Accelerator Physics Lecture Modules

1. Introduction/Timing
2. Beam propagation basics
3. Simple Harmonic Oscillator

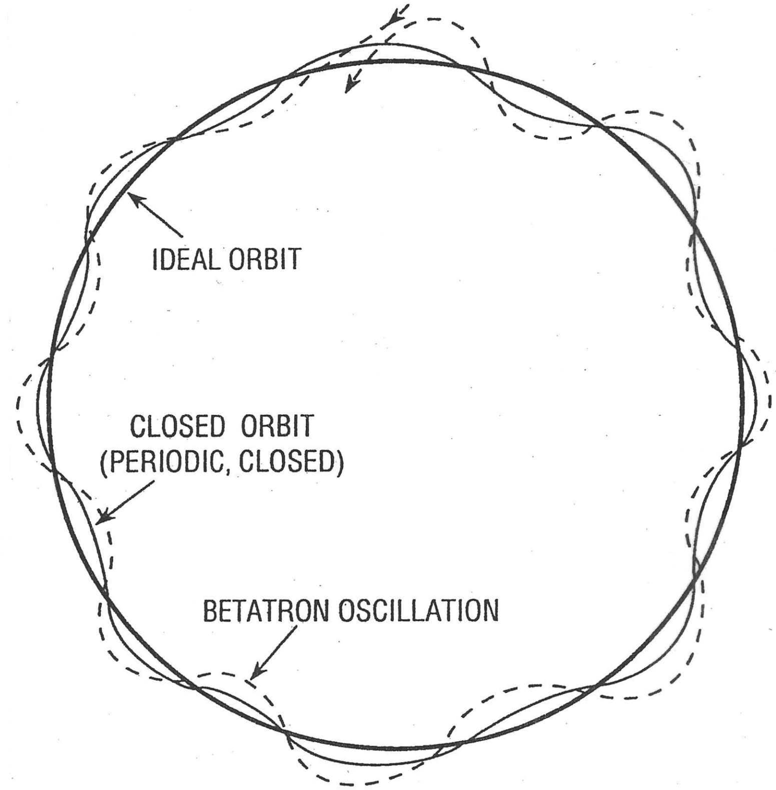
4. Betatron oscillations (transverse motion)
5. Dispersion
6. Closed Orbit Distortion

7. Synchrotron oscillations
8. Properties of synchrotron radiation

9. Radiation damping
10. Equilibirum emittance

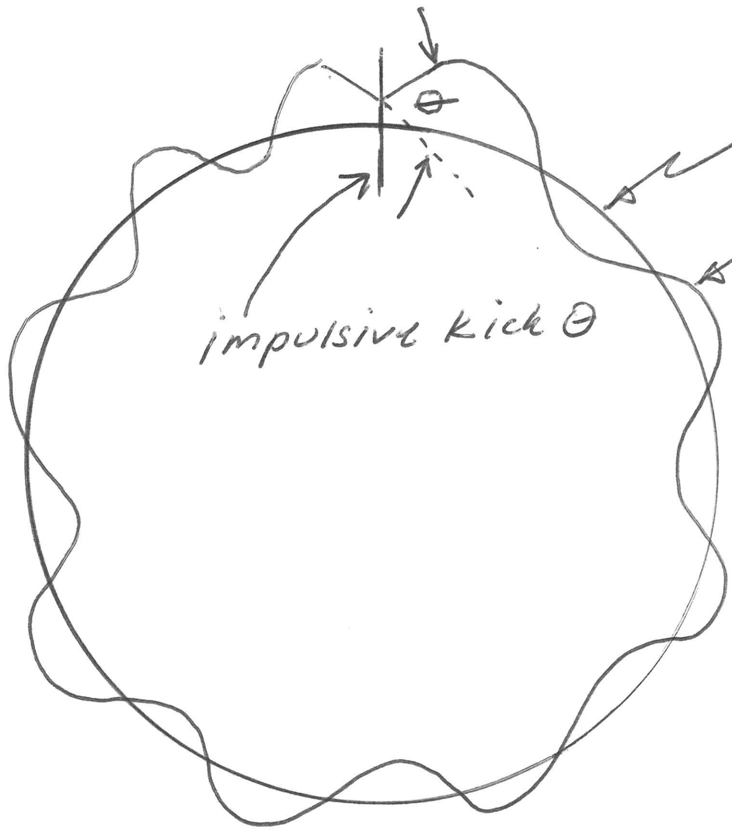
Closed Orbit Distortions

- Betatron oscillations do not 'close'
- The betatron function does 'close'
  - Twiss parameters (α, β, γ) repeat because focusing function $K(s)$ repeats
- The Dispersion function also 'repeats'
  - boundary conditions on $D(s)$, $D'(s)$
- What about the 'Closed Orbit'
  - Closed Orbit also repeats (by definition)



