Accelerator Physics Lecture Modules

1. Introduction/Timing
2. Beam propagation basics
3. Simple Harmonic Oscillator

4. Betatron oscillations (transverse motion)
5. Dispersion
6. Closed Orbit Distortion

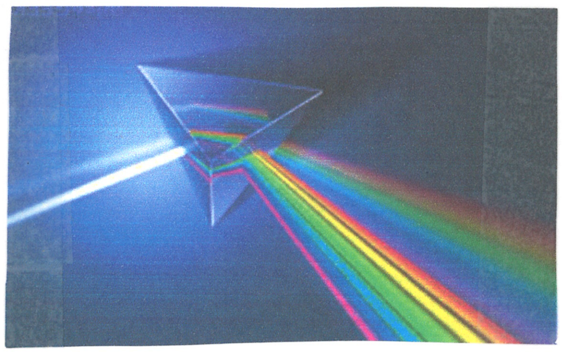
7. Synchrotron oscillations
8. Properties of synchrotron radiation

9. Radiation damping
10. Equilibirum emittance

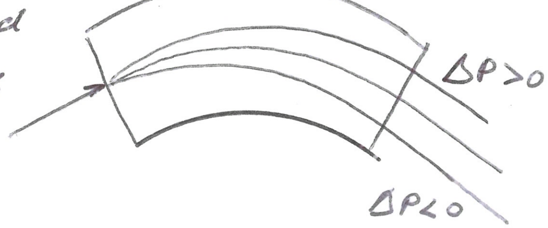
Dispersion

Dispersion means to <u>separate</u> or <u>scatter</u>

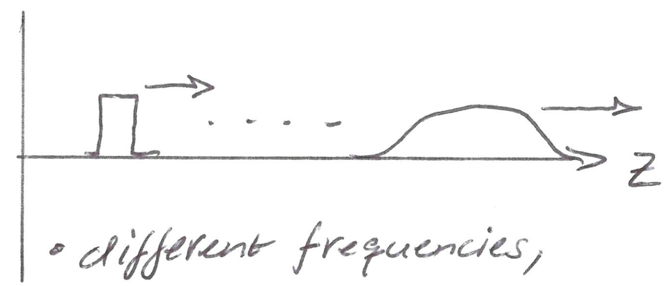
Prism
$n(\lambda)$



Rainbow



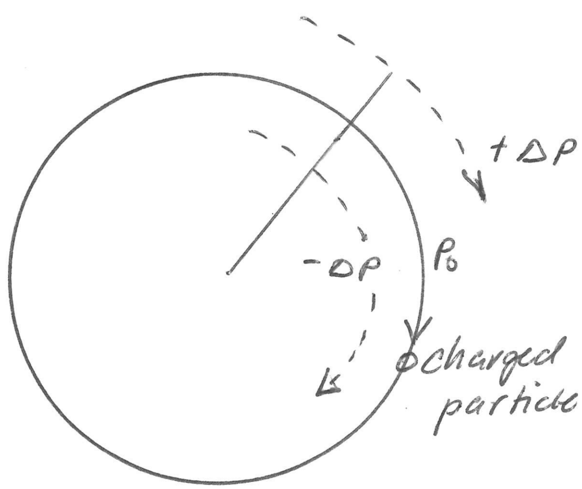
Transmission Line



- different frequencies, different speeds
