mathbf{P}(x,y)\exp\left[i\frac{k}{2}\left(\frac{1}{d_0}+\frac{1}{d_i}-\frac{1}{f}\right)(x^2+y^2)\right]$$

$$\times \exp\left[-ik\left(\left(\frac{x_0}{d_0}+\frac{x_i}{d_i}\right)x + \left(\frac{y_0}{d_0}+\frac{y_i}{d_i}\right)y\right)\right]dxdy$$

physical meaning is not clear!

Then, h is given by;

$$\mathbf{h}(x_0, y_0 : x_i, y_i) = \iint \mathbf{U'}_1(x,y)\,\mathbf{P}(x,y)\exp\left(i\frac{k}{2di}\left(x^2+y^2\right)\right)\exp\left(i\frac{k}{2d_0}\left(x^2+y^2\right)\right)$$

$$\times\exp\left(-i\frac{k}{2f}\left(x^2+y^2\right)\right)\exp\left[\frac{ik}{2d_0}\left[x_0x+y_0y\right]\right]\exp\left[\frac{ik}{2di}\left[xx_i+yy_i\right]\right]dxdy$$

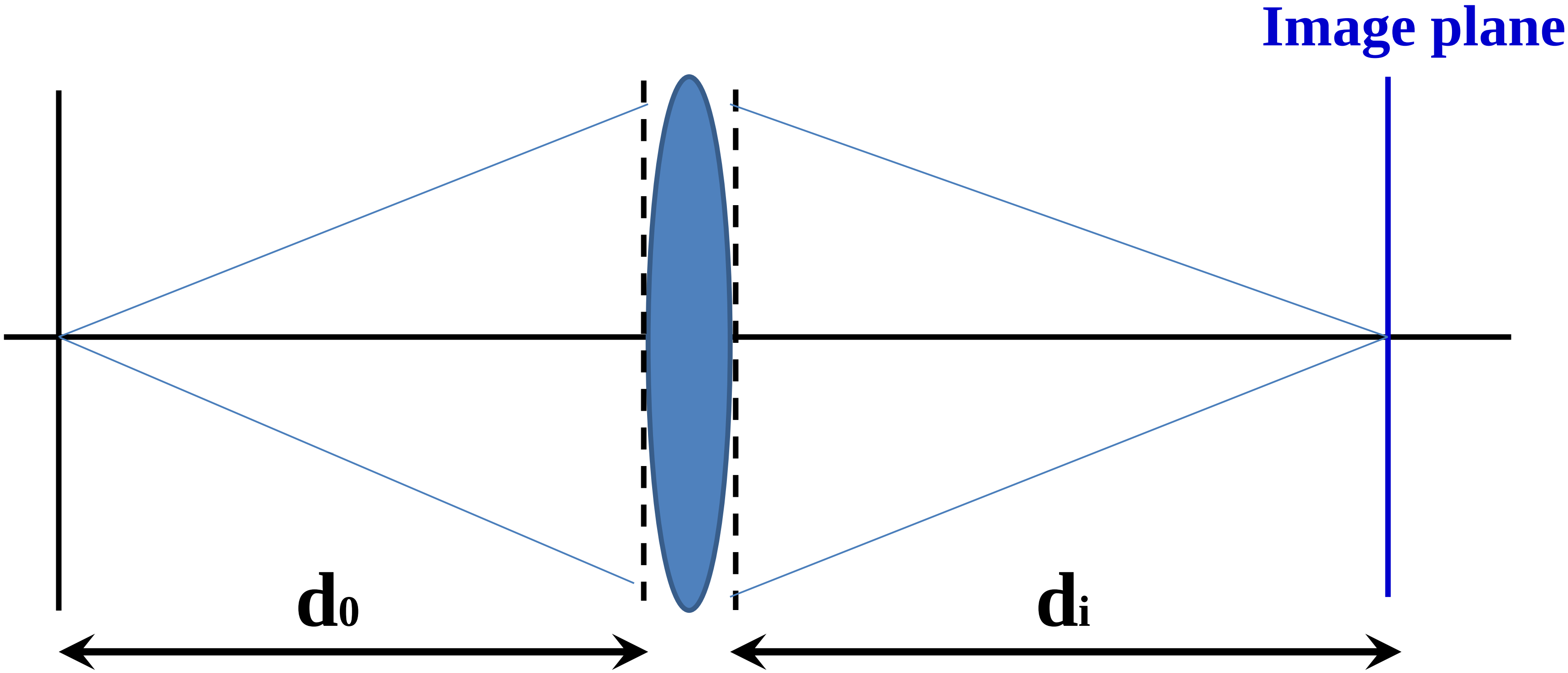
$$=\left(\text{Quadratic phase factor}\right)\times\iint\mathbf{P}(x,y)\exp\left[i\frac{k}{2}\left(\frac{1}{d_0}+\frac{1}{d_i}-\frac{1}{f}\right)\left(x^2+y^2\right)\right]$$

$$\times\exp\left[-ik\left(\left(\frac{x_0}{d_0}+\frac{x_i}{d_i}\right)x+\left(\frac{y_0}{d_0}+\frac{y_i}{d_i}\right)y\right)\right]dxdy$$

Excise
What is physical meaning of this term?

Answer ;

Return to geometrical optics;



$$\frac{1}{d_0}+\frac{1}{d_i}-\frac{1}{f}=0$$

Lens equation !

Then phase factor $\exp\left[i\frac{k}{2}\left(\frac{1}{d_0}+\frac{1}{d_i}-\frac{1}{f}\right)\left(x^2+y^2\right)\right]$ **be 1**

Then, h becomes

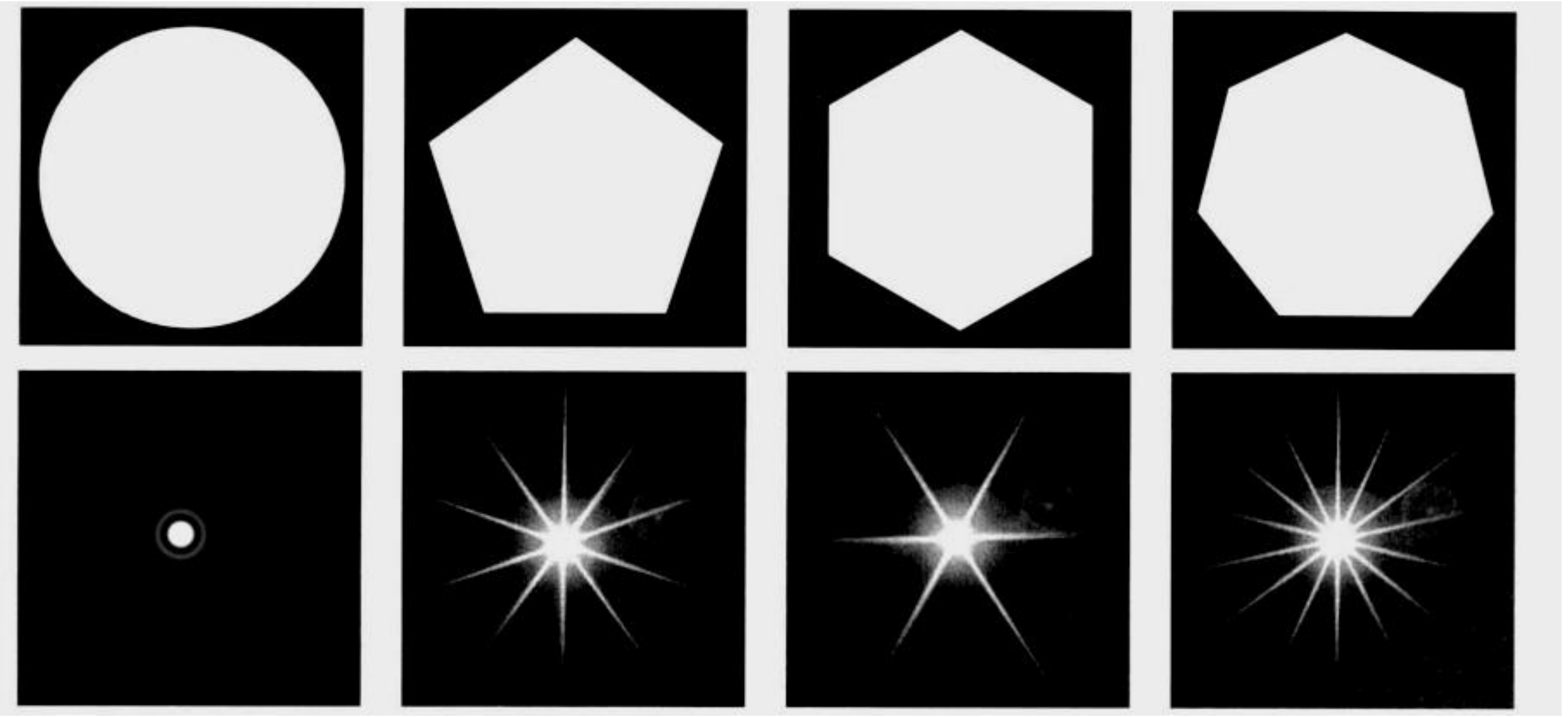
$$\mathbf{h}\left(x_0,y_0:x_i,y_i\right)=\iint \mathbf{P}(x,y)\exp\left[-ik\left(\left(\frac{x_0}{d_0}+\frac{x_i}{d_i}\right)x+\left(\frac{y_0}{d_0}+\frac{y_i}{d_i}\right)y\right)\right]dxdy$$

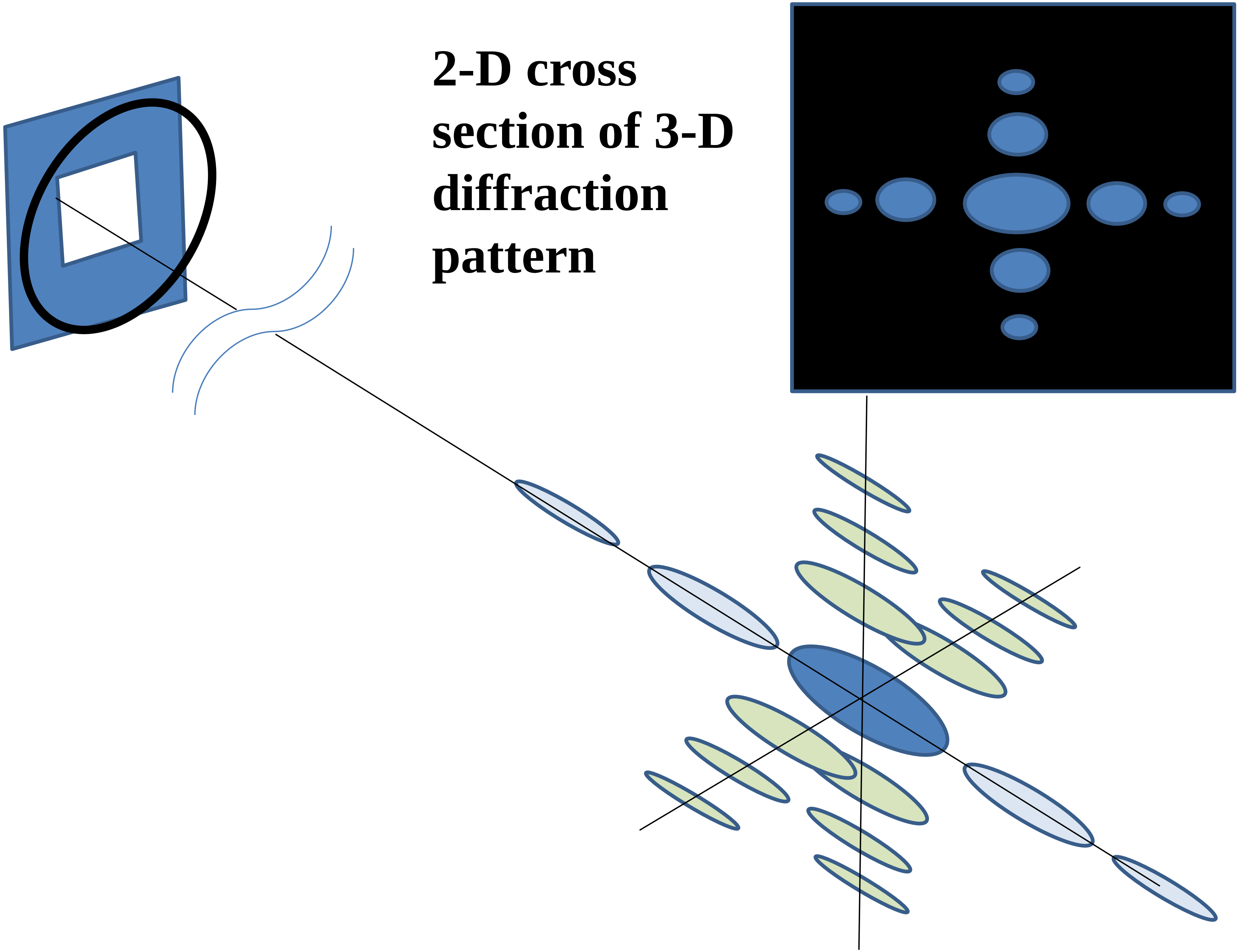
Introducing magnification M; $M\equiv\frac{d_i}{d_0}$

$$\mathbf{h}\left(x_0,y_0:x_i,y_i\right)\cong\iint \mathbf{P}(x,y)\exp\left[-\frac{ik}{d_i}\left(\left(x_i+Mx_0\right)x+\left(y_i+My_0\right)y\right)\right]dxdy$$

The Fraunhofer diffraction will be appear on image plane with magnification in geometrical optics!