Closed Orbit Distortions (COD)

- Dipole errors distort the closed orbit – 'kick'



for closure need

$$\begin{cases} x_1 = x_0 \\ x_1' = x_0' + \theta \end{cases}$$

at location of kick θ

Closed Orbit Distortion Formula

From Dispersion (closed orbit) Calculation

$$D(s) = \frac{x(s)}{\frac{\Delta p}{p}} = \frac{\sqrt{\beta(s)}}{2\sin\pi Q} \oint \frac{\sqrt{\beta(\tilde{s})}}{R(\tilde{s})} \cos(\phi(s) - \phi(\tilde{s}) + \pi Q)\, d\tilde{s}$$

Re-arrange $\frac{\Delta p}{p}$ normalization

$$x(s) = \frac{1}{2\sin\pi Q} \oint \underbrace{\left[\frac{\frac{\Delta p}{p}}{R(\tilde{s})}\right]}_{\text{Kick distribution}} \sqrt{\beta(s)\beta(\tilde{s})} \cos(\phi(s) - \phi(\tilde{s}) + \pi Q)\, d\tilde{s}$$

observation point s

perturbation points $\tilde{s}$ → integrate

Replace the kick distribution with impulse: $\frac{\frac{\Delta p}{p}}{R(s)} \rightarrow \theta(s)$

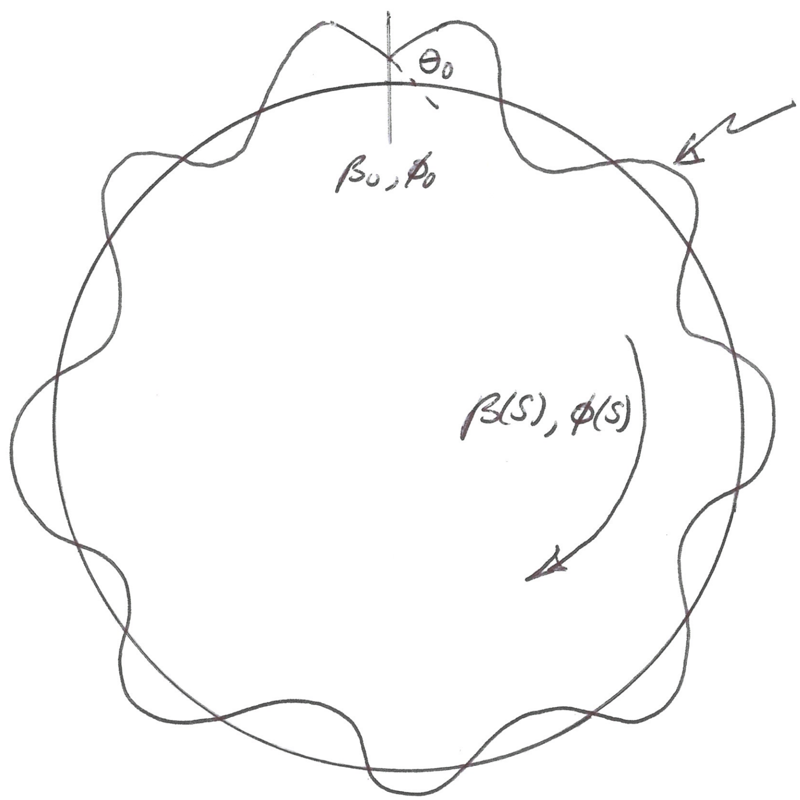
$$\boxed{x_1 = \frac{1}{2\sin\pi Q} \cdot \underbrace{\theta_0}_{\text{Kick}} \cdot \sqrt{\beta_1 \beta_0} \cos(\phi_1 - \phi_0 + \pi Q)}$$ COD formula