- wave propagation
- fiber optics

Storage Ring Dispersion

'Dispersion' in a storage ring refers to shift of the closed orbit



Lorentz Force Law

$$F = \frac{dP}{dt} = q\,\vec{V} \times \vec{B}$$

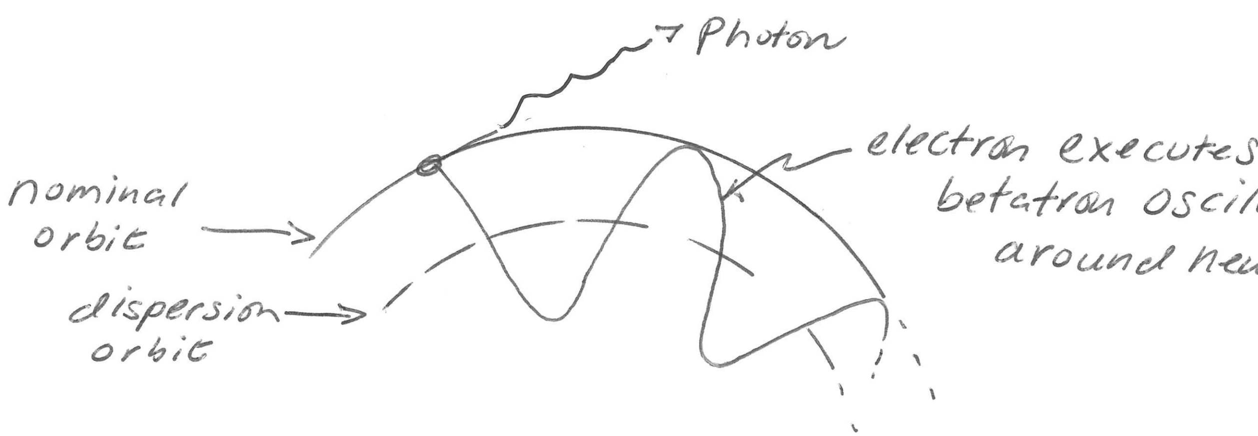
integrate $P_0 = q R_0 B$

$$\boxed{R_0 = \frac{P_0}{qB}}$$

$P = P_0 + \Delta P$

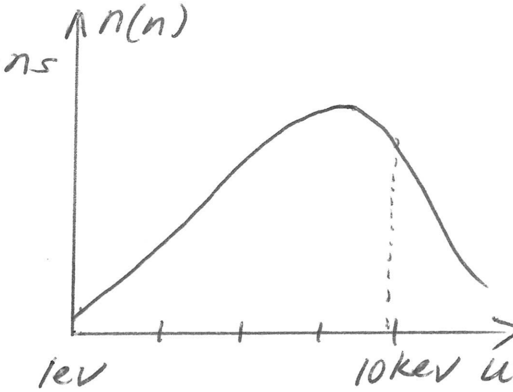
$$\Delta R = \frac{\Delta P}{qB}$$

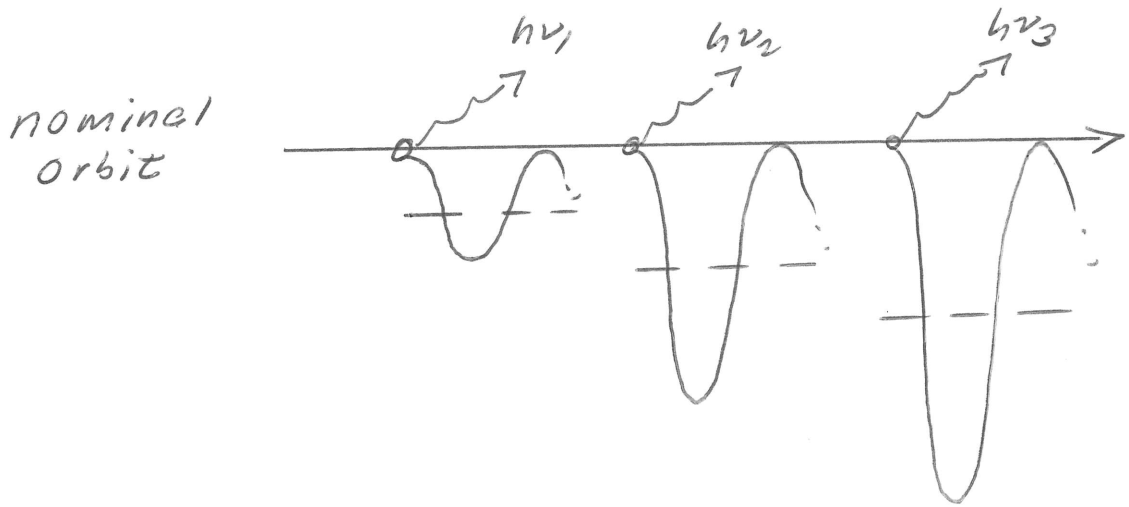
separation proportional to ΔP



Storage Ring Dispersion

Electron beam emits a broad spectrum of photons
∴ broad spread of oscillation amplitudes
(and phases)