Diffraction patterns for several apertures




2-D cross
section of 3-D
diffraction
pattern

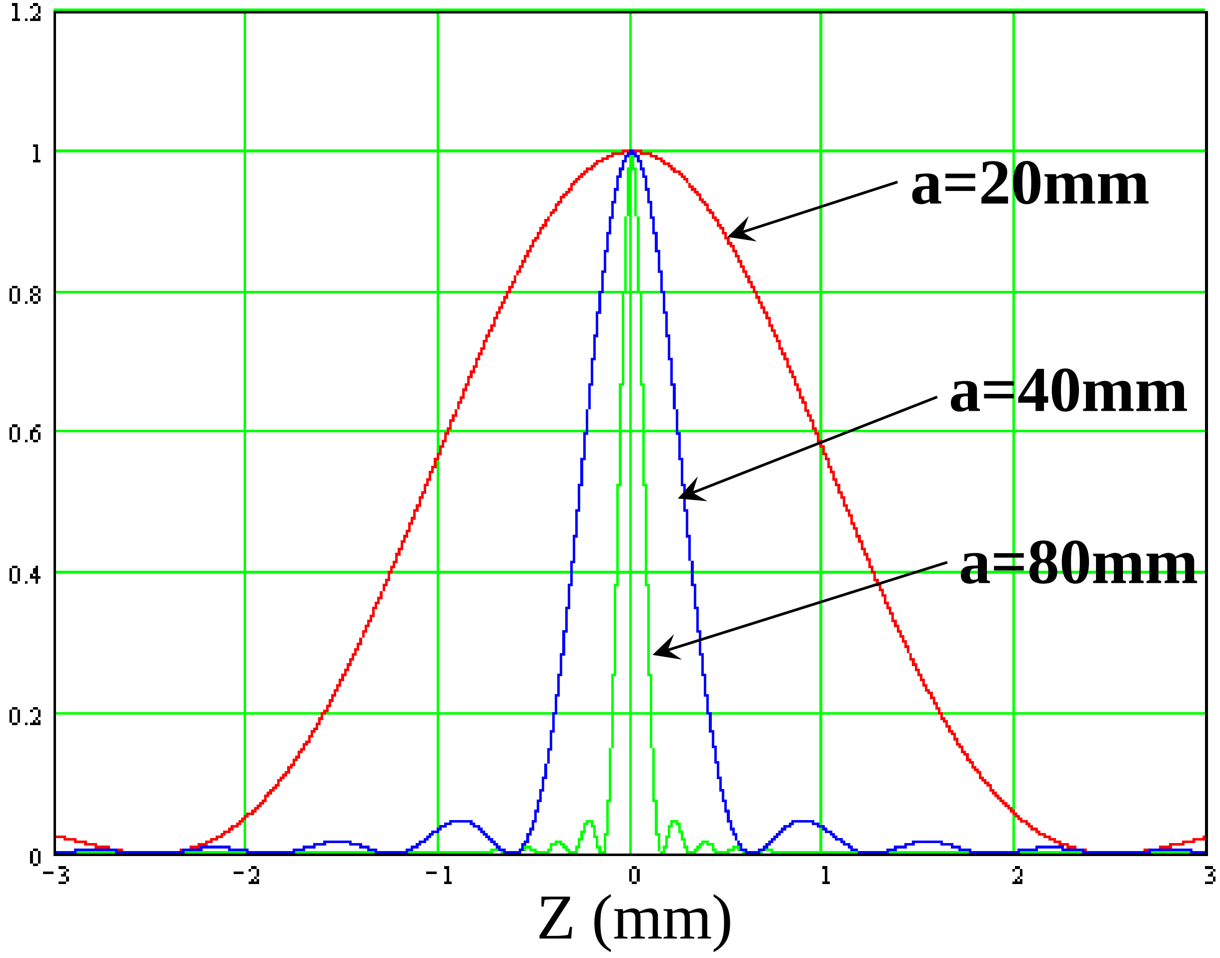
Intensity of longitudinal on axis

$$N=\frac{a^2}{\lambda\cdot f}\ , \qquad U(z)=\frac{2\pi}{\lambda}\left(\frac{a}{f}\right)^2 z$$

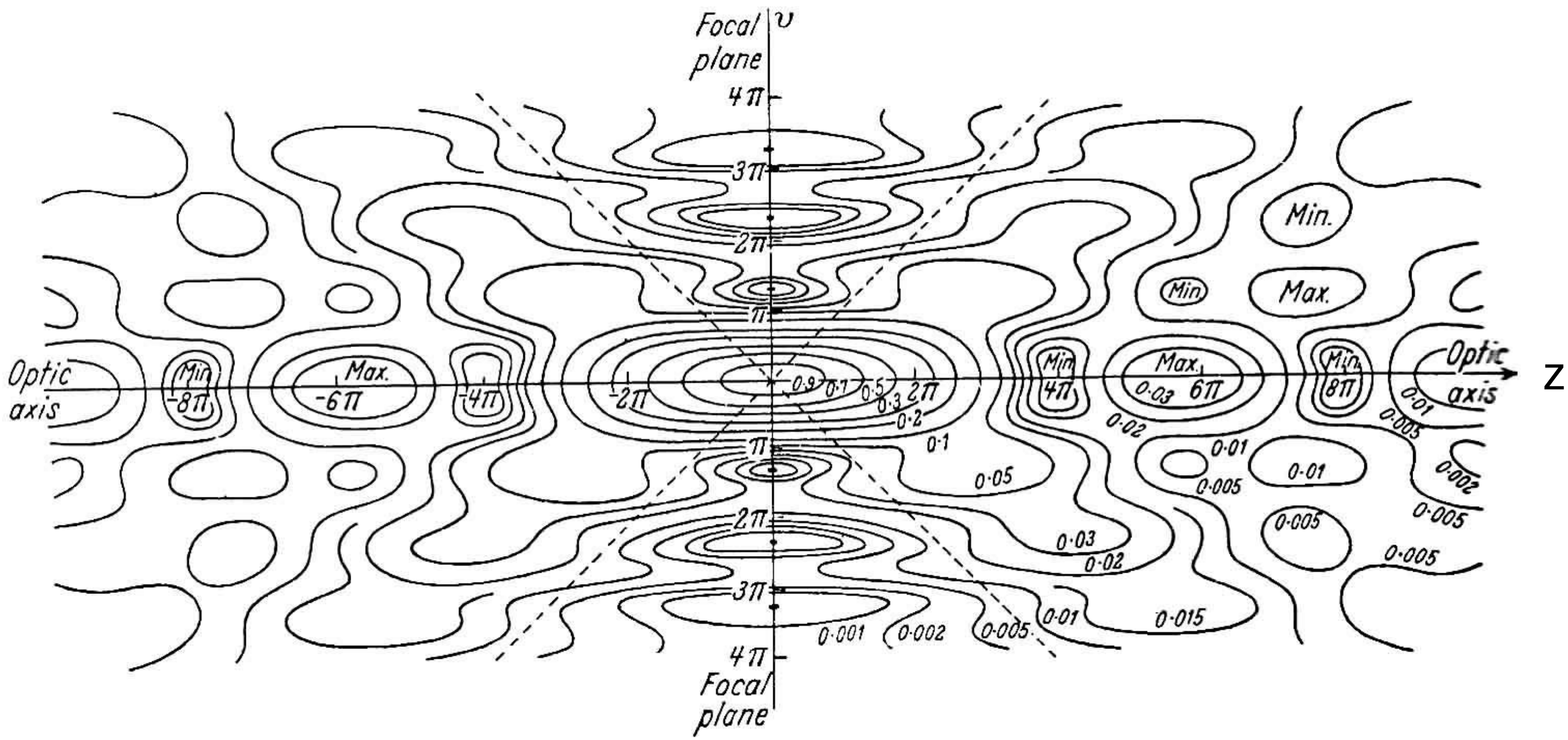
$$V(z)=\frac{U(z)}{1+\frac{U(z)}{2\pi N}}$$

$$I(z)=\left(1-\frac{V(z)}{2\pi N}\right)\left(\frac{\sin\left(\frac{V(z)}{4}\right)}{\frac{V(z)}{4}}\right)^2$$

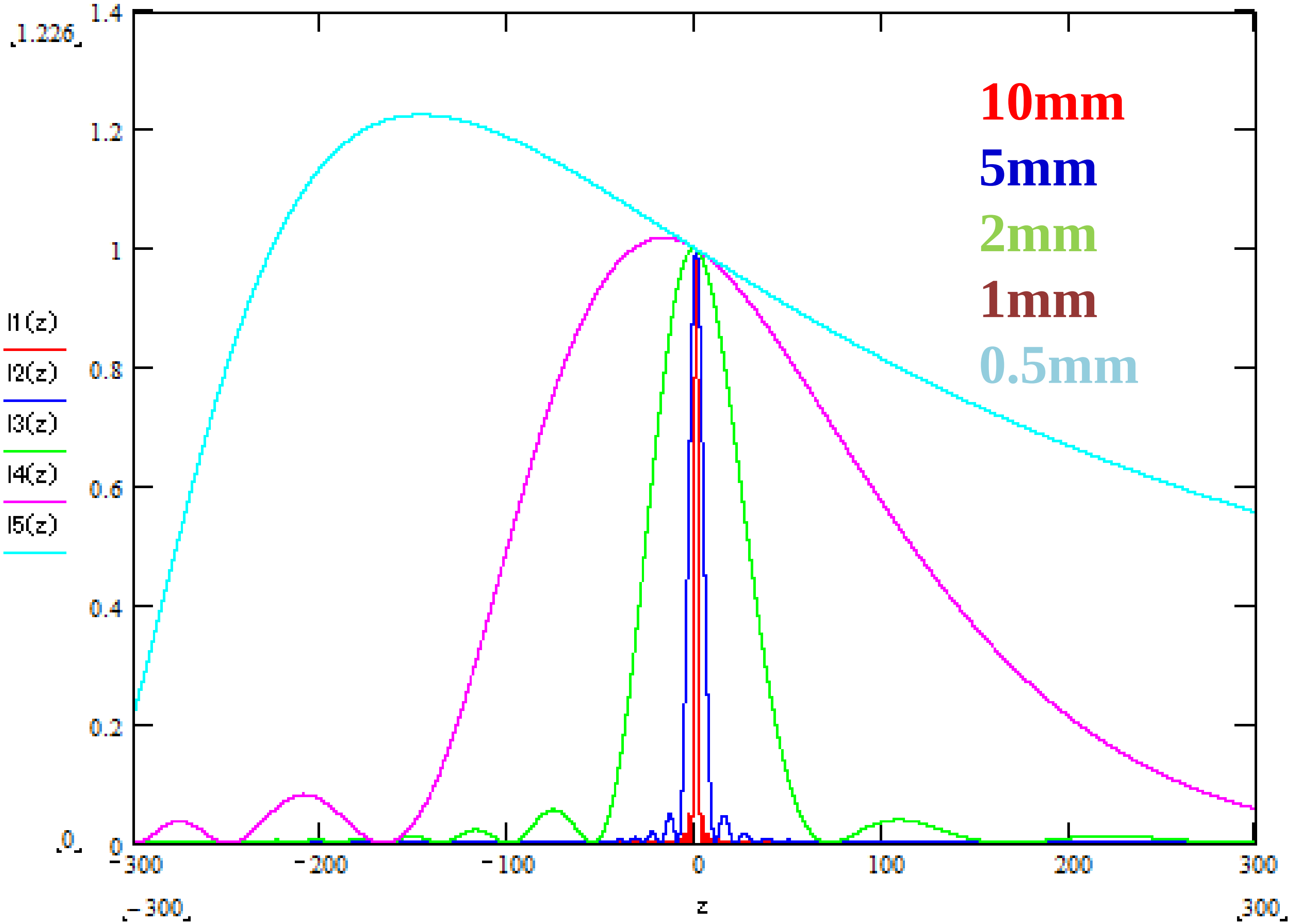
Longitudinal diffraction pattern for lens f=1000mm, a=80, 40, 20mm



Cross section of longitudinal diffraction pattern



Focal shift dependence to change of aperture diameter due to longitudinal diffraction

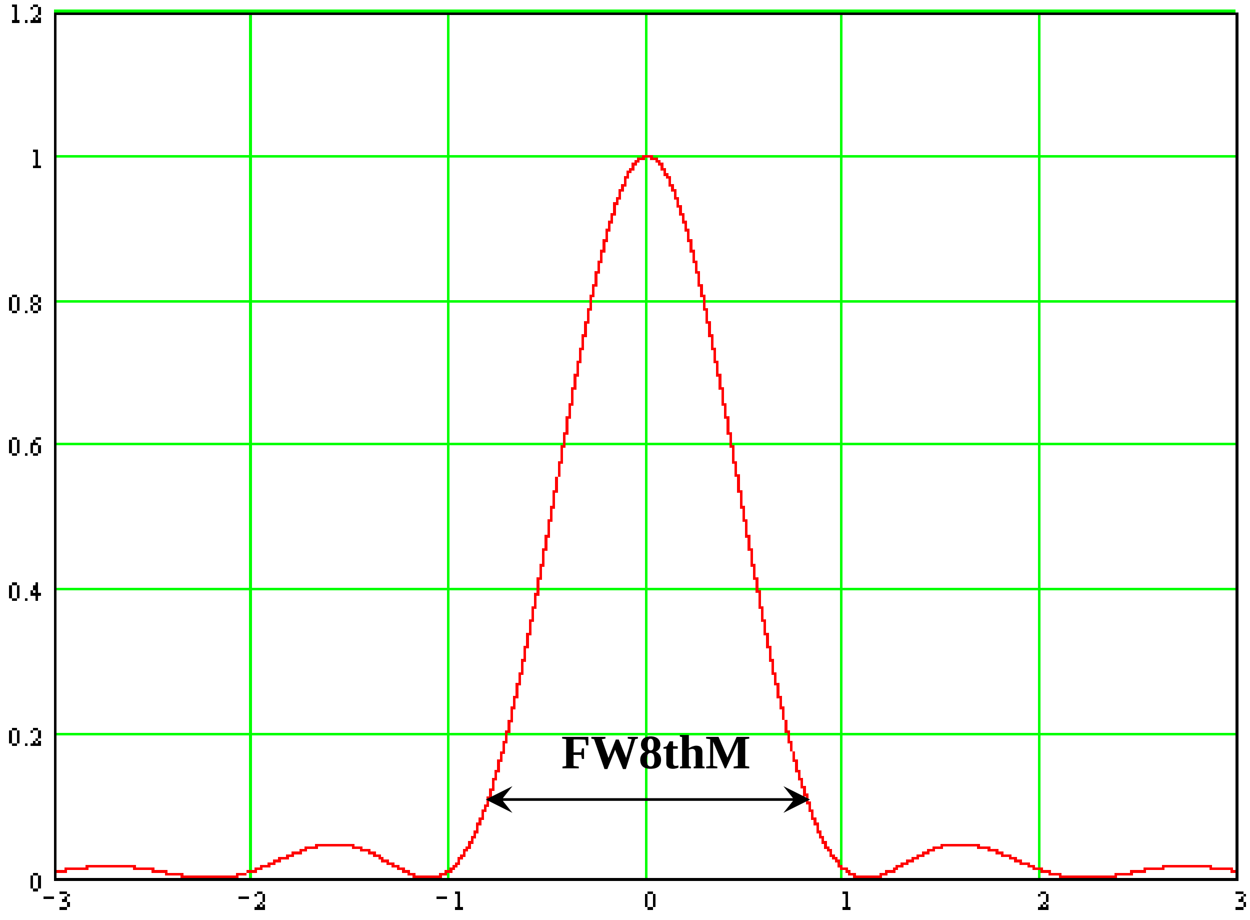


Inside and outside beam images around focus point

The diffraction PSF is smeared with aberrations, but we still can feel discontinuity of longitudinal diffraction pattern from this observations.

f -53 f-43 f-33 f-23 f-13 f f+12 f+22 f+32 f+42 f+53

Definition of field depth=Full Width at 8th Maximum



Z (mm)

Filed depth
≠
coherent length of light
(length of wave pocket)

Ensemble of incoherent lights come from source point makes Field depth.