Kick

observation

Kick location

Closed Orbit from a Kick

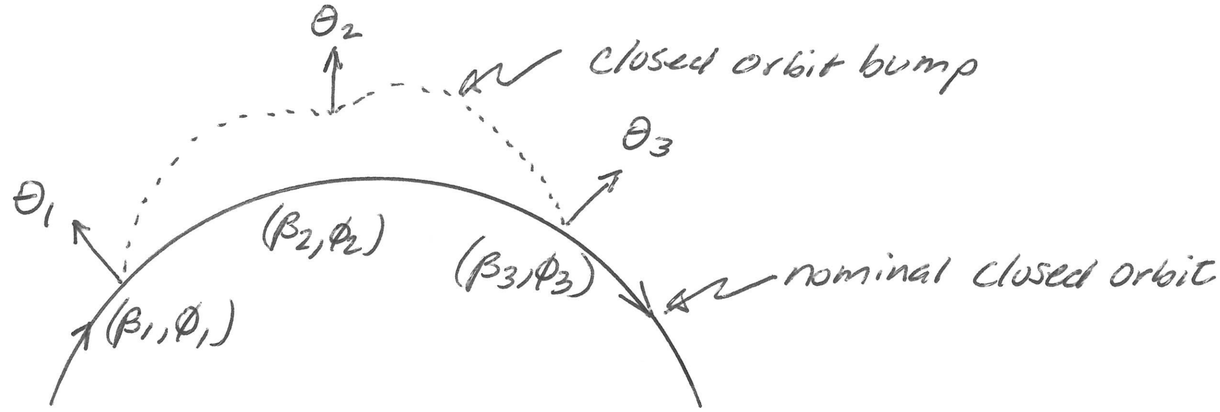


$$x_{COD}(s) = \frac{\theta_0}{2\sin\pi Q}\sqrt{\beta_0\beta(s)}\cos\left(\left|\phi(s)-\phi_0\right| + \pi Q\right)$$

integer resonance

note abs bars

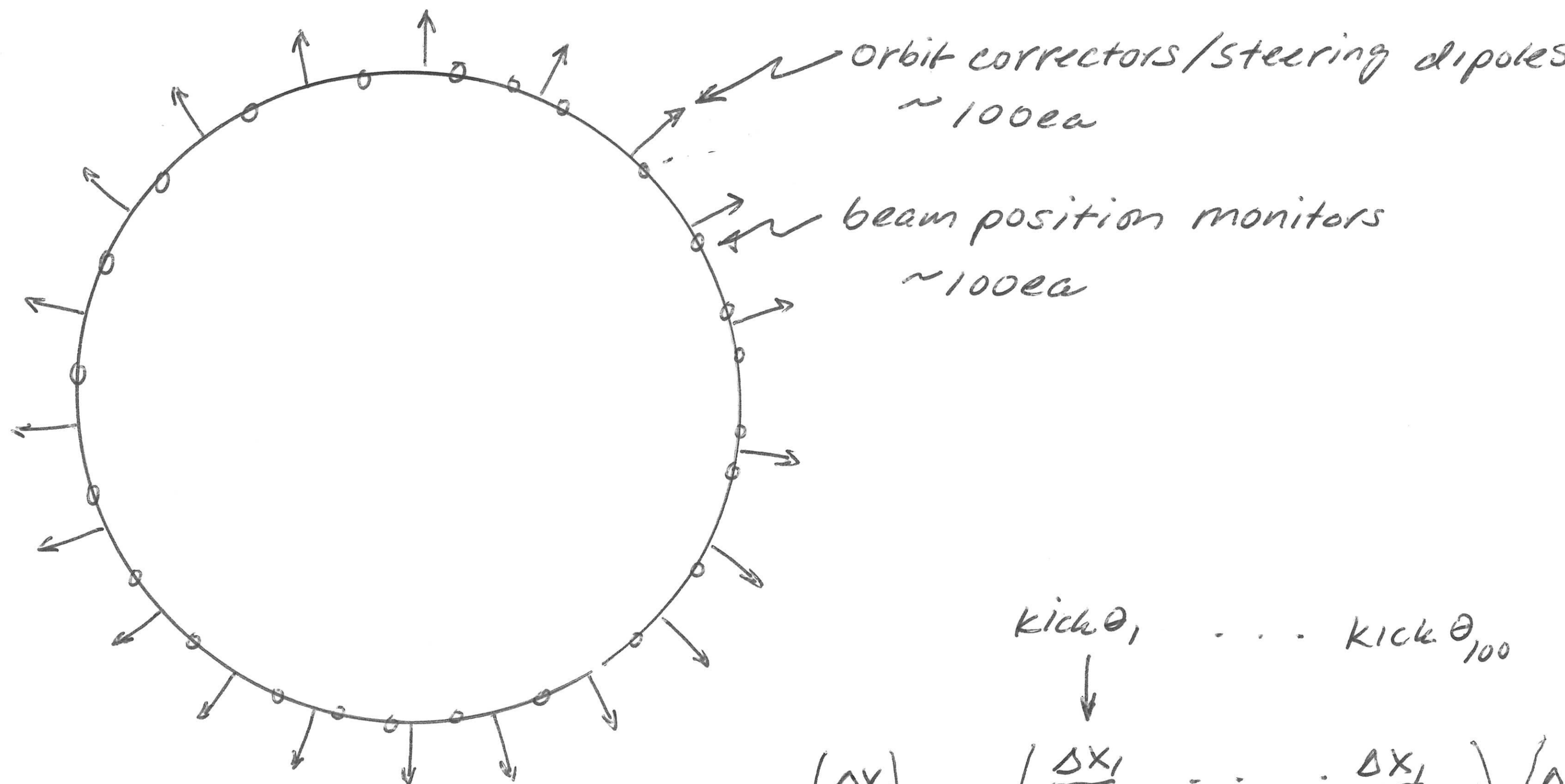
Closed Orbit 'Bumps'