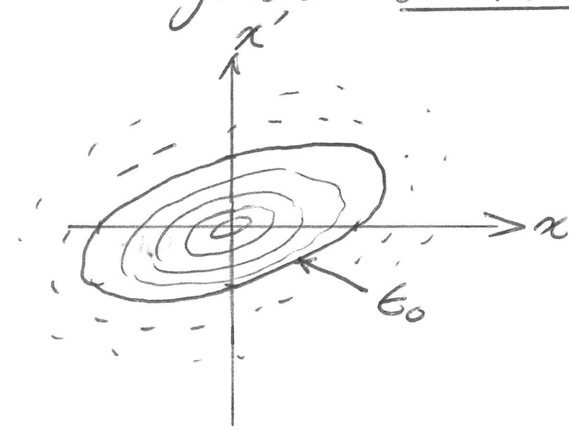
$h\nu_1 < h\nu_2 < h\nu_3$

Dispersion orbit

$$x_D = D \cdot \frac{\Delta p}{p} \quad \left(\text{or } \eta \cdot \frac{\Delta p}{p}\right)$$

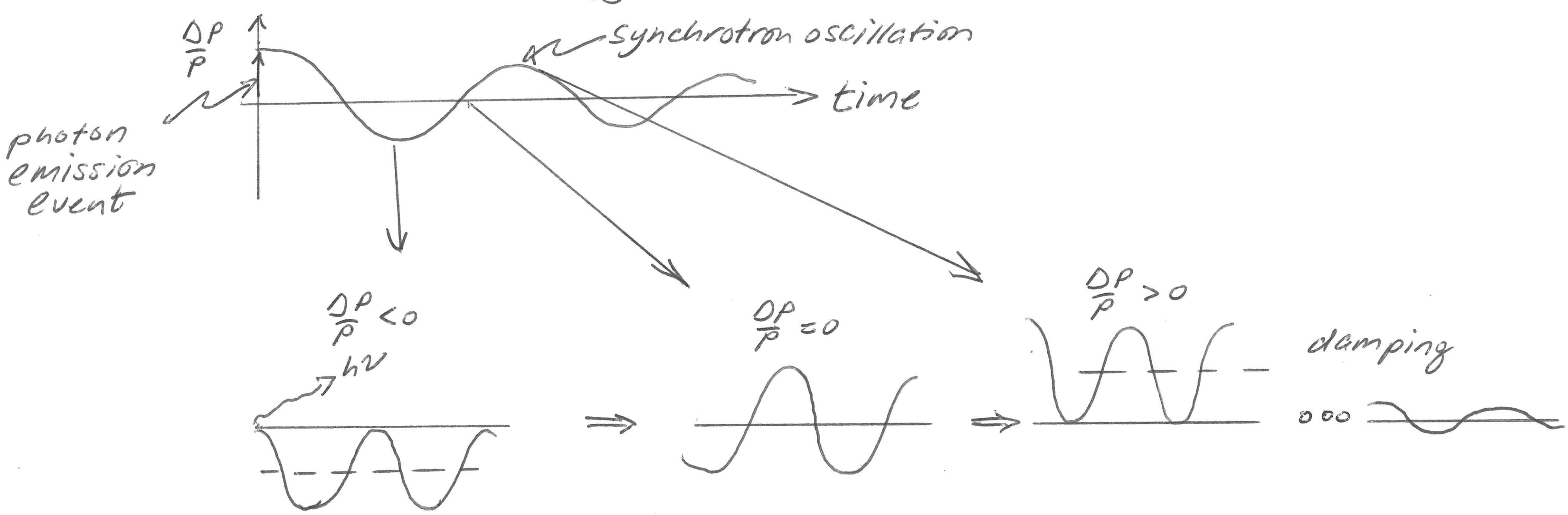
Collective spread of betatron oscillation amplitudes generate emittance

$$f(x,x') = \frac{1}{2\pi\sigma_x\sigma_{x'}} e^{-\frac{\gamma x^2 + 2\alpha x x' + \beta x'^2}{\epsilon_0}}$$