In generally, from this view point Field depth should say as Incoherent field depth

Coherent field depth = length of wave pocket

Impulsive response function in inverse space and CTF, OTF, MTF

Let's go to the inverse space!

Let us introduce very important parameter, Spatial frequency f_x, f_y by

$$f_x = \frac{2\pi x}{\lambda d_i}$$

$$f_y = \frac{2\pi y}{\lambda d_i}$$

$$\mathbf{h}\left(x_0, y_0 : x_i, y_i\right) = \iint \mathbf{P}(x, y) \exp\left[-\frac{ik}{d_i}\left(\left(x_i + Mx_0\right)x + \left(y_i + My_0\right)y\right)\right] dxdy$$

$$= M \iint \mathbf{P}(f_x, f_y) \exp\left[-i\left(\left(x_i + Mx_0\right)f_x + \left(y_i + My_0\right)f_y\right)\right] df_x df_y$$

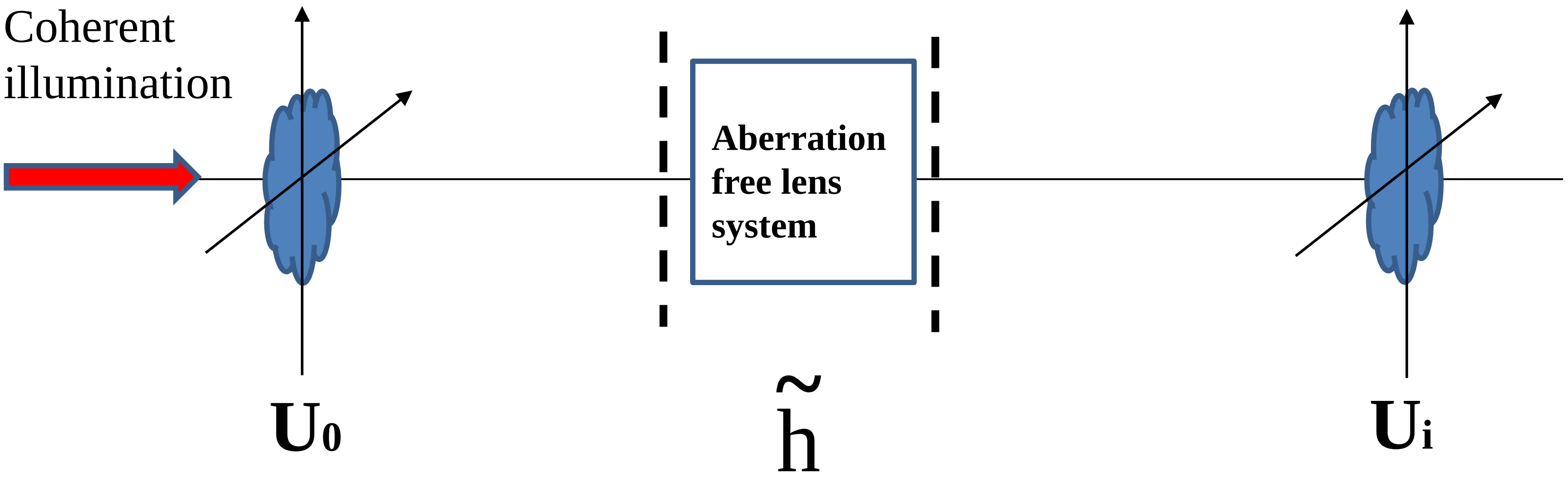
Then, we introduce the spatial invariant response function by;

$$\tilde{\mathbf{h}} = \frac{1}{M}\mathbf{h}$$

$$\tilde{\mathbf{h}}\left(x_0, y_0 : x_i, y_i\right) = \iint \mathbf{P}(f_x, f_y) \exp\left[-i\left(\left(x_i + Mx_0\right)f_x + \left(y_i + My_0\right)f_y\right)\right] df_x df_y$$

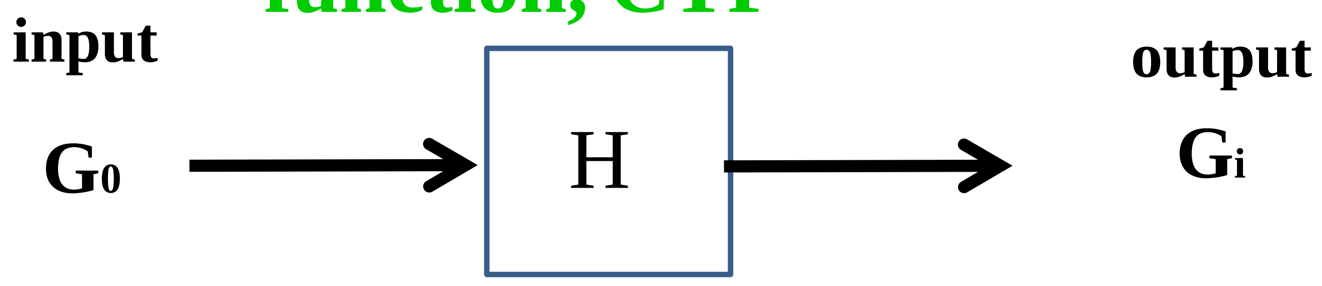
Now we stand in front of entrance for the inverse space!

Consider propagation of Field of light



Let us consider in inverse space

Coherent transfer function, CTF



What is coherent illumination?

Disturbance of illumination U_c is represented by:

$$\mathbf{U}_c = \frac{A\,\mathbf{exp}(ikz)}{i\lambda z}\,\mathbf{exp}\left[i\frac{k}{2z}\left(x^2+y^2\right)\right]$$

if z becomes very large, U_c becomes plane wave.

Example: CW Laser such as He-Ne Laser

$G_0 = \mathscr{F}(U_0)$ $\mathscr{F}(\quad)$: Fourier transform

$G_i = \mathscr{F}(U_i)$

$H = \mathscr{F}(\tilde{h})$

Then

$G_i = HG_0$

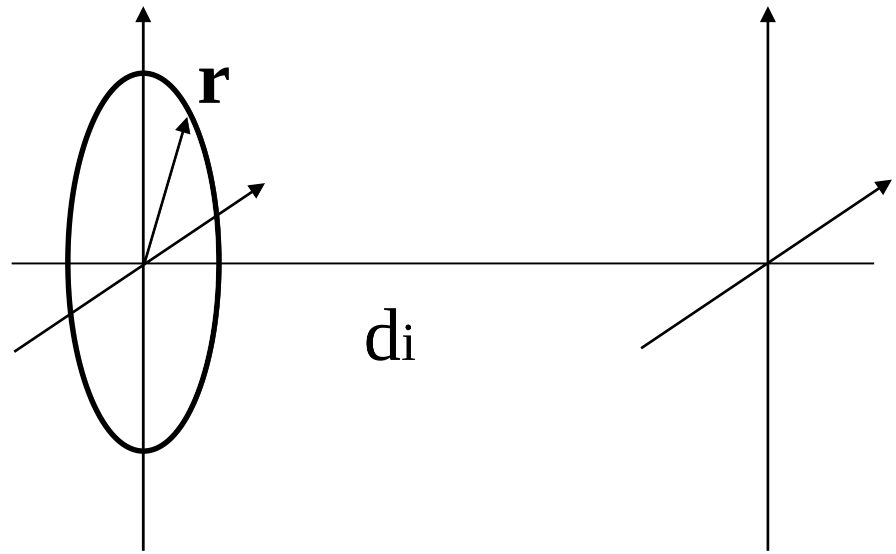
$$\tilde{h} = \mathcal{F}(P(f_x,f_y))$$

Then,

$$H = \mathcal{F}(\mathcal{F}(P(f_x,f_y))) = P(f_x,f_y)$$

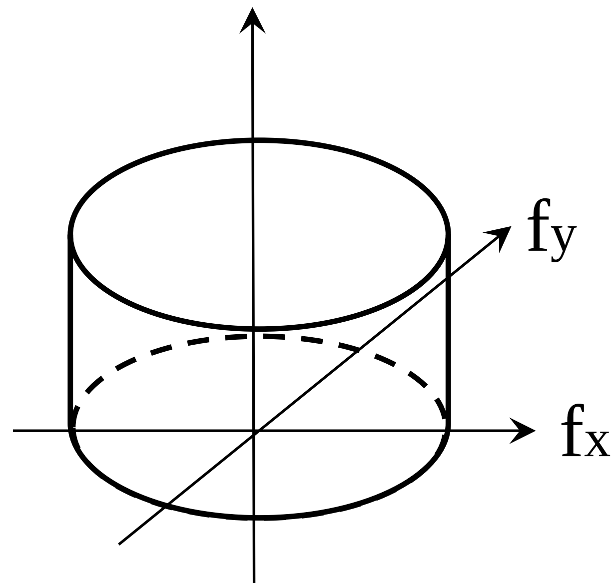
Coherent transfer function (CTF) is pupil function it self !

Properties of CTF



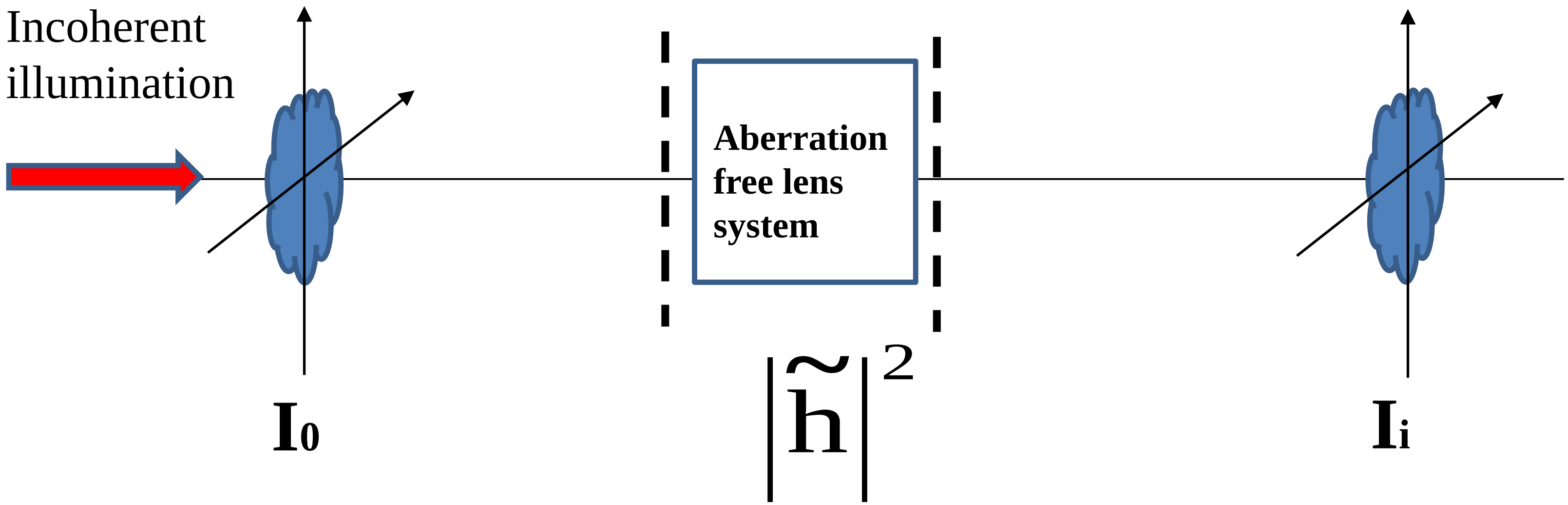
Lens has pupil of radius r and focal length d_i

1. Cut off frequency: $f_0 = \dfrac{2\pi r}{\lambda d_i}$

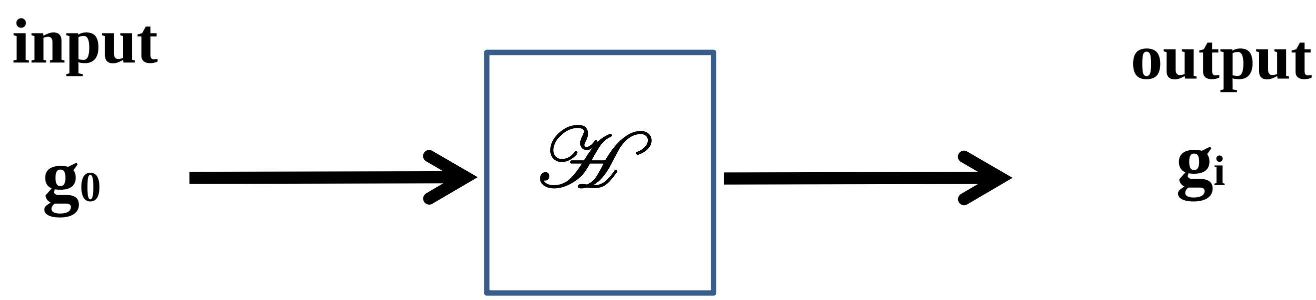


2. Inside of cut off frequency, information of amplitude, phase will transfer without distortion

Consider propagation of intensity of light



Optical transfer function, OTF



$g_0 = \mathscr{F}(I_0)$ $\mathscr{F}(\quad)$: Fourier transform normalized by it's value at $f_x, f_y = 0$

$g_i = \mathscr{F}(I_i)$

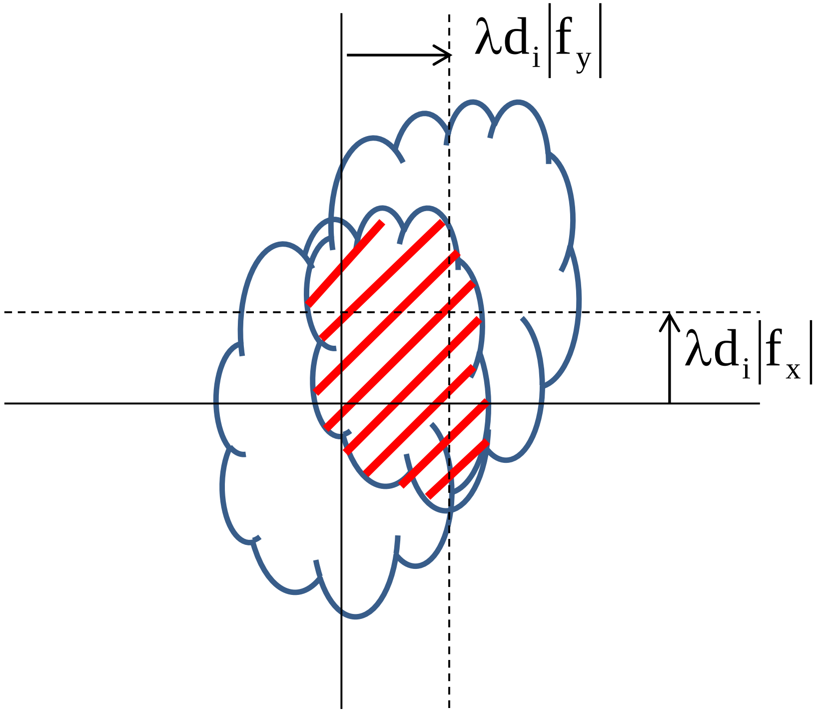
$\mathscr{H} = \mathscr{F}\left(\left| \tilde{h} \right|^2 \right)$