3 equations with 3 variables $(\theta_1, \theta_2, \theta_3)$

- Steer or 'point' a photon beam
- steer counter-propagating beams into collision
- 3 bumps
  4 bumps
  5 bumps
  ⋮
  closed orbit control (feedback)

The Orbit Response Matrix



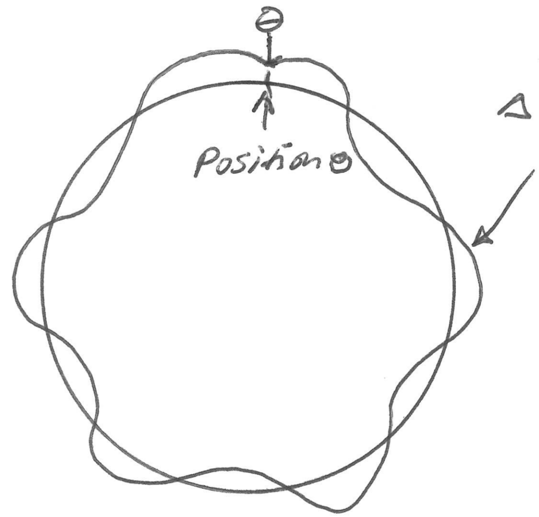
$$\boxed{\begin{aligned} \vec{\Delta X} &= R\,\vec{\Delta\theta} \\ \vec{\Delta\theta} &= R^{-1}\,\vec{\Delta X} \end{aligned}}$$

inverse response matrix

$$\underbrace{\begin{pmatrix} \Delta X_1 \\ \vdots \\ \Delta X_{100} \end{pmatrix}}_{\text{BPMS}} = \underbrace{\begin{array}{ccc} \text{kick}\,\theta_1 & \cdots & \text{kick}\,\theta_{100} \\ \begin{pmatrix} \frac{\Delta X_1}{\Delta\theta_1} & \cdots & \frac{\Delta X_1}{\Delta\theta_{100}} \\ \vdots & \frac{\partial X_i}{\partial\theta_j} & \vdots \\ \frac{\Delta X_{100}}{\Delta\theta_1} & \cdots & \frac{\Delta X_{100}}{\Delta\theta_{100}} \end{pmatrix} \end{array}}_{\text{Response Matrix}} \underbrace{\begin{pmatrix} \Delta\theta_1 \\ \vdots \\ \Delta\theta_{100} \end{pmatrix}}_{\text{Correctors}}$$

Review I

- Dipole kicks generate Closed Orbit Distortions (COD)



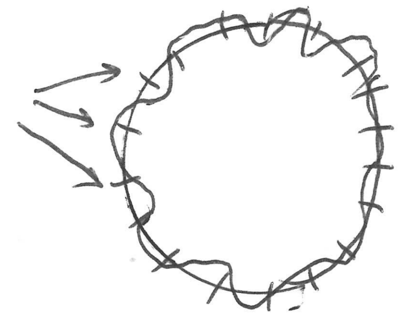
$$\Delta x_1 = \frac{1}{2\sin\pi Q} \cdot \theta_0 \cdot \sqrt{\beta_1 \beta_0} \cos(|\phi_1 - \phi_0| + \pi Q)$$

- Use corrector magnets to steer the beam (closed bump)
  - photon beams
  - beams in collision

Review II

- If a quadrupole moves it makes a 'kick' $B = \frac{dB}{dx} \cdot \Delta x$

- Many quads moving makes many kicks
  each has an amplification factor



- Use orbit response matrix for feedback correction

$$\begin{pmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{pmatrix} = \begin{pmatrix} \text{Response} \\ \text{Matrix} \end{pmatrix} \begin{pmatrix} \Delta\theta_1 \\ \vdots \\ \Delta\theta_m \end{pmatrix}$$

$$\Delta\vec{X} = R\Delta\vec{\theta}$$

$$\Delta\vec{\theta} = R^{-1}\Delta X$$

- slow feedback (creep, lunar, ID's)
- fast feedback (vibration compensation)