Dispersion and Emittance

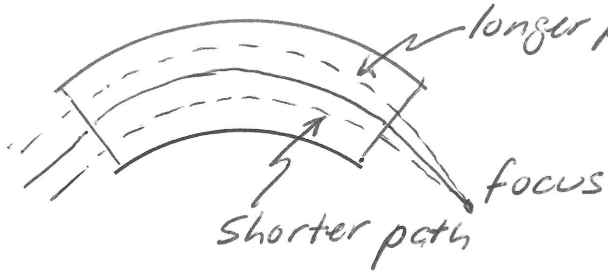
Earlier we saw energy perturbations oscillate in time



$$\boxed{\Delta x = \Delta x_\beta + \Delta x_D}$$

- need to integrate over energy spread and dispersion
- then balance excitation with damping to get emittance

Calculation of Dispersion Function



Effective focussing $\frac{1}{R^2}$ ← was $-\frac{\nabla B}{B\rho}$ for quads

$$x''(s) + \left(\frac{1}{R^2}\right)x(s) = 0$$

$\left.\begin{array}{l} x_h(s) \\ x_h'(s) \end{array}\right\}$ homogeneous solution

For off-momentum particles

$$x'' + \frac{1}{R^2}x = \frac{1}{R}\frac{\Delta p}{p}$$ ← Driving term (particular solution)

$$x = x_h + x_p$$

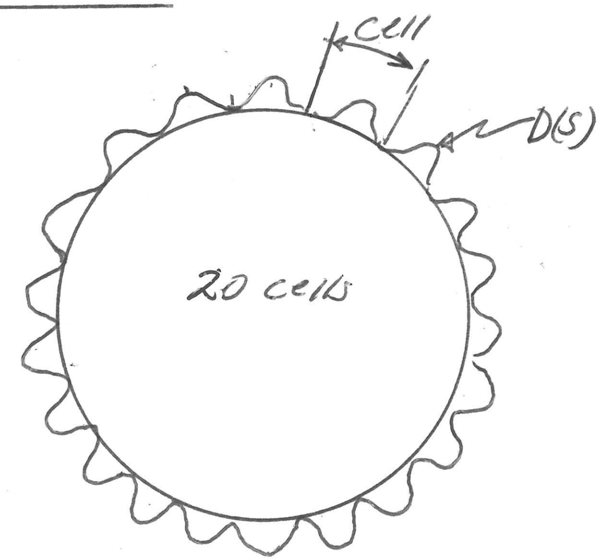
$$x_{particular} \cong (\text{forcing function}) * (\text{homogeneous solution})$$

$$\boxed{D \triangleq \frac{x_p(s)}{\frac{\Delta p}{p}} = \int \frac{1}{R(\tilde{s})} G(\tilde{s}/s)\, d\tilde{s}}$$ we are not going to do it!

↑ radius ↑ Greens function (Impulse Response)

Periodic Dispersion Function

$$x'' + \Big(\underbrace{\tfrac{1}{R^2}}_{\text{Dipoles}} + \underbrace{K^2}_{\text{Quads}}\Big)x = \underbrace{\tfrac{1}{R}\tfrac{\Delta p}{p}}_{\text{Driving term}}$$



1) Calculate betafunctions as before (α, β, γ)