Then

$g_i = \mathscr{H} g_0$

OTF is given by autocorrelation of CTF

$$\mathscr{H}(\mathbf{f_x},\mathbf{f_y}) = \frac{\iint H(\xi,\eta)H(\xi-f_x,\eta-f_y)\,d\xi\, d\eta}{\iint \left|H(\xi,\eta)\right|^2 d\xi\, d\eta}$$

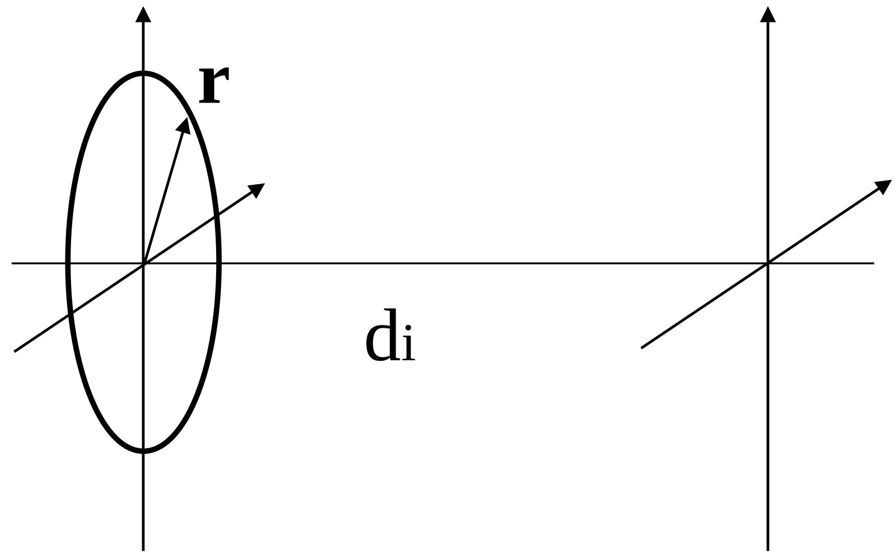


Geometrical meaning of OTR

$$\mathscr{H}(\mathbf{f_x},\mathbf{f_y}) = \frac{\text{area of overlap}\left(f_x, f_y\right)}{\text{total area}}$$

Example of OTR

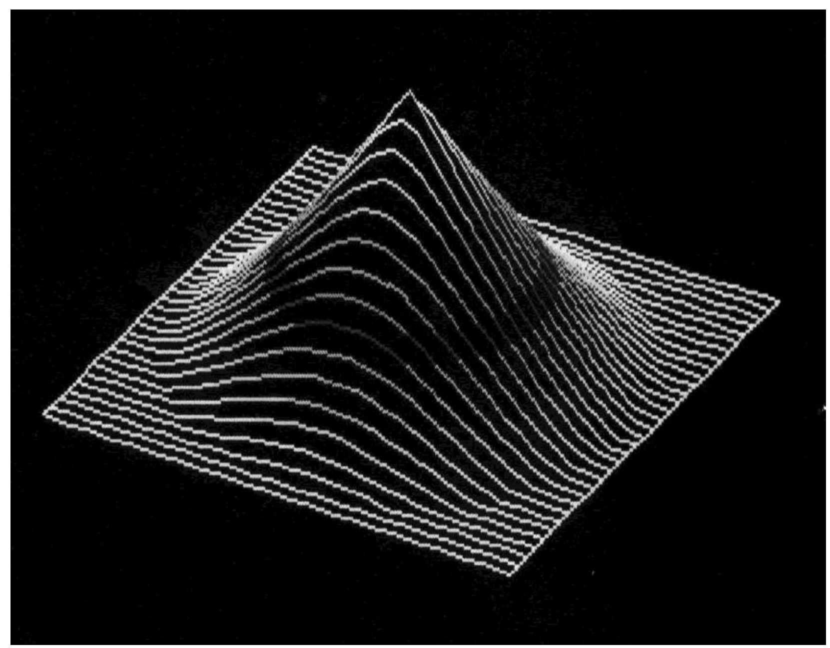
Lens has pupil of radius r and focal length d_i



Cut off frequency of OTR

twice of CTF cut off frequency

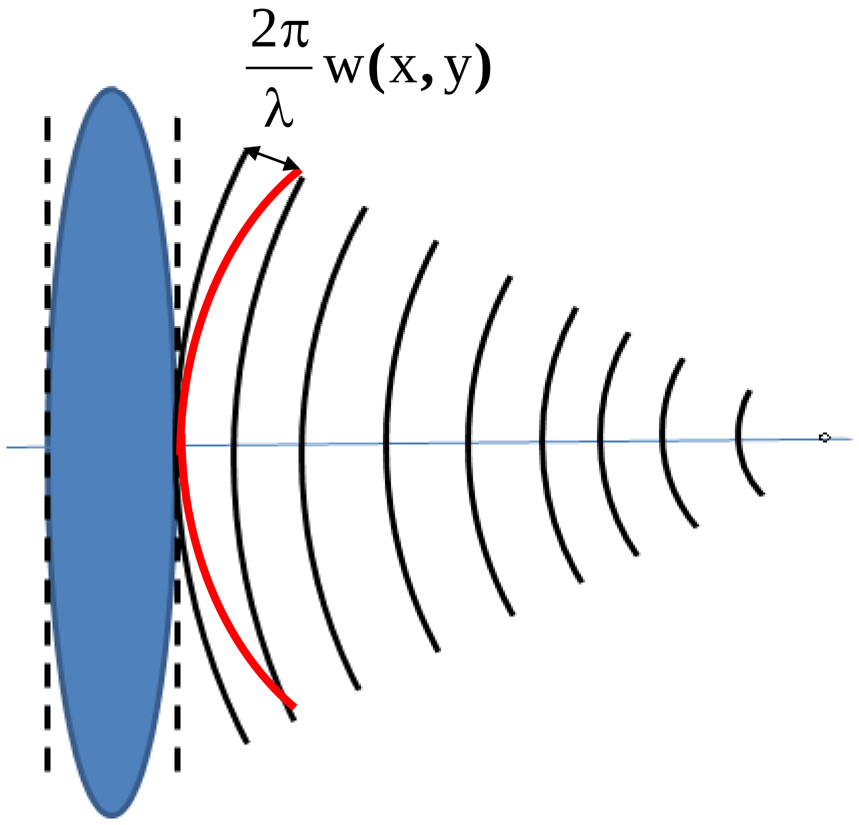
$$f_0 = 2 \times \frac{2\pi r}{\lambda d_i}$$



Influence of aberration

Influence of aberration to frequency response

Phase transmittance



Amplitude transmittance: A(x,y)

Generalized pupil function

$$p(x, y) = A(x, y)P(x, y)\,\mathbf{exp}\left(i\frac{2\pi}{\lambda} w(x, y) \right)$$

CTF is given by

$$H(f_x, f_y) = A\left(\frac{\lambda d_i}{2\pi} f_x, \frac{\lambda d_i}{2\pi} f_y\right)P\left(\frac{\lambda d_i}{2\pi} f_x, \frac{\lambda d_i}{2\pi} f_y\right)\mathbf{exp}\left(i\frac{2\pi}{\lambda} w\left(\frac{\lambda d_i}{2\pi} f_x, \frac{\lambda d_i}{2\pi} f_y \right)\right)$$

OTF is again given by autocorrelation of CTF

OTF with aberration is sometimes complex, so we use absolute value of OTF, It is called Modulation Transfer Function, MTF

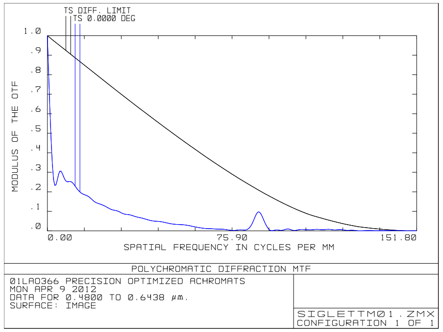
Important general properties of MTF:

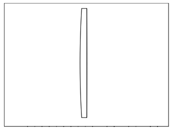
1. **MTF having aberration is always smaller than aberration-free MTF (diffraction limited MTF)**

2. **Cut-off frequency is not changed by aberration**

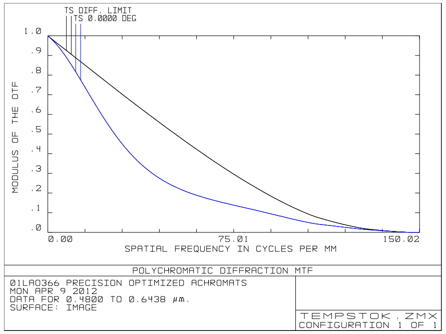
3. **Zero cross of MTF corresponding to inverse of contrast.**

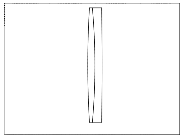
Singlet D=80mm f=1000mm



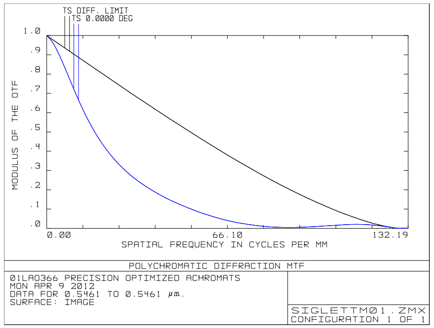


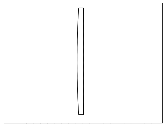
Doublet D=80mm f=1000mm



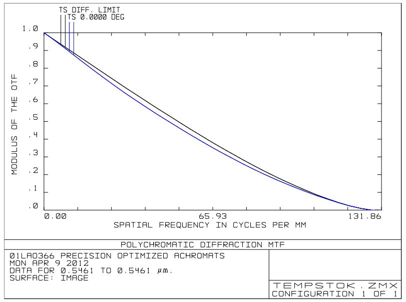


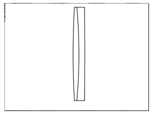
Singlet D=80mm f=1000mm
λ=0.55μm



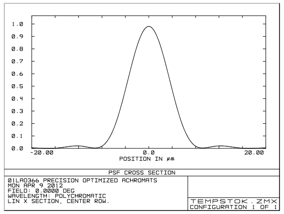


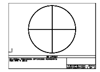
Doublet D=80mm f=1000mm
λ=0.55μm

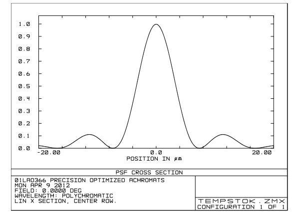


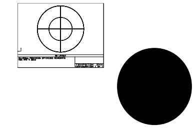


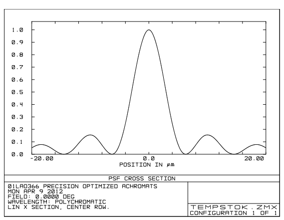
Appodization or super resolution

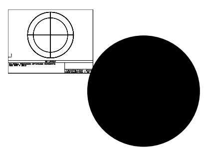


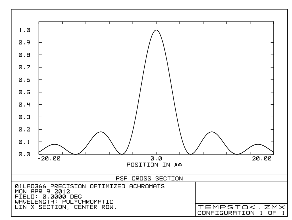


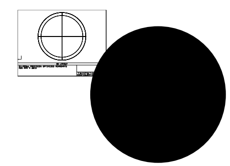


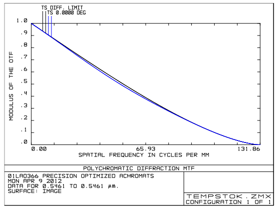








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MON APR 9 2012
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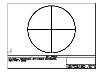
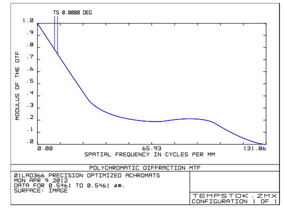
CONFIGURATION 1 OF 1

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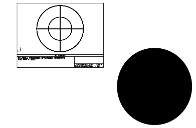

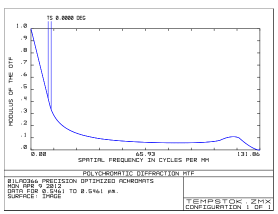
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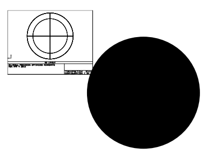

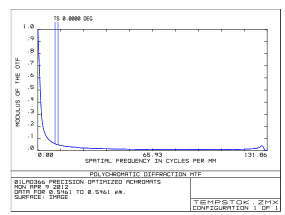
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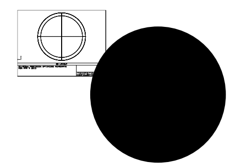

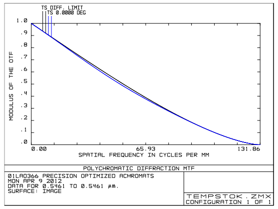

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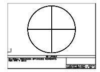

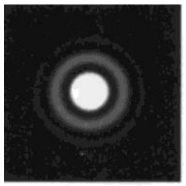

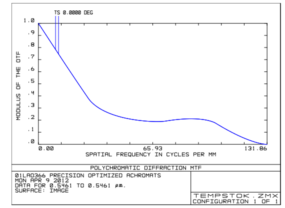

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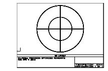

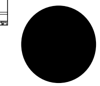

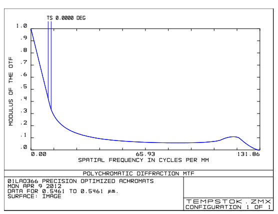

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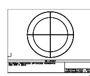

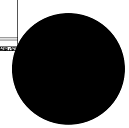

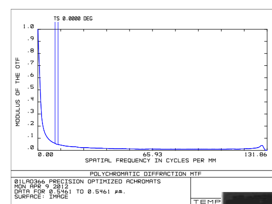

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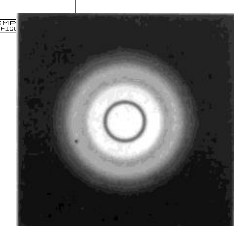
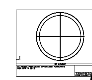
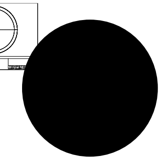
CONFIGU

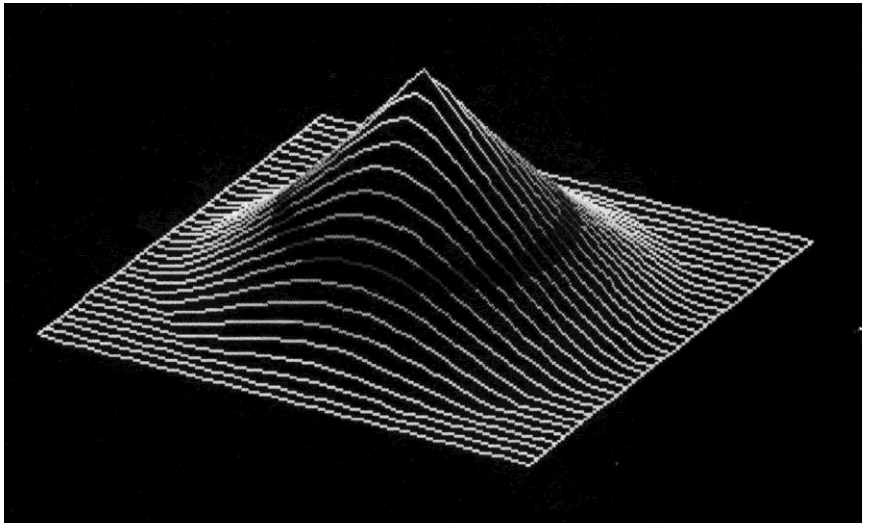




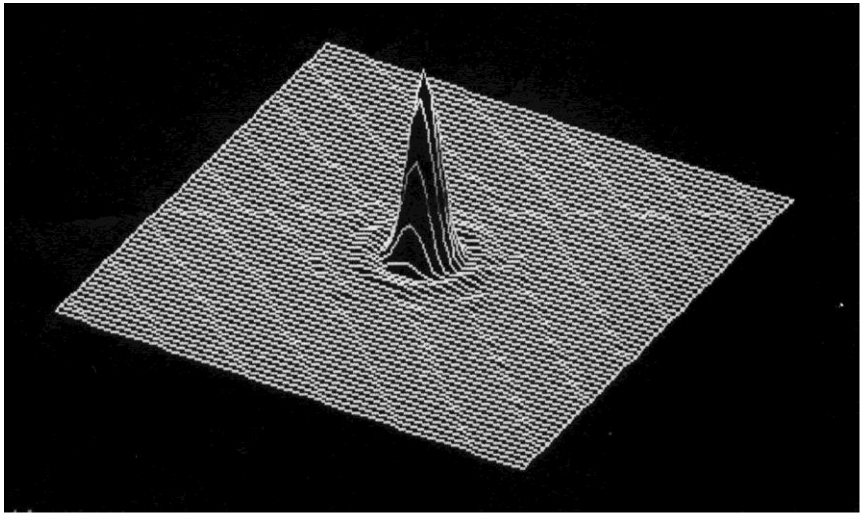


Diffraction without wavefront error

Surface plot of MTF

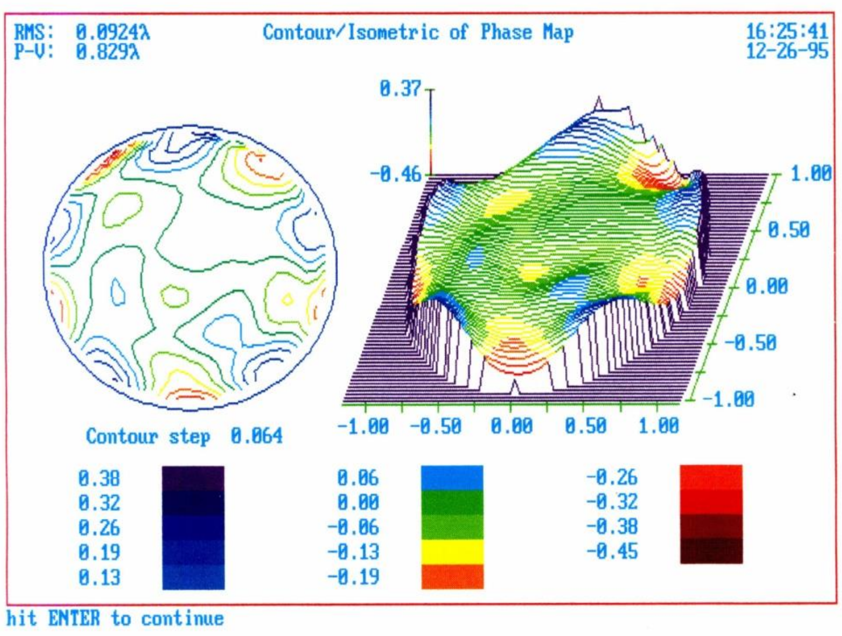


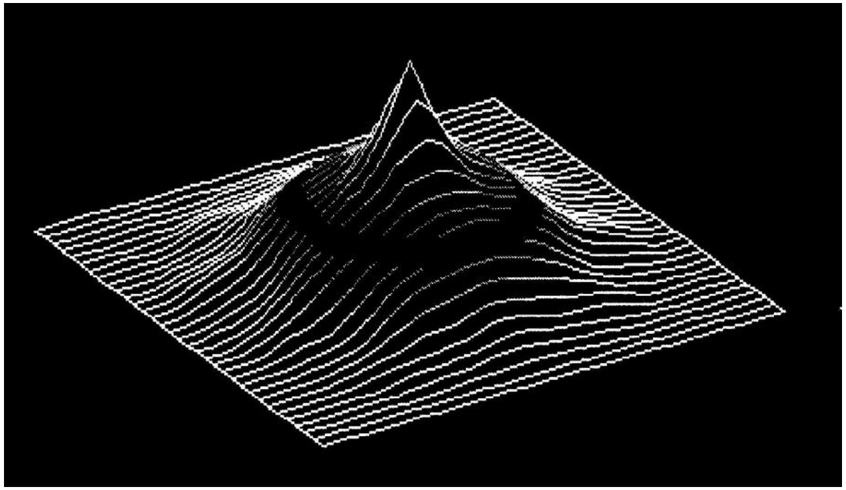
PSF plot

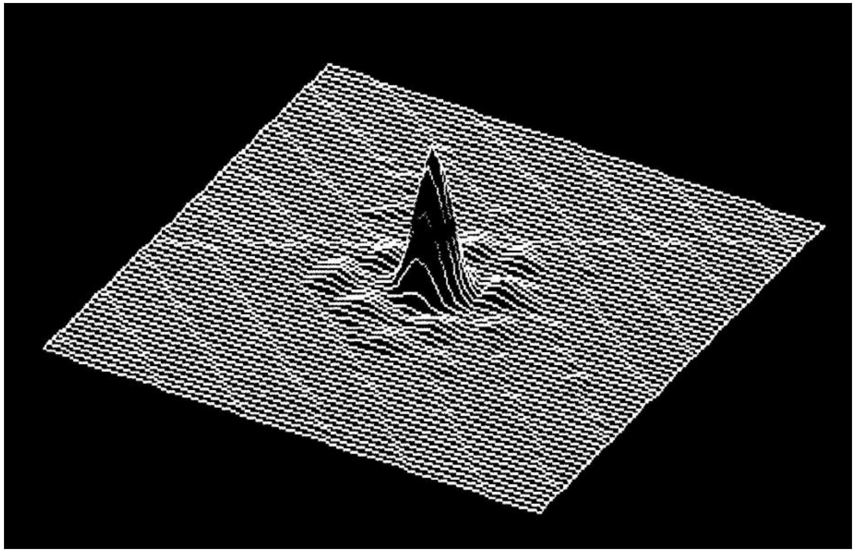


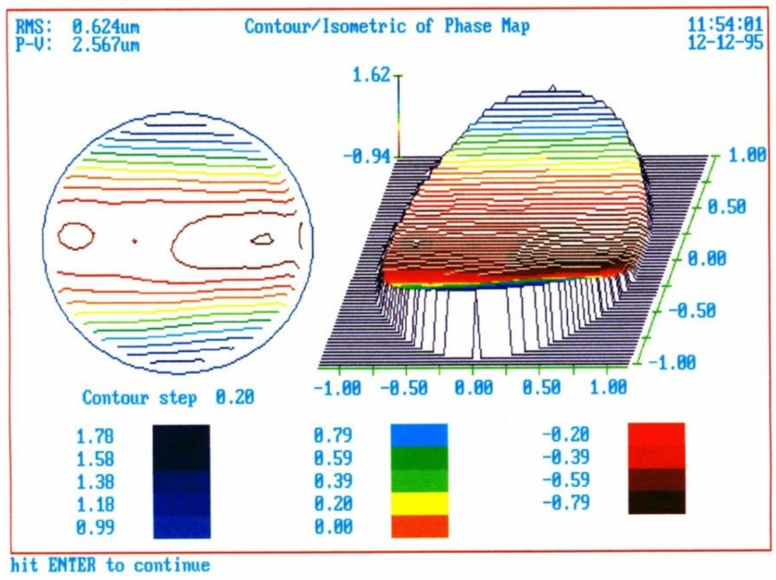
Diffraction with wavefront error

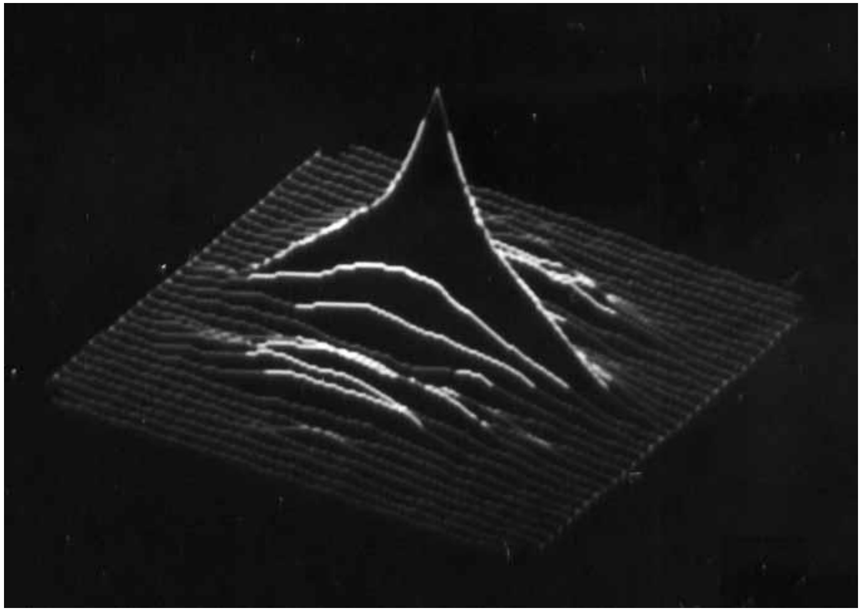
p-v: 0.82λ,   RMS : 0.092λ

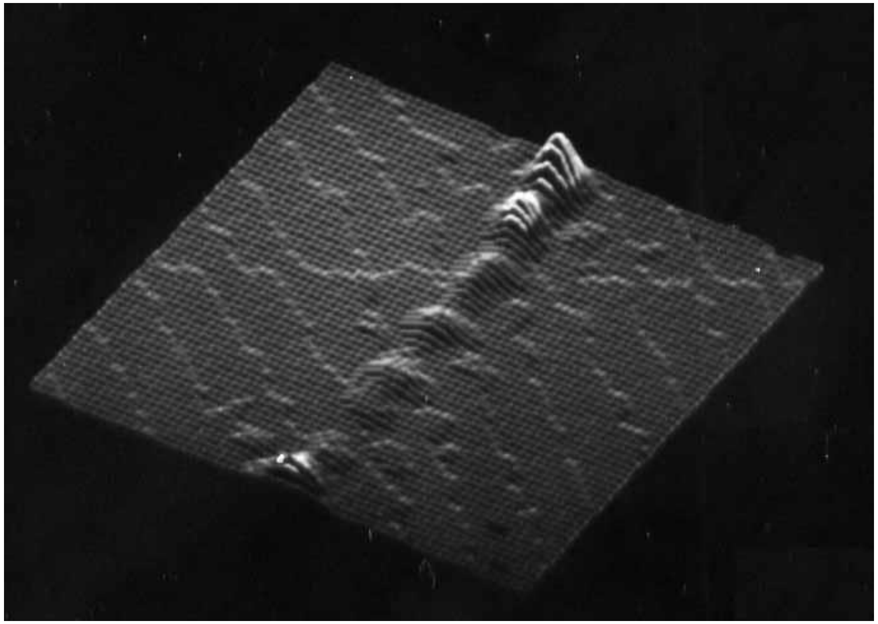


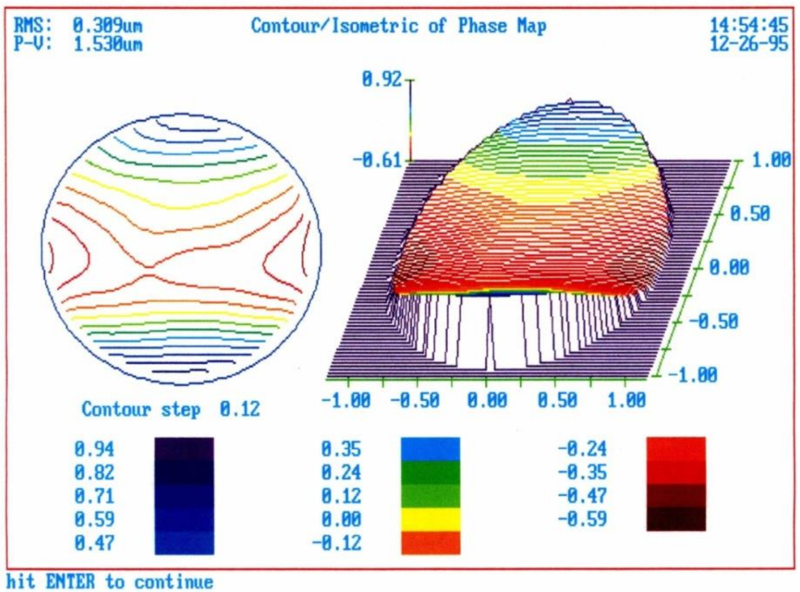


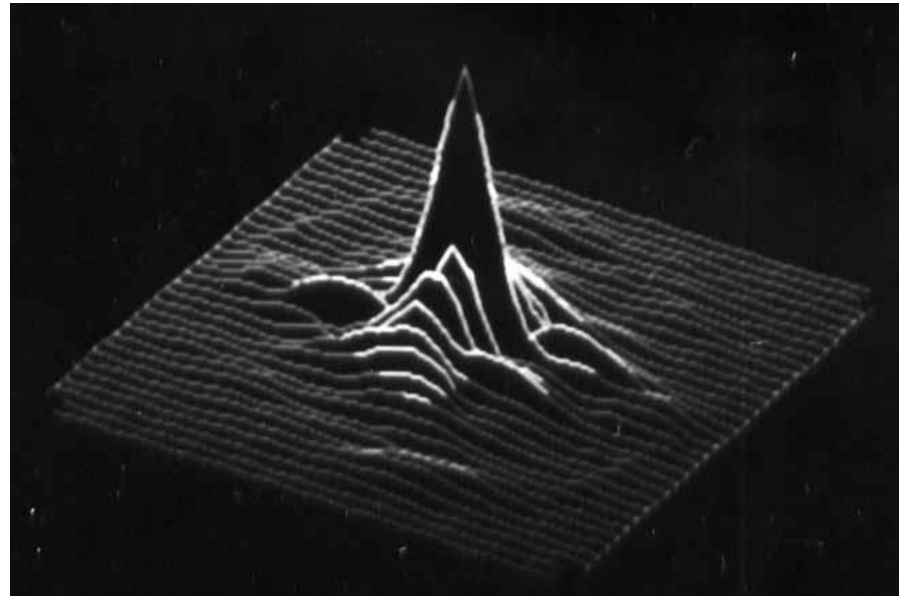


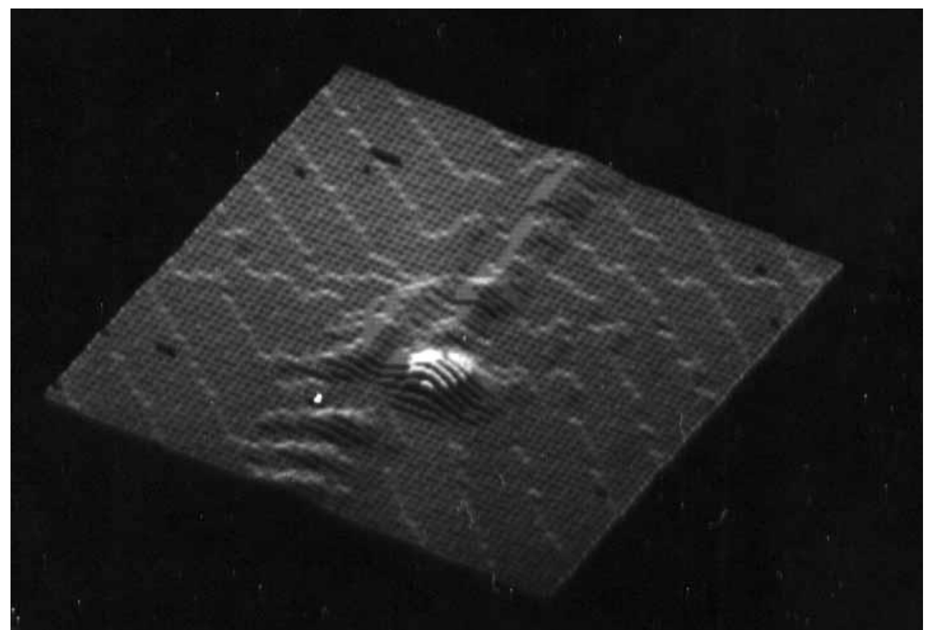

RMS: 0.624um
P-V: 2.567um
Contour/Isometric of Phase Map
11:54:01
12-12-95
1.62
-0.94
1.00
0.50
0.00
-0.50
-1.00
-1.00 -0.50 0.00 0.50 1.00
Contour step 0.20
1.78
1.58
1.38
1.18
0.99
0.79
0.59
0.39
0.20
0.00
-0.20
-0.39
-0.59
-0.79
hit ENTER to continue






RMS: 0.309um
P-V: 1.530um
Contour/Isometric of Phase Map
14:54:45
12-26-95
0.92
-0.61
1.00
0.50
0.00
-0.50
-1.00
-1.00 -0.50 0.00 0.50 1.00
Contour step 0.12
0.94
0.82
0.71
0.59
0.47
0.35
0.24
0.12
0.00
-0.12
-0.24
-0.35
-0.47
-0.59
hit ENTER to continue