2) Apply closure condition with one-turn map

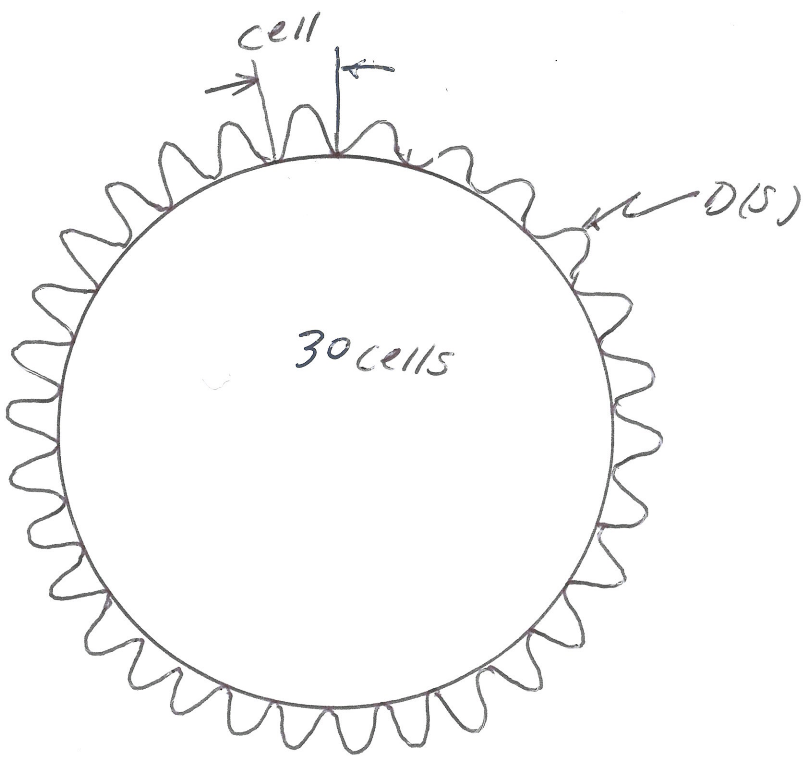
$$\begin{bmatrix} D \\ D' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 2\pi Q & \beta \sin 2\pi Q & \text{Greens} \\ \frac{1}{\beta}\sin 2\pi Q & \cos 2\pi Q & \text{Greens} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D \\ D' \\ 1 \end{bmatrix}$$

$\underbrace{\qquad\qquad}_{\text{homogeneous}}$ $\underbrace{\qquad}_{\text{particular}}$

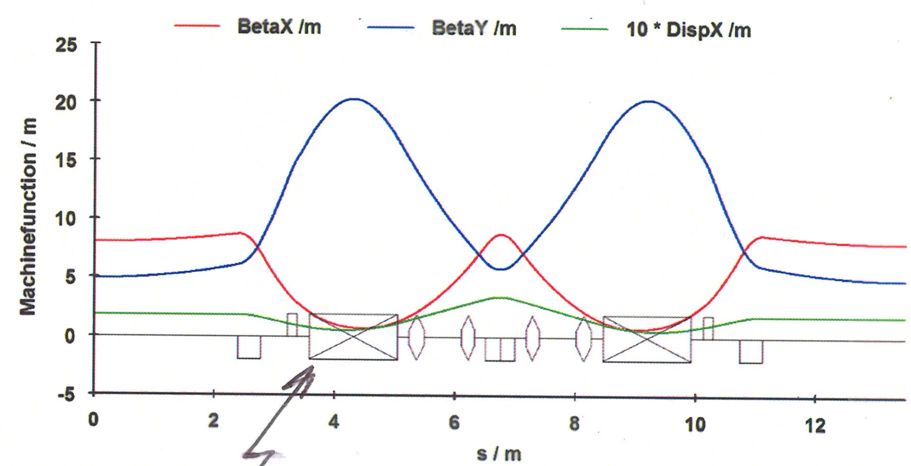
yields

$$\boxed{D(s) = \underbrace{\frac{\sqrt{\beta(s)}}{2\sin\pi Q}}_{\text{resonance}} \oint \underbrace{\frac{\sqrt{\beta(\tilde{s})}}{R(\tilde{s})}}_{\text{want small }\beta\text{ in dipoles}} \cos\big(\underbrace{\phi(s)}_{\text{observation}} - \underbrace{\phi(\tilde{s})}_{\text{dipole}} + \pi Q\big)\, d\tilde{s}}$$