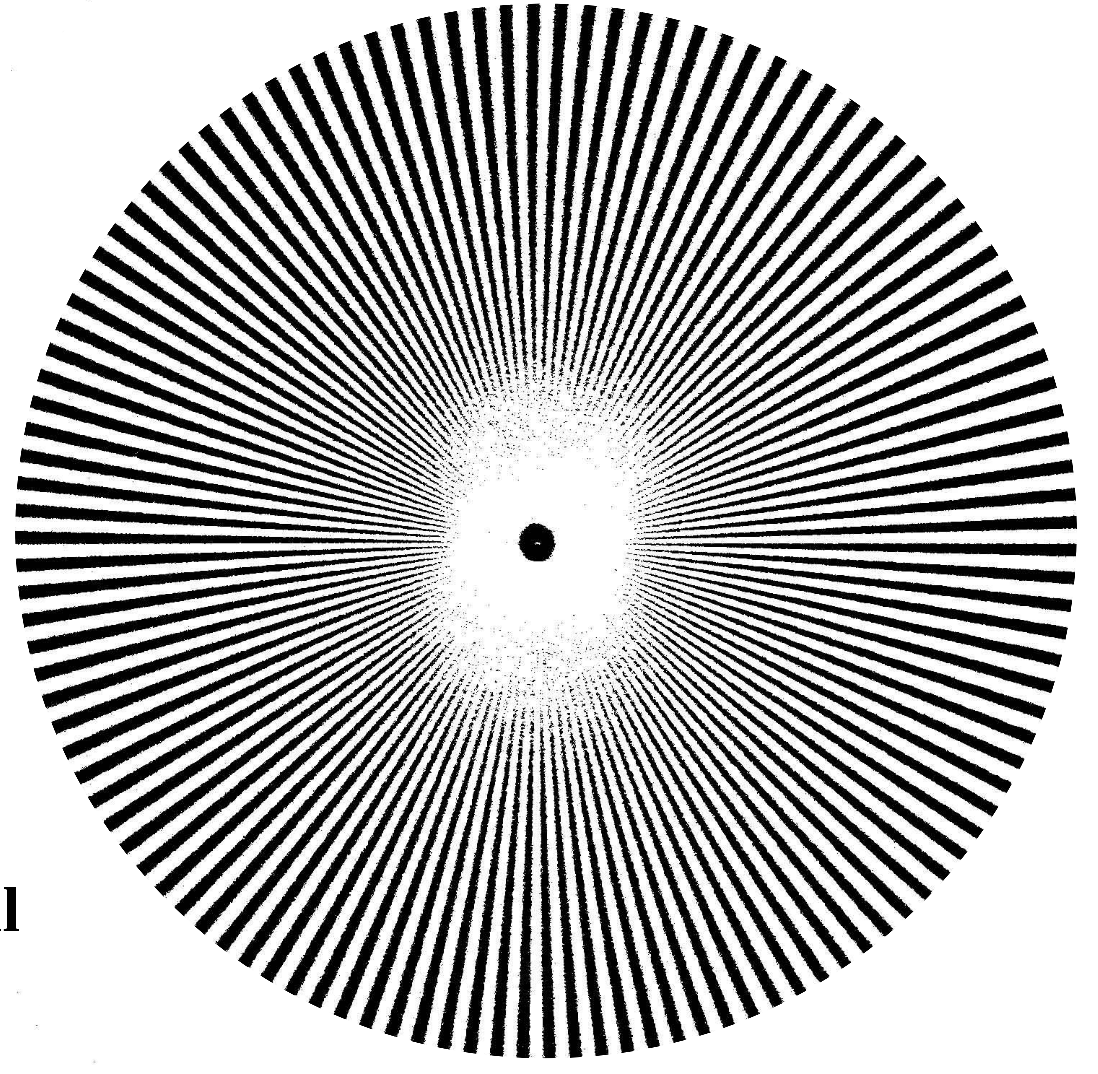




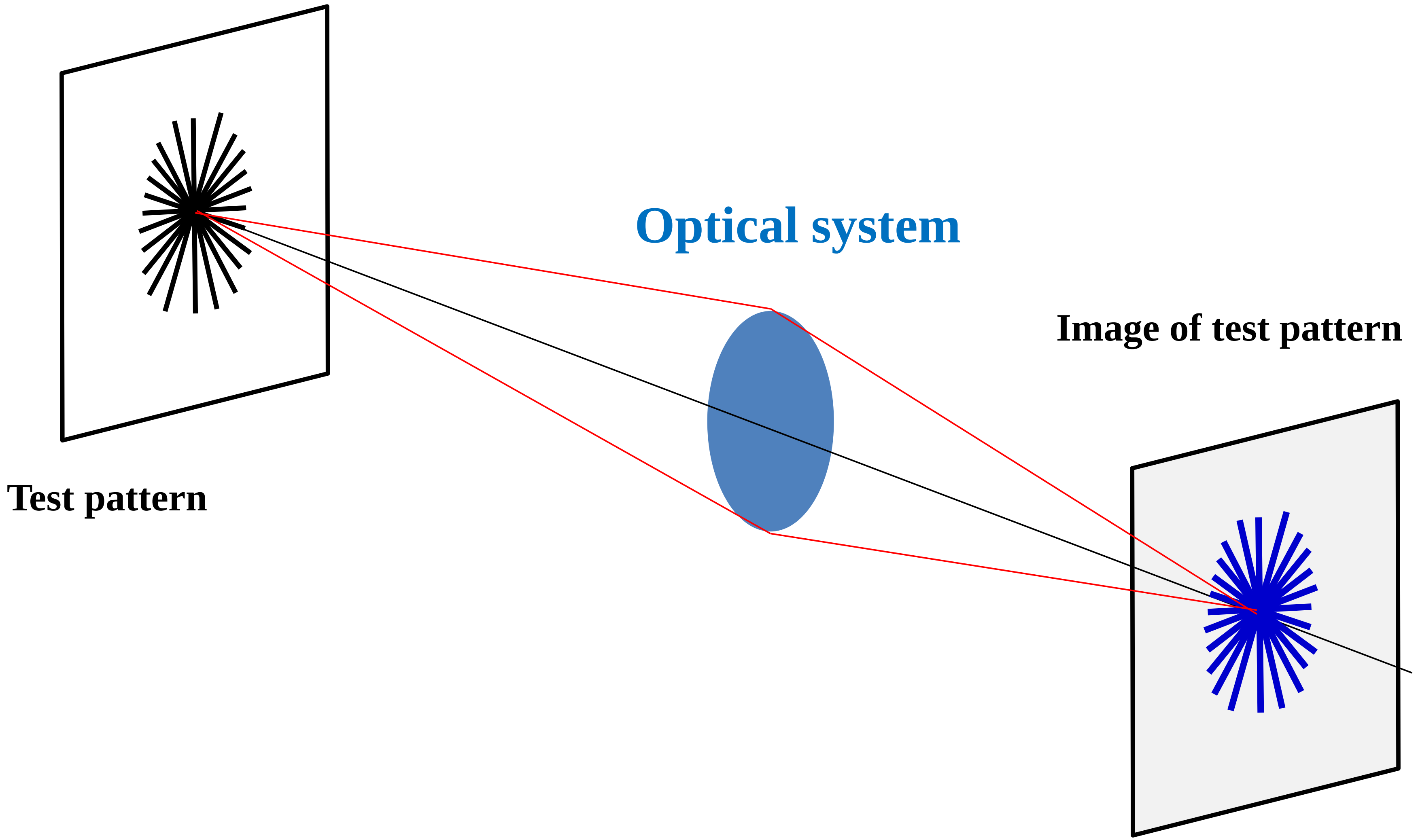
MTF measurement

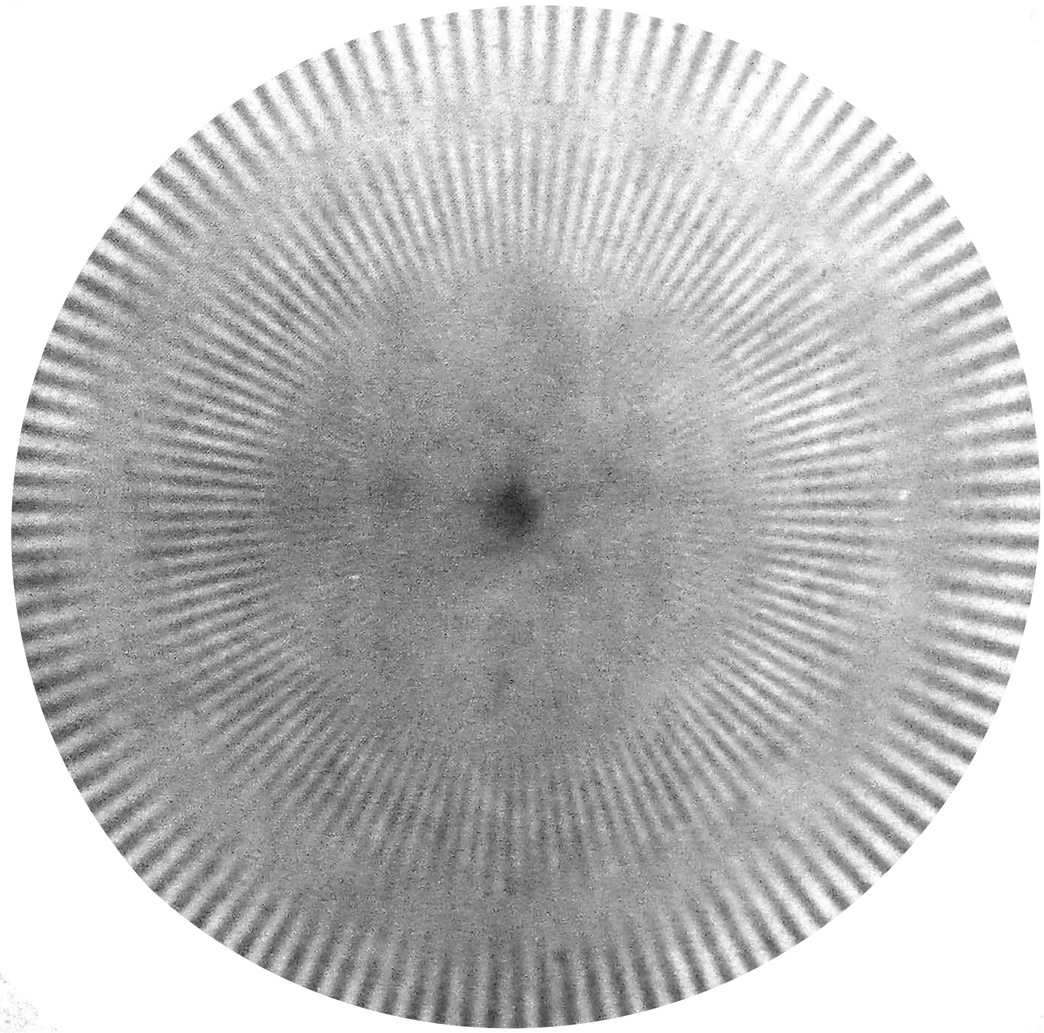
Test pattern

Spoke chart

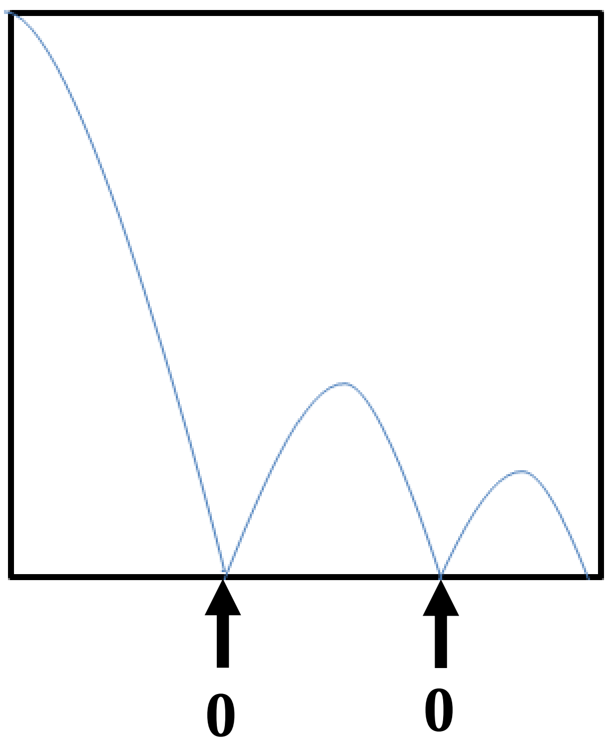


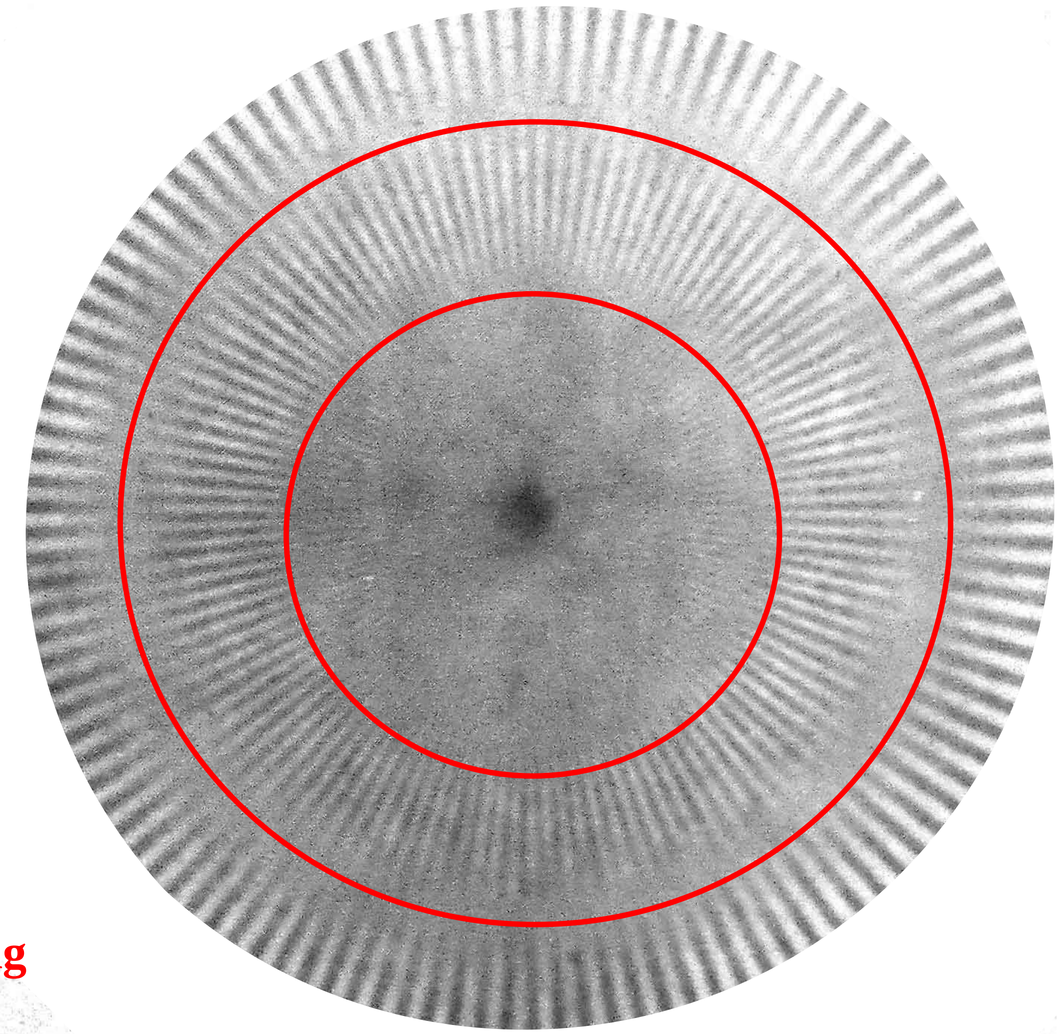
From center to edge, spatial frequency will change higher to lower


Optical system
Image of test pattern
Test pattern

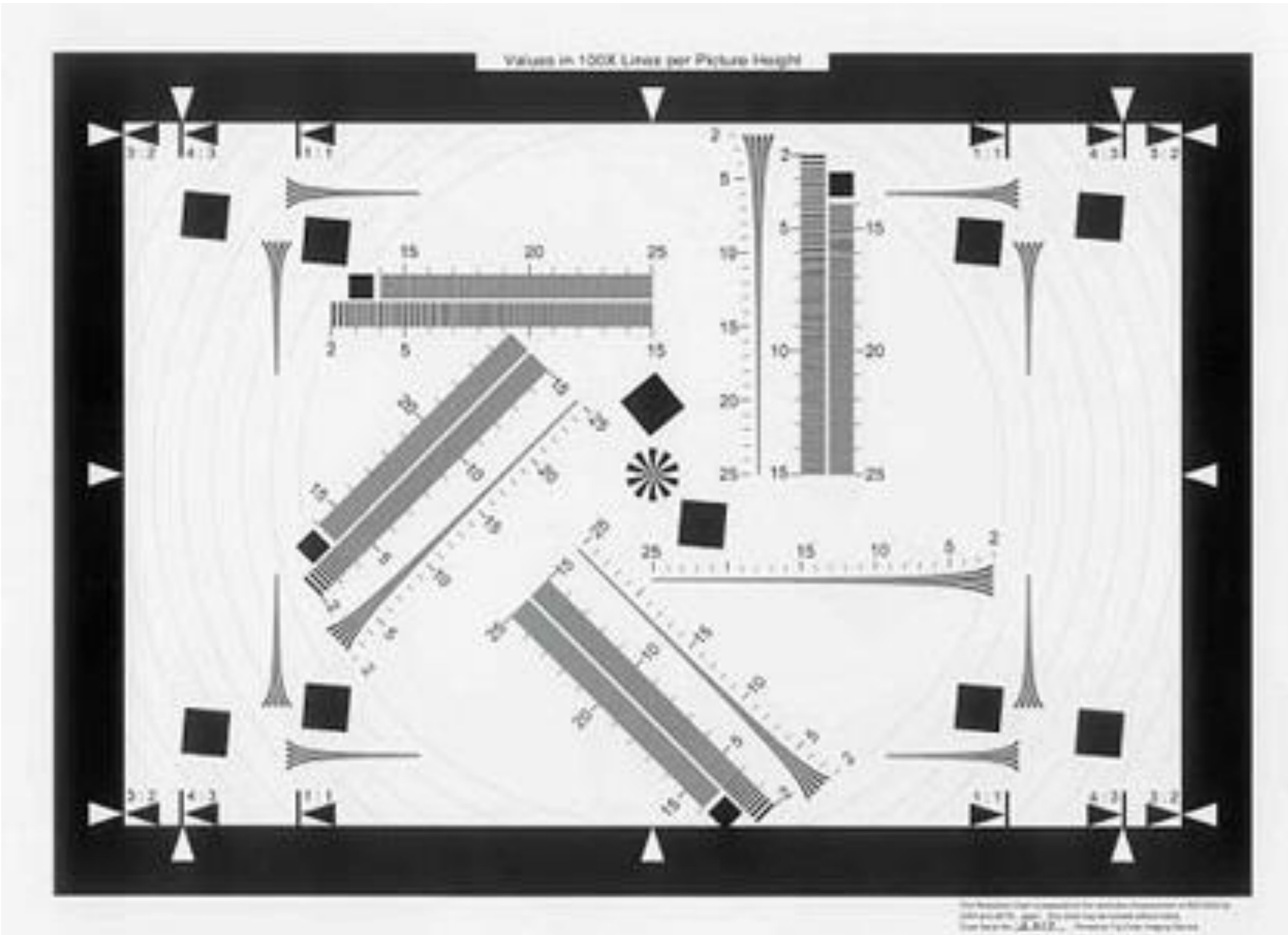


Corresponding MTF

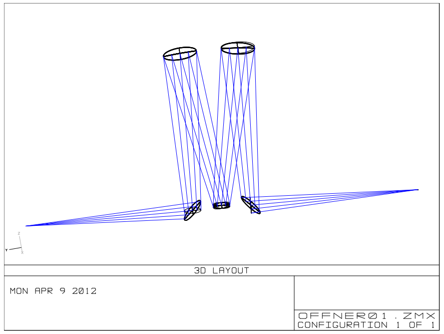




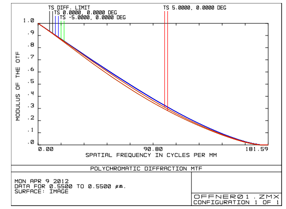
Information at corresponding spatial frequencies are not transmitted