Periodic Dispersion Function



Double-Bend Cell

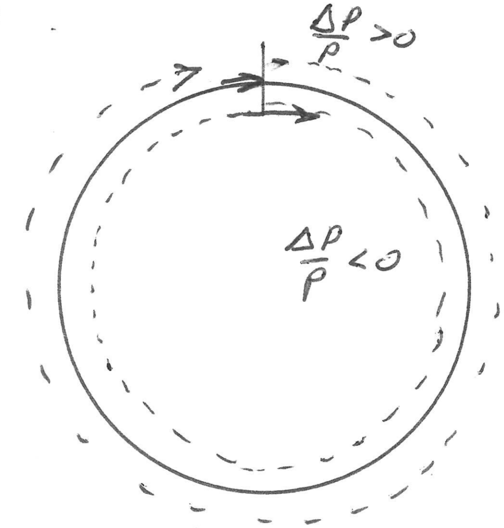


minimize D (and D′)
in bending magnets
where photons are emitted

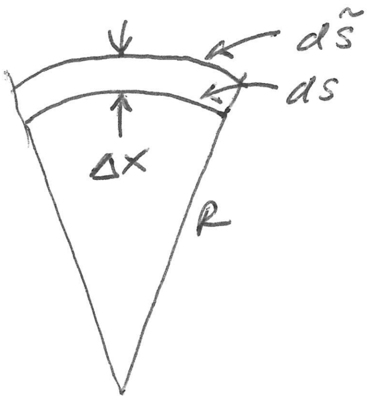
- also minimize D in undulators

$$D(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi Q} \oint \frac{\sqrt{\beta(\tilde{s})}}{R(\tilde{s})} \cos(\phi(s) - \phi(\tilde{s}) + \pi Q)\, d\tilde{s}$$

Momentum Compaction Factor



- High-energy particles take the outside track
- Low-energy particles take the inside track
- At the same speed low-energy arrives first



$$d\tilde{s} = \left(1 + \frac{\Delta x}{R}\right) ds \quad \text{where } \Delta x = D \cdot \frac{\Delta p}{p}$$

$$\tilde{L} = \oint \Big[\underbrace{1}_{L_0} + \frac{D}{R} \cdot \frac{\Delta p}{p}\Big] ds$$

$$\tilde{L} - L_0 = \Delta L = \frac{\Delta p}{p} \oint \frac{D(s)}{R(s)} ds$$

$$\alpha = \frac{\Delta L}{L} \Big/ \frac{\Delta p}{p} = \oint \frac{D}{R} ds$$

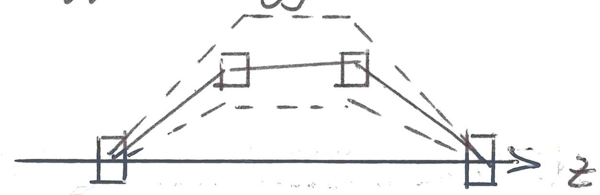
1) momentum compaction influences beam size properties (σ_s, σ_x)

2) momentum compaction influence beam stability $\frac{1}{\sqrt{\alpha}}$

3) for proton machines $\eta \triangleq \alpha - \frac{1}{\gamma^2}$ $\qquad \frac{1}{\sqrt{\eta}}$ when $\eta = 0$ "transition"

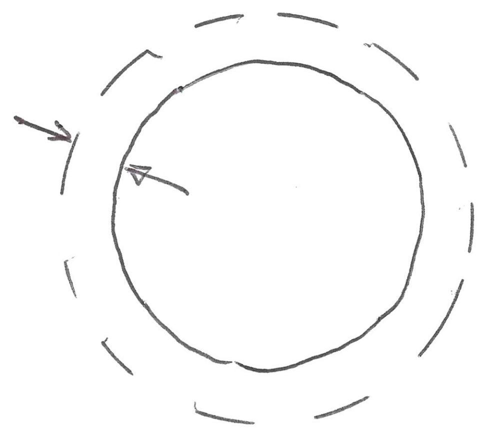
Review

- Dispersion refers to the off-energy orbit
  - transport line chicane (bunch compressor)



  - Storage ring $\Delta x \triangleq D \frac{\Delta p}{p}$

(we will use D extensively)



- Dispersion is a particular solution of $x'' + \frac{1}{R^2} x = \frac{\Delta p}{p} \cdot \frac{1}{R}$

Driving force

- Dispersion is periodic in cells, storage ring

- $\alpha = \frac{\Delta L}{L} \Big/ \frac{\Delta p}{p}$ momentum compaction factor