Values in 100X Lines per Picture Height

Streak camera reflective input optics

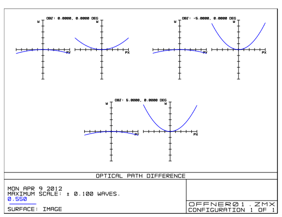


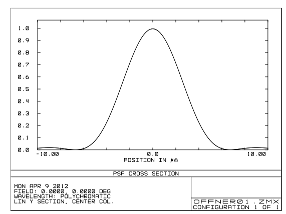
Optical performance of reflective relay



OPD<0.055µm

2σ PSF width   5.06µm



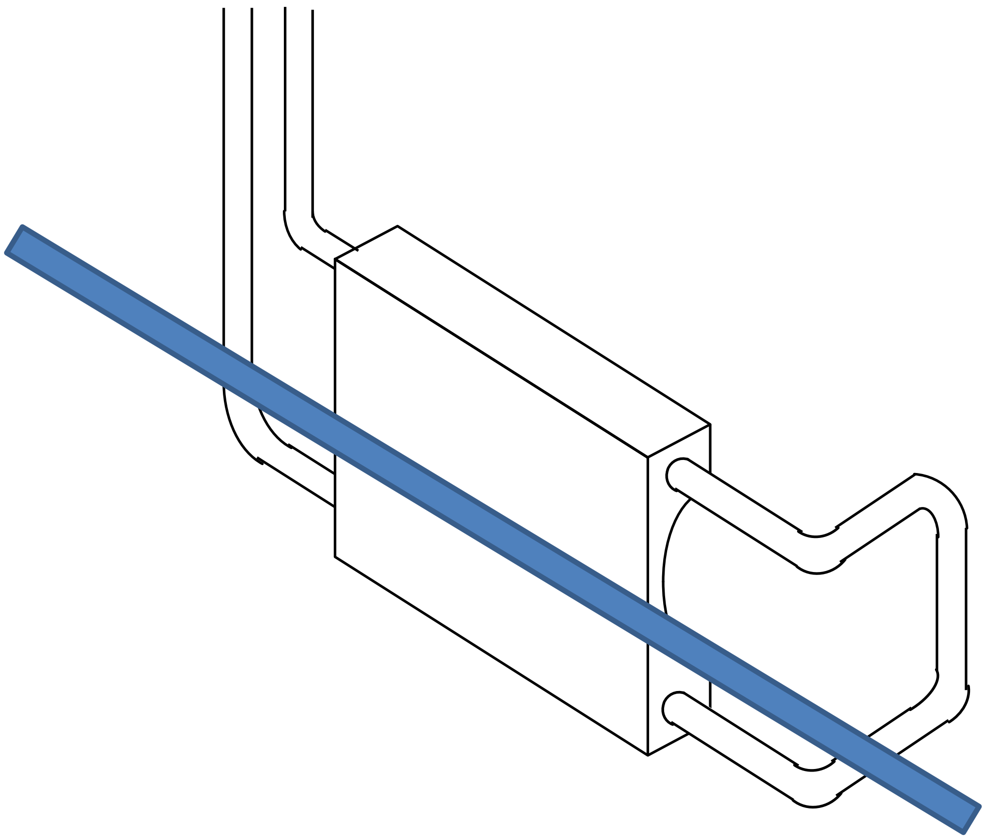


2σ=5.06µm

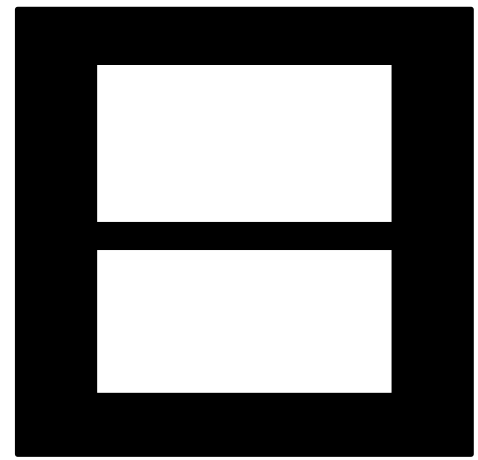
Interesting property of double aperture

Interesting property of double aperture

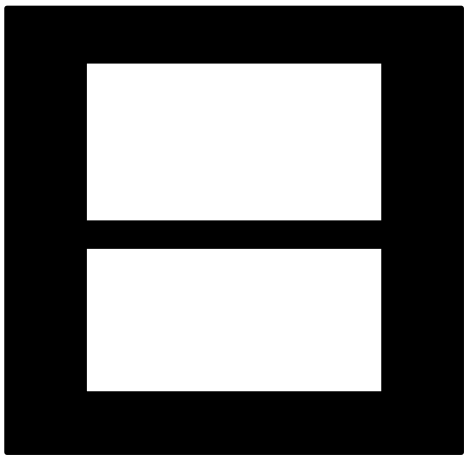
Extraction mirror with cold finger

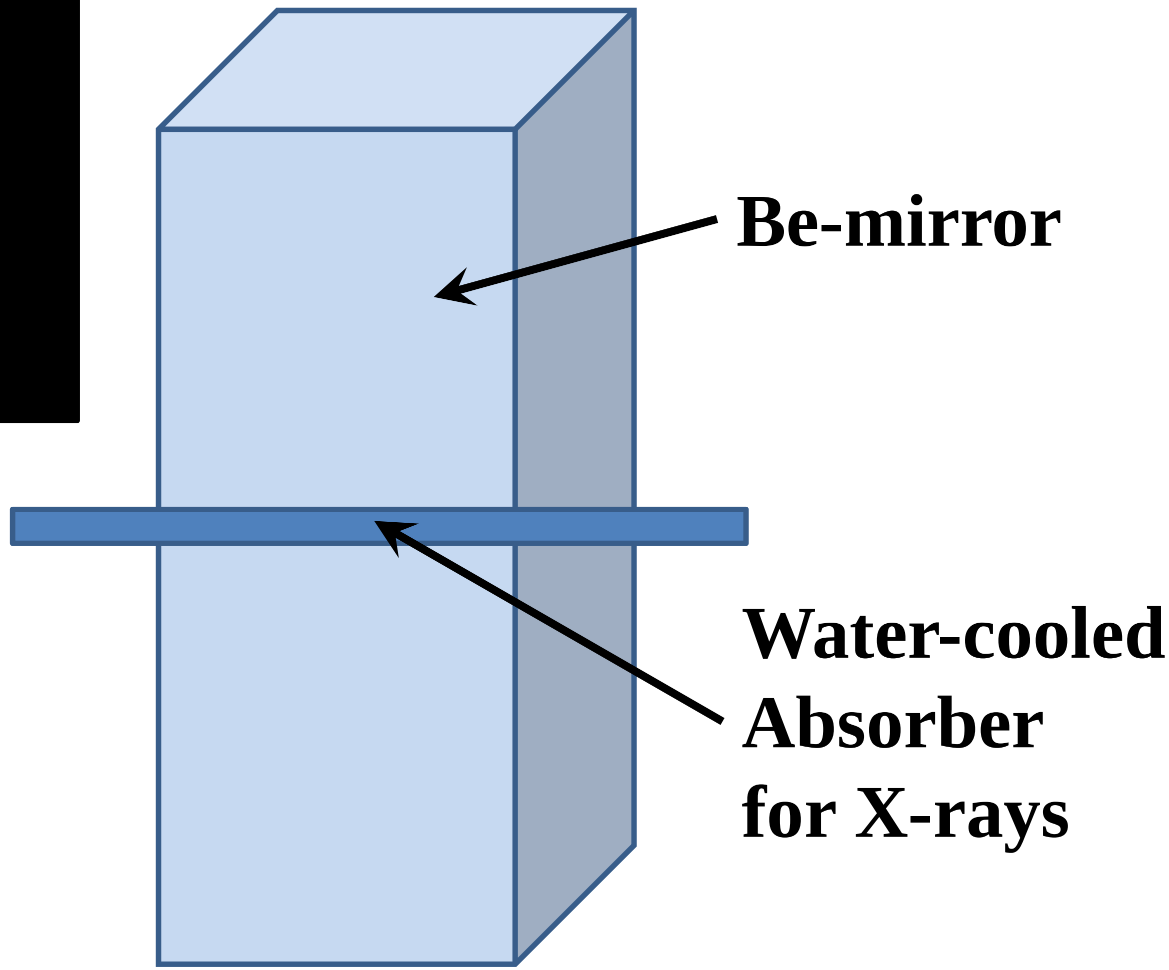


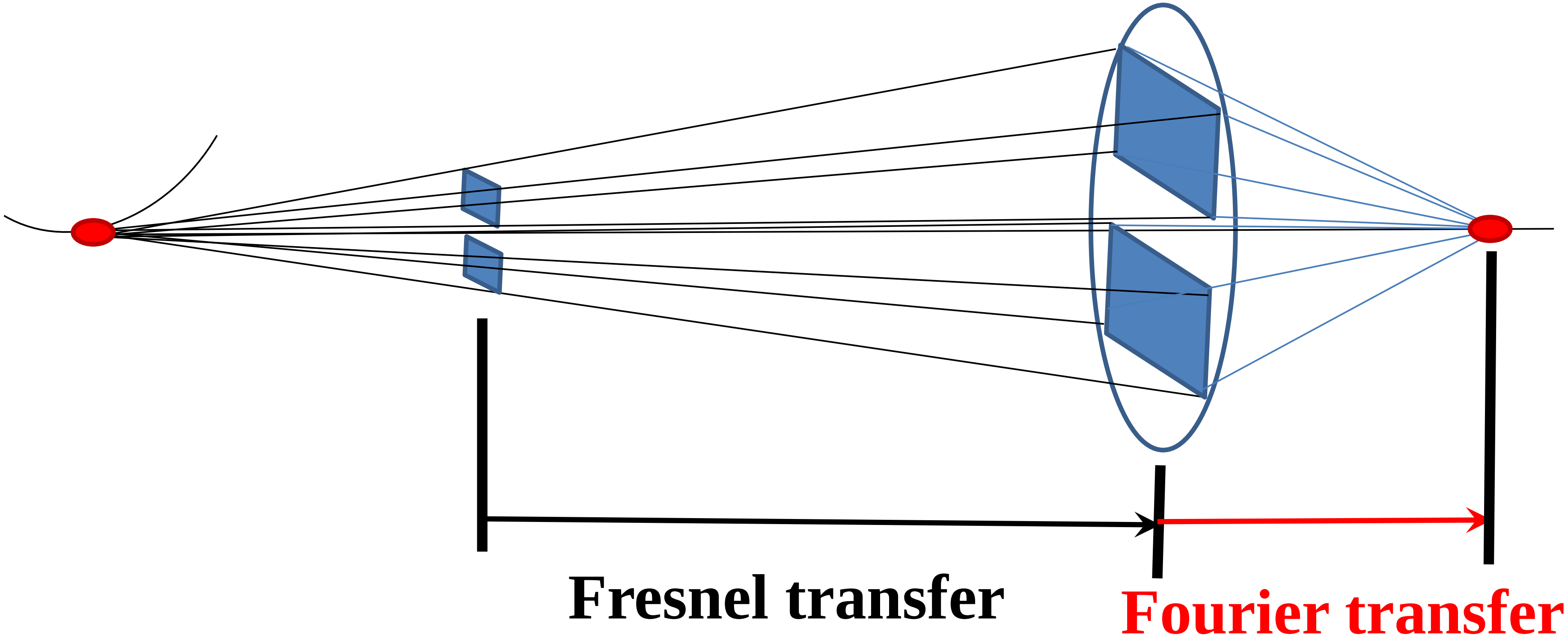
Cold finger

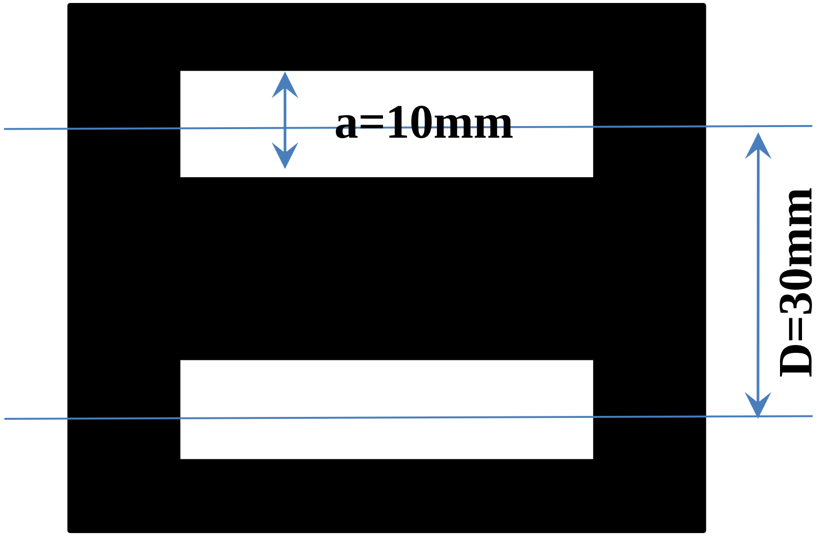


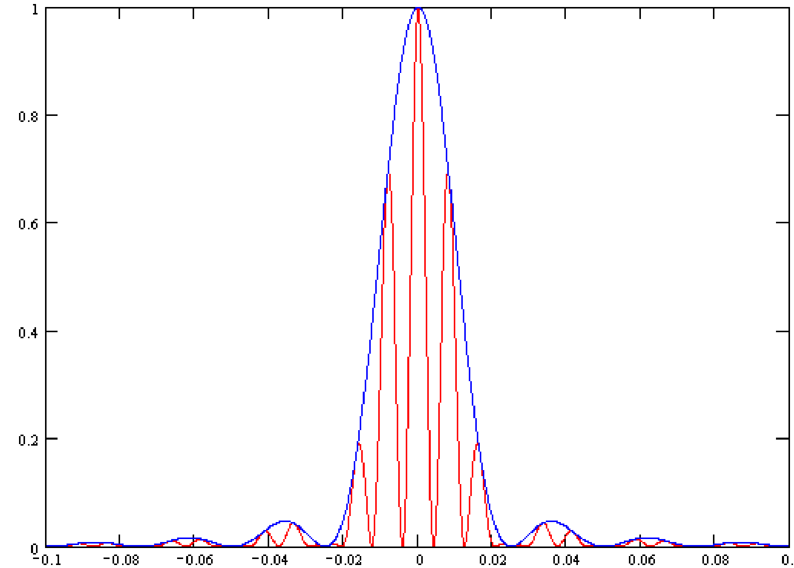
Extraction mirror with X-ray absorber

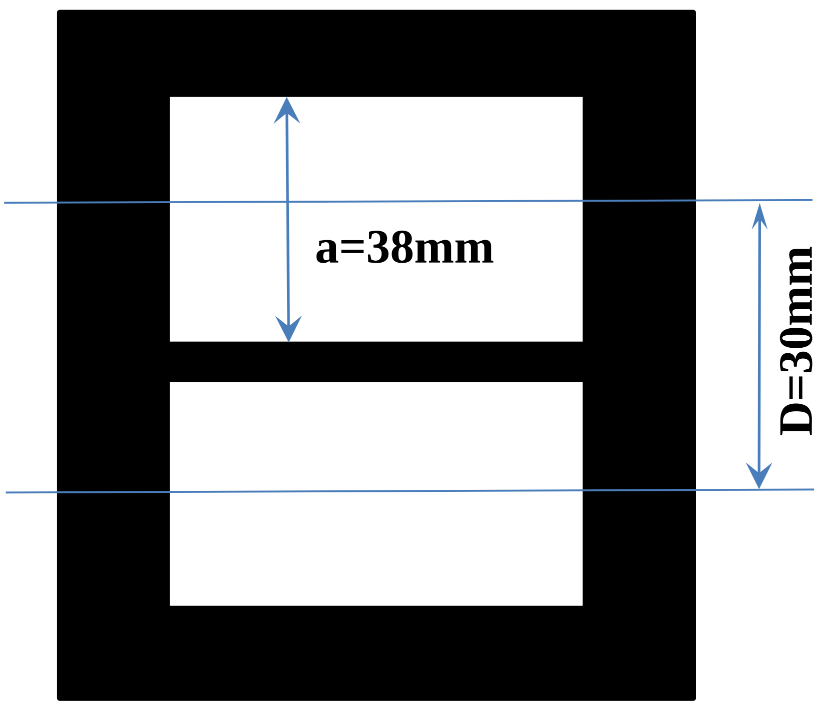


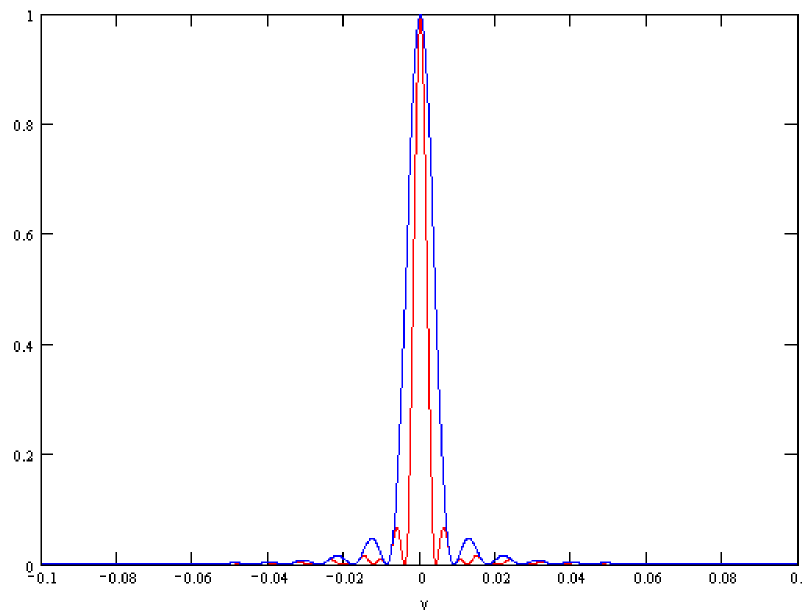



Fresnel transfer
Fourier transfer


a=10mm
D=30mm


1
0.8
0.6
0.4
0.2
0
−0.1
−0.08
−0.06
−0.04
−0.02
0
0.02
0.04
0.06
0.08
0.1


a=38mm
D=30mm


1
0.8
0.6
0.4
0.2
0
−0.1
−0.08
−0.06
−0.04
−0.02
0
0.02
0.04
0.06
0.08
0.1
v

1. **Width of Diffraction envelope is dominated by single aperture height.**

2. **Inside diffraction envelope is modulated by interference of double aperture.**

3. **PSF including interference is almost same width of diffraction with full aperture, but contrast is more worth the single aperture case due to surrounding fringes.** ***Seems still better than large thermal deformation of mirror.***

Conclusions

1. **Good extraction mirror**
   Identification of extraction mirror deformation is important
2. **Good optical path design**
   Optical path having no focusing components or double optical path
3. **Good lens or reflector for focusing system**
   Do not use singlet lens even monochromatic light!
4. **Good alignment**

MTF measurement is very helpful to know performance of your optical